This appendix derives the values of the pay-off matrix that depicts the duopoly game between Cyber and Net in Chapter 13 of the text.

**COST AND DEMAND CONDITIONS**

The cost of producing magazines is the same for both Cyber and Net. The average total cost curve \((ATC)\) and the marginal cost curve \((MC)\) for each firm are shown in Fig. 1(a). The market demand curve for magazines \((D)\) is shown in Fig. 1(b). Each firm produces an identical magazine product, so one firm’s magazine is a perfect substitute for the other’s. The market price of each firm’s product, therefore, is identical. The quantity demanded depends on that price — the higher the price, the lower is the quantity demanded.

Notice that in this industry, there is room for only two firms. For each firm the *minimum efficient scale* of production is 300,000 magazines a month. When the price equals the average total cost of production at the minimum efficient scale, total industry demand is 600,000 magazines a month. Thus there is no room for three firms in this industry. If there were only one firm in the industry, it would make an enormous profit and invite competition. If there were three firms, at

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**FIGURE A13.1**

Costs and Demand

(a) Individual firm  
Part (a) shows the costs facing Cyber and Net, two duopolists who publish magazines. Each firm faces identical costs. The average total cost curve for each firm is \(ATC\), and the marginal cost curve is \(MC\). For each firm the minimum efficient scale of product is 300,000 units a month, and the average total cost of producing that output is $6,000 a unit. Part (b) shows the industry demand curve. At a price of $6, the quantity demanded is 600,000 units a month. There is room for only two firms in this industry.
least one of them would make a loss. Thus the number of firms that an industry can sustain depends on the relationship between cost and the industry’s demand conditions.

**COLLUDING TO MAXIMISE PROFITS**

Let’s begin by working out the payoffs to the two firms if they collude to make the maximum industry profit — the profit that would be made by a single monopoly. The calculations that the two firms will perform are exactly the same calculations that a monopoly performs. The only additional thing that the duopolists have to do is to agree on how much of the total output each of them will produce.

The price and quantity that maximises industry profit for the duopolists is shown in Fig. 2. Part (a) shows the situation for each firm and part (b) for the industry as a whole. The curve labelled $\text{MR}$ is the industry marginal revenue curve. The curve labelled $\text{MC}_I$ is the industry marginal cost curve if each firm produces the same level of output. That curve is constructed by adding together the outputs of the two firms at each level of marginal cost.

That is, at each level of marginal cost, industry output is twice as much as the output of each individual firm. Thus the curve $\text{MC}_I$ in part (b) is twice as far to the right as the curve $\text{MC}$ in part (a).

To maximise industry profit, the duopolists agree to restrict output to the rate that makes the industry marginal cost and marginal revenue equal. That output rate, as shown in part (b), is 400,000 magazines a month. The highest price for which the 400,000 units can be sold is $9 each. Let’s suppose that Cyber and Net agree to split the market equally so that each firm produces 200,000 magazines a month. The average total cost ($\text{ATC}$) of producing 200,000 units a month is $8, so the profit per unit is $1 and the total profit is $0.2 million (200,000 units $\times$ $1$ per unit). The profit of each firm is represented by the blue rectangle in Fig. 2(a).

We have just described one possible outcome for the duopoly game: the two firms collude to produce the monopoly profit-maximising output and divide that output equally between themselves. From the industry point of view, this solution is identical to a monopoly. A duopoly that operates in this way is

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**FIGURE A13.2**
Colluding to Make Monopoly Profits

If Cyber and Net come to a collusive agreement, they can act as a single monopolist and maximize profit. To maximize profit, the firms first calculate the industry marginal cost curve, $\text{MC}_I$ (part b), which is the horizontal sum of the two firms’ marginal cost curves, $\text{MC}$ (part a). Next they calculate the industry marginal revenue, $\text{MR}$. They then choose the output rate that makes marginal revenue equal to marginal cost (400,000 units a month). They agree to sell that output for a price of $9, the price at which 400,000 magazines are demanded.

Each firm has the same costs, so each produces half the total output — 200,000 units a month. Average total cost is $8 a unit, so each firm makes a profit of $0.2 million a month (blue rectangle) — 200,000 units multiplied by $1,000 profit per unit.
indistinguishable from a monopoly. The profit that is made by a monopoly is the maximum profit that can be made by colluding duopolists.

