that they should render themselves indeterminate. Wherefore all things are deter-
mined from a necessity of the divine nature, not only to exist, but to exist and act in
a certain manner, and there is nothing contingent.

2.4 Problems in Reasoning

In reasoning we advance from premises known (or affirmed for the purpose)
to conclusions. We construct arguments of our own every day, in deciding
how we shall act, in judging the conduct of others, in defending our moral or
political convictions, and so on. Skill in devising good arguments (and in
deciding whether a proffered argument is good) is of enormous value, and
this skill can be improved with practice. Ancient games of reasoning, such as
chess and go, exercise that skill, and there are some widely known commercial
games (Clue and Mastermind are examples) that also have this merit.

Problems may be contrived which are designed to test and strengthen logical
skills; some of these are presented in this section. Such problems are far neater
than those that arise in real life, of course. But solving them may require extended
reasoning in patterns not very different from those employed by a detective, a
journalist, or a juror. Chains of inferences will be needed, in which subconclusions
are used as premises in subsequent arguments. Finding the solution may require
the creative recombination of information given earlier or discovered. Contrived
problems can prove frustrating—but solving them, like every successful applica-
tion of reasoning, is quite satisfying. In addition to being models for the employ-
ment of reason, logical games and puzzles are good fun. “The enjoyment of the
doubtful,” wrote the philosopher John Dewey, “is a mark of the educated mind.”

One type of reasoning problem is the common brainteaser, in which, using
only the clues provided, we must determine the names or other facts about
certain specified characters. Here is a simple example:

In a certain flight crew, the positions of pilot, copilot, and flight engineer are held
by three persons, Allen, Brown, and Carr, though not necessarily in that order. The
copilot, who is an only child, earns the least. Carr, who married Brown’s sister,
earns more than the pilot. What position does each of the three persons hold?

To solve such problems we look first for a sphere in which we have
enough information to reach some conclusions going beyond what is given in
the premises. In this case we know most about Carr: he is not the pilot, be-
because he earns more than the pilot; and he is not the copilot because the copi-
lot earns the least. By elimination we may infer that Carr must be the flight
engineer. Using that subconclusion we can determine Brown’s position.
Brown is not the copilot because he has a sister and the copilot is an only child;
he is not the flight engineer because Carr is. Brown must therefore be the pilot.
Allen, the only one left, must therefore be the copilot.
When problems of this type become more complex, it is useful to construct a graphic display of the alternatives, called a matrix, which we fill in as we accumulate new information. The helpfulness of such a matrix will be seen in solving the following problem:

Alonzo, Kurt, Rudolf, and Willard are four creative artists of great talent. One is a dancer, one is a painter, one is a singer, and one is a writer, though not necessarily in that order.

1. Alonzo and Rudolf were in the audience the night the singer made his debut on the concert stage.

2. Both Kurt and the writer have had their portraits painted from life by the painter.

3. The writer, whose biography of Willard was a best-seller, is planning to write a biography of Alonzo.

4. Alonzo has never heard of Rudolf.

What is each man’s artistic field?

To remember the facts asserted in these premises, as well as the subconclusions that may be inferred from them, would be a demanding task. Written notes could become a confusing clutter. We need a method for storing and exhibiting the information given and the intermediate conclusions drawn, keeping it all available for use as the number of inferences increases and the chain of arguments lengthens. The matrix we construct allows us to represent all the relevant possibilities and to record each inference drawn.

For this problem the matrix must display an array of the four persons (in four rows) and the four artistic professions (in four columns) that they hold. It would look like this:

<table>
<thead>
<tr>
<th></th>
<th>Dancer</th>
<th>Painter</th>
<th>Singer</th>
<th>Writer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alonzo</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurt</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rudolf</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Willard</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When we conclude that one of those individuals (named at the left of one of the rows) cannot be the artist whose profession is at the top of one of the columns, we write an N (for “No”) in the box to the right of that person’s name and in the column headed by that profession. We can immediately infer, from premise (1), that neither Alonzo nor Rudolph is the singer, so we place an N to the right of their names, in the third (singer) column. We can infer from premise (2) that Kurt
is neither the painter nor the writer, so we enter an N to the right of his name in
the second (painter) and the fourth (writer) columns. From premise (3) we see
that the writer is neither Alonzo nor Willard, so we enter an N to the right of their
names in the fourth column. The entries we have made thus far are all justified by
the information given originally, and our matrix now looks like this:

<table>
<thead>
<tr>
<th></th>
<th>Dancer</th>
<th>Painter</th>
<th>Singer</th>
<th>Writer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alonzo</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Kurt</td>
<td></td>
<td>N</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Rudolf</td>
<td>N</td>
<td></td>
<td></td>
<td>N</td>
</tr>
<tr>
<td>Willard</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

From the information now clearly exhibited, we can conclude by elimination
that Rudolf must be the writer, so we enter a Y (for “Yes”) in the box to the right of
Rudolf’s name in the fourth (writer) column, and we place an N in the other
boxes to the right of his name. The array now makes it evident that the painter
must be either Alonzo or Willard, and we can eliminate Alonzo in this way:
Rudolf had his portrait painted by the painter (from premise 2), and Alonzo has
never heard of Rudolf (from premise 4)—therefore Alonzo cannot be the painter.
So we enter an N to the right of Alonzo’s name under column 2 (painter). We may
conclude that Alonzo must be the dancer, so we enter a Y to the right of Alonzo’s
name in the first (dancer) column. In that same column we can now enter an N for
both Kurt and Willard. The only possible category remaining for Kurt is singer,
and therefore we enter a Y in that box for him, and an N in the singer column for
Willard. By elimination, we conclude that Willard must be the painter and put a Y
in the last empty box in the matrix. Our completed graphic display looks like this:

<table>
<thead>
<tr>
<th></th>
<th>Dancer</th>
<th>Painter</th>
<th>Singer</th>
<th>Writer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alonzo</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Kurt</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Rudolf</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Willard</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

Our matrix now filled in, the full solution is evident: Alonzo is the dancer;
Kurt is the singer; Rudolf is the writer; Willard is the painter.

Some brainteasers of this kind, requiring solutions on several dimensions,
are very challenging and almost impossible to solve without using a matrix.
In the real world, we are often called upon to reason from some present state of affairs to its causes, from what is to what was. Scientists—especially archeologists, geologists, astronomers, and physicians—commonly confront events or conditions whose origins are problematic. Reasoning that seeks to explain how things must have developed from what went before is called *retrograde analysis.* For example, to the amazement of astronomers, comet Hyakutake, streaking by the earth in 1996, was found to be emitting variable X-rays a hundred times stronger than anyone had ever predicted a comet might emit. A comet expert at the Max Planck Institute in Germany remarked, “We have our work cut out for us in explaining these data—but that’s the kind of problem you love to have.”

We do love to have them, and for that reason problems in retrograde analysis are often devised for amusement. In the real world, logical problems arise within a theoretical framework that is supplied by scientific or historical knowledge; but in contrived problems that framework must be provided by the problem itself. Some rules or laws must be set forth within which logical analysis can proceed. The chessboard is the setting for the most famous of all problems in retrograde analysis; the rules of chess provide the needed theoretical context. No skill in playing chess is required, but readers who are not familiar with the rules of chess may skip the illustration that follows.

Retrograde problems in chess commonly take this form: An arrangement of pieces on the chessboard is given; it was reached in a game of chess in which all the rules of the game were obeyed. What move, or series of moves, has just been completed? An example of such a problem follows. The diagram presents a position reached in an actual game of chess, all moves in that game having been made in accordance with the rules of chess. The black king has just moved.
For the purpose of analysis, the rows are numbered from bottom to top, 1 to 8, and the columns are lettered from left to right, a to h. Each square on the board can then be identified by a unique letter–number combination: The black king is on a8, the white pawn on h2, and so on. The problem is this: The last move was made by black. What was that move? And what was white’s move just before that? Can you reason out the solution before reading the next paragraph?

Solution: Because the two kings may never rest on adjacent squares, the black king could not have moved to its present position from b7 or from b8; therefore we may be certain that the black king has moved from a7, where it was in check.

That much is easily deduced. But what preceding white move could have put the black king in check? No move by the white bishop (on g1) could have done it, because there would have been no way for that bishop to move to that square, g1, without the black king having been in check with white to move. Therefore it must be that the check was discovered by the movement of a white piece that had been blocking the bishop’s attack and was captured by the black king on its move to a8. What white piece could have been on that black diagonal and moved from there to the white square in the corner? Only a knight that had been on b6. We may therefore be certain that before black’s last move (the black king from a7 to a8), white’s last move was that of a white knight from b6 to a8.

Problems of reasoning that confront us in the real world are rarely this tidy. Many real problems are not described accurately, and their misdescription may prove so misleading that no solution can be reached. In cases of that kind, some part or parts of the description of the problem need to be rejected or replaced. However, we cannot do this when we are seeking to solve logical puzzles of the sort presented here.