CHEATING ON A COLLUSIVE AGREEMENT

Under a collusive agreement, the colluding firms restrict output to make their joint marginal revenue equal to their joint marginal cost. They set the highest price for which the quantity produced can be sold — a price higher than marginal cost. In such a situation, each firm recognises that if it cheats on the agreement and raises its output, even though the price will fall below that agreed to, more will be added to revenue than to cost, so its profit will increase. Since each firm recognises this fact, there is a temptation for each firm to cheat. There are two possible cheating situations: one in which one firm cheats and one in which both firms cheat. What happens if one of the firms cheats on the agreement?

ONE FIRM CHEATS

What is the effect of one firm cheating on a collusive agreement? How much extra profit does the cheating firm make? What happens to the profit of the firm that sticks to the agreement in the face of cheating by the other firm? Let’s work out the answers to these questions.

There are many different ways for a firm to cheat. We will work out just one possibility. Suppose that Cyber convinces Net that there has been a fall in industry demand and that it cannot sell its share of the output at the agreed price. It tells Net that it plans to cut its price in order to sell the agreed 200,000 magazines each month. Since the two firms produce a virtually identical product, Net has no alternative but to match the price cut of Cyber.

In fact, there has been no fall in demand and the lower price has been calculated by Cyber to be exactly the price needed to sell the additional output that it plans to produce. Net, though lowering its price in line with that of Cyber, restricts its output to the previously agreed level.

Figure 3 illustrates the consequences of Cyber cheating in this way: part (a) shows what happens to Net (the complier); part (b) shows what happens to Cyber (the cheat); and part (c) shows what is happening in the industry as a whole.

FIGURE A13.3
Cheating on a Collusive Agreement

In part (a), one firm complies with the agreement. In part (b), the other firm cheats by raising output above the agreed limit to 300,000 magazines a month. Either firm can be the complier and the other the cheat. In part (c), the effects on the industry price of the actions of the cheat are shown. As a result of cheating, industry output rises to 500,000 units a month and the market price falls to $7.50 — the price at which 500,000 magazines can be sold.

Part (a) describes the complier’s situation. Output remains at 200,000 units, and average total cost remains at $8.00 per unit. The firm loses $0.50 per magazine and makes a total loss of $0.1 million (red rectangle). Part (b) describes the cheat’s situation. Average total cost is $6.00 per unit, and profit per magazine is $1.50, so the cheat’s total profit is $0.45 million (blue rectangle).
Suppose that Cyber decides to raise output from 200,000 to 300,000 units a month. It recognises that if Net sticks to the agreement to produce only 200,000 units a month, total output will be 500,000 a month, and given demand in part (c), the price will have to be cut to $7.50 a unit.

Net continues to produce 200,000 units a month at a cost of $8 a unit, and incurs a loss of $0.50 a unit or $0.1 million a month. This loss is represented by the red rectangle in part (a). Cyber produces 300,000 units a month at an average total cost of $6 each. With a price of $7.50, Cyber makes a profit of $1.50 a unit and therefore a total profit of $0.45 million. This profit is the blue rectangle in part (b).

We have now described a second possible outcome for the duopoly game — one of the firms cheats on the collusive agreement. In this case, the industry output is larger than the monopoly output and the industry price is lower than the monopoly price. The total profit made by the industry is also smaller than the monopoly’s profit. Cyber (the cheat) makes a profit of $0.45 million and Net (the complier) incurs a loss of $0.1 million. The industry makes a profit of $0.35 million. Thus the industry profit is $0.5 million less than the maximum profit would be with a monopoly outcome. But that profit is distributed unevenly. Cyber makes an even bigger profit than it would under the collusive agreement, while Net makes a loss.

We have just worked out what happens if Cyber cheats and Net complies with the collusive agreement. There is another similar outcome that would arise if Net cheated and Cyber complied with the agreement. The industry profit and price would be the same but in this case Net (the cheat) would make a profit of $0.45 million and Cyber (the complier) would incur a loss of $0.1 million.

There is yet another possible outcome: both firms cheat on the agreement.