In the real world, moreover, even when they are described accurately, problems may be incomplete in that something not originally available may be essential for the solution. The solution may depend on some additional scientific discovery, or some previously unimagined invention or equipment, or the search of some as-yet-unexplored territory. In the statement of a logical puzzle, as in the writing of a good murder mystery, all the information that is sufficient for the solution must be given; otherwise we feel that the mystery writer, or the problem maker, has been unfair to us.

Finally, the logical puzzle presents a sharply formulated question (for example, which member of the artistic foursome is the singer? What were black’s and white’s last moves?) whose answer, if given and proved, solves the problem definitively. But that is not the form in which many real-world problems arise. Real problems are often identified, initially at least, only by the recognition of some inconsistency or the occurrence of an unusual event, or
perhaps just by the feeling that something is amiss, rather than by a well-formed question seeking a clearly defined answer. In spite of these differences, contrived problems and puzzles are useful in strengthening our reasoning skills. And they are fun.

**EXERCISES**

The following problems require reasoning for their solution. To prove that an answer is correct requires an argument (often containing subsidiary arguments) whose premises are contained in the statement of the problem—and whose final conclusion is the answer to it. If the answer is correct, it is possible to construct a valid argument proving it. In working these problems, readers are urged to concern themselves not merely with discovering the answers but also with formulating arguments to prove that those answers are correct.

1. In a certain mythical community, politicians never tell the truth, and nonpoliticians always tell the truth. A stranger meets three natives and asks the first of them, “Are you a politician?” The first native answers the question. The second native then reports that the first native denied being a politician. The third native says that the first native is a politician. How many of these three natives are politicians?

2. Of three prisoners in a certain jail, one had normal vision, the second had only one eye, and the third was totally blind. The jailor told the prisoners that, from three white hats and two red hats, he would select three and put them on the prisoners’ heads. None could see what color hat he wore. The jailor offered freedom to the prisoner with normal vision if he could tell what color hat he wore. To prevent a lucky guess, the jailor threatened execution for any incorrect answer. The first prisoner could not tell what hat he wore. Next the jailor made the same offer to the one-eyed prisoner. The second prisoner could not tell what hat he wore either. The jailor did not bother making the offer to the blind prisoner, but he agreed to extend the same terms to that prisoner when he made the request. The blind prisoner said:

   I do not need to have my sight;
   From what my friends with eyes have said,
   I clearly see my hat is _____!

How did he know?
3. On a certain train, the crew consists of the brakeman, the fireman, and the engineer. Their names, listed alphabetically, are Jones, Robinson, and Smith. On the train are also three passengers with corresponding names, Mr. Jones, Mr. Robinson, and Mr. Smith. The following facts are known:

a. Mr. Robinson lives in Detroit.

b. The brakeman lives halfway between Detroit and Chicago.

c. Mr. Jones earns exactly $40,000 a year.

d. Smith once beat the fireman at billiards.

e. The brakeman’s next-door neighbor, one of the three passengers mentioned, earns exactly three times as much as the brakeman.

f. The passenger living in Chicago has the same name as the brakeman.

What is the engineer’s name?

4. The employees of a small loan company are Mr. Black, Mr. White, Mrs. Coffee, Miss Ambrose, Mr. Kelly, and Miss Earnshaw. The positions they occupy are manager, assistant manager, cashier, stenographer, teller, and clerk, though not necessarily in that order. The assistant manager is the manager’s grandson, the cashier is the stenographer’s son-in-law, Mr. Black is a bachelor, Mr. White is twenty-two years old, Miss Ambrose is the teller’s stepsister, and Mr. Kelly is the manager’s neighbor.

Who holds each position?

5. Benno Torelli, genial host at Miami’s most exclusive nightclub, was shot and killed by a racketeer gang because he fell behind in his protection payments. After considerable effort on the part of the police, five suspects were brought before the district attorney, who asked them what they had to say for themselves. Each of them made three statements, two true and one false. Their statements were

**Lefty:** I did not kill Torelli. I never owned a revolver in all my life. Spike did it.

**Red:** I did not kill Torelli. I never owned a revolver. The others are all passing the buck.

**Dopey:** I am innocent. I never saw Butch before. Spike is guilty.
2.4 Problems in Reasoning

Spike: I am innocent. Butch is the guilty one. Lefty did not tell the truth when he said I did it.

Butch: I did not kill Torelli. Red is the guilty one. Dopey and I are old pals.

Whodunnit?

6. Mr. Short, his sister, his son, and his daughter are fond of golf and often play together. The following statements are true of their foursome:
   a. The best player’s twin and the worst player are of the opposite sex.
   b. The best player and the worst player are the same age.

Which one of the foursome is the best player?

7. Daniel Kilraine was killed on a lonely road, 2 miles from Pontiac, Michigan, at 3:30 A.M. on March 17 of last year. Otto, Curly, Slim, Mickey, and the Kid were arrested a week later in Detroit and questioned. Each of the five made four statements, three of which were true and one of which was false. One of these persons killed Kilraine.

   Their statements were

   Otto: I was in Chicago when Kilraine was murdered. I never killed anyone. The Kid is the guilty one. Mickey and I are pals.
   Curly: I did not kill Kilraine. I never owned a revolver in my life. The Kid knows me. I was in Detroit the night of March 17.
   Slim: Curly lied when he said he never owned a revolver. The murder was committed on St. Patrick’s Day. Otto was in Chicago at this time. One of us is guilty.
   Mickey: I did not kill Kilraine. The Kid has never been in Pontiac. I never saw Otto before. Curly was in Detroit with me on the night of March 17.
   The Kid: I did not kill Kilraine. I have never been in Pontiac. I never saw Curly before. Otto erred when he said I am guilty.

Whodunnit?

8. Six balls confront you. Two are red; two are green; two are blue. You know that in each color pair, one ball is heavier than the other. You also know that all three of the heavier balls weigh the same, as do all
three of the lighter balls. The six balls (call them R1, R2, G1, G2, B1, and B2) are otherwise indistinguishable. You have only a balance scale; if equal weights are placed on the two sides of your scale, they will balance; if unequal weights are placed on the two sides, the heavier side will go down. With no more than two weighings on that balance scale, how can you identify the heavier and the lighter balls in all three pairs?

9. In the same mythical community described in Exercise 1, a stranger meets three other natives and asks them, “How many of you are politicians?” The first native replies, “We are all politicians.” The second native says, “No, just two of us are politicians.” The third native then says, “That isn’t true either.”

Is the third native a politician?

10. Imagine a room with four walls, with a nail placed in the center of each wall, as well as in the ceiling and floor, six nails in all. The nails are connected to each other by strings, each nail connected to every other nail by a separate string. These strings are of two colors, red or blue, and of no other color. All these strings obviously make many triangles, because any three nails may be considered the apexes of a triangle.

Can the colors of the strings be distributed so that no one triangle has all three sides (strings) of the same color? If so, how? And if not, why not?

**CHALLENGE TO THE READER**

Here is a final reasoning problem whose solution requires the construction of a set of sustained arguments. It isn’t easy—but solving it is well within your power and will give you great pleasure.

You are presented with a set of twelve metal balls, apparently identical in every respect: size, color, and so on. In fact, eleven of them are identical, but one of them is “odd”: It differs from all the rest in weight only; it is either heavier, or lighter, than all the others. You are given a balance scale, on which the balls can be weighed against one another. If the same number of balls are put on each side of the balance, and the “odd” ball is on one side, that side will go down if the odd ball is heavier, or up if the odd ball is lighter; the two sides will balance if the odd ball is not among those weighed and the same number of balls are placed on each side. You are allowed three weighings only; any removal or addition of a ball constitutes a separate weighing.
Your challenge is this: Devise a set of three weighings that will enable you to identify the odd ball wherever it may lie in a random mixing of the twelve balls, and that will enable you to determine whether the odd ball is heavier or lighter than the rest.

SUMMARY

In this chapter we have discussed techniques for the analysis of arguments, and some of the difficulties confronted in that process.

In Section 2.1 we explained the paraphrasing of an argumentative passage, in which the essential propositions may be reworded (or supplied if they are assumed but missing), and in which premises and conclusions are put into most intelligible order.

In Section 2.2 we explained the diagramming of an argument, in which the propositions of an argument are represented by numbers, and the relations of the premises and conclusions are then exhibited graphically in two dimensions, by showing on a page the relations of those numbered propositions.

In Section 2.3 we discussed complex argumentative passages, in which the conclusions of subarguments may serve as premises for further arguments, and whose complete analysis generally requires an intricate diagram, or an extensive paraphrase.

In Section 2.4 we discussed contrived problems of reasoning, which often mirror the complexities confronted by many different kinds of investigation in real life, and whose solutions require the construction of extended sets of arguments and subarguments.

End Notes

2G. H. Hardy, A Mathematician’s Apology (Cambridge: Cambridge University Press, 1940).
CHAPTER 2 Analyzing Arguments


12Thomas Aquinas, Summa Theologiae, I, Question 96, Article 2, circa 1265.

13Ibid., Article 3.


20Readers who find retrograde analysis enjoyable will take delight in a collection of such problems, compiled by the logician Raymond Smullyan, and entitled The Chess Mysteries of Sherlock Homes (New York: Alfred A. Knopf, 1979).