If both firms cheat by raising their output and lowering the price, the collusive agreement completely breaks down. The limit to the breakdown of the agreement is the competitive equilibrium. Neither firm will want to cut the price below $6 (minimum average total cost), for to do so will result in losses. Part (a) shows the situation facing each firm. At a price of $6, the firm's profit-maximizing output is 300,000 units a month. At that output rate, price equals marginal cost, and it also equals average total cost. Economic profit is zero. Part (b) describes the situation in the industry as a whole. The industry marginal cost curve ($MC_I$) — the horizontal sum of the individual firms’ marginal cost curves ($MC$) — intersects the demand curve at 600,000 magazines a month and at a price of $6. This output and price is the one that would prevail in a competitive industry.
**BOTH FIRMS CHEAT**

Suppose that instead of just one firm cheating on the collusive agreement, both firms cheat. In particular, suppose that each firm behaves in exactly the same way as the cheating firm that we have just analysed. Each firm tells the other that it is unable to sell its output at the going price and that it plans to cut its price. But since both firms cheat, each will propose a successively lower price. They will only stop proposing lower prices when the price has reached $6. That is the price that equals minimum average cost. At a price of less than $6, each firm will make a loss. At a price of $6, each firm will cover all its costs and make zero economic profit. Also, at a price of $6, each firm will want to produce 300,000 units a month, so that the industry output will be 600,000 units a month. Given the demand conditions, 600,000 units can be sold at a price of $6 each.

The situation just described is illustrated in Fig. 4. Each firm, shown in part (a) of the figure, is producing 300,000 units a month, and this output level occurs at the point of minimum average total cost ($6 per unit). The market as a whole, shown in part (b), operates at the point at which the demand curve \((D)\) intersects the industry marginal cost curve. This marginal cost curve is constructed as the horizontal sum of the marginal cost curves of the two firms. Each firm has lowered its price and increased its output in order to try to gain an advantage over the other firm. They have each pushed this process as far as they can without incurring losses.

We have now described a third possible outcome of this duopoly game — both firms cheat. If both firms cheat on the collusive agreement, the output of each firm is 300,000 units a month and the price is $6. Each firm makes zero profit.

**THE PAYOFF MATRIX AND EQUILIBRIUM**

Now that we have described the strategies and payoffs in the duopoly game, let’s summarise the strategies and the payoffs in the form of the game’s payoff matrix and then calculate the equilibrium.

Figure 5 sets out the payoff matrix for this game. It is constructed in exactly the same way as the payoff matrix for the prisoners’ dilemma in the Toolkit on game theory (pp. 13.8–13.9). The squares show the payoffs for the two firms — Net and Cyber. In this case, the payoffs are profits. (In the case of the prisoners’ dilemma, the payoffs were losses.)

**FIGURE A13.5**

Duopoly Payoff Matrix

Each square shows the payoffs from a pair of actions. For example, if both firms comply with the collusive agreement, the payoffs are recorded in the square at the bottom-right corner of the table. Net’s payoff is shown in the red triangle and Cyber’s in the blue triangle. The equilibrium is a Nash equilibrium in which both firms cheat.

The table shows that if both firms cheat (top left), they achieve the perfectly competitive outcome — each firm makes zero economic profit. If both firms comply (bottom right), the industry makes the monopoly profit and each firm earns a profit of $0.2 million. The top-right and bottom-left squares show what happens if one firm cheats while the other complies. The firm that cheats collects a profit of $0.45 million and the one that complies makes a loss of $0.1 million.

This duopoly game is, in fact, the same as the prisoners’ dilemma that we examined in Chapter 13; it is a duopolist’s dilemma. You will see this once you have determined what the equilibrium of this game is.

To find the equilibrium, let’s look at things from the point of view of Net. Net reasons as follows: suppose that Cyber cheats. If we comply with the agreement, we make a loss of $0.1 million. If we also cheat, we make a zero profit. Zero profit is better than a $0.1 million loss, so it will pay us to cheat. But suppose Cyber complies with the agreement. If we cheat, we will make a profit of $0.45 million, and if we comply, we will make a profit of $0.2 million. A $0.45 million profit is
better than a $0.2 million profit so it would again pay us to cheat. Thus regardless of whether Cyber cheats or complies, it pays us to cheat. Net’s dominant strategy is to cheat.

Cyber comes to the same conclusion as Net. Therefore both firms will cheat. The equilibrium of this game then is that both firms cheat on the agreement. Although there are only two firms in the industry, the price and quantity are the same as in a competitive industry. Each firm makes zero profit.

Although we have done this analysis for only two firms, it would not make any difference (other than to increase the amount of arithmetic) if we were to play the game with three, four, or more firms. In other words, though we have analysed duopoly, the game theory approach can also be used to analyse oligopoly. The analysis of oligopoly is much harder, but the essential ideas that we have learned apply to oligopoly.