Learning Objectives

In this chapter, you learn:

- How to use payoff tables and decision trees to evaluate alternative courses of action
- How to use several criteria to select an alternative course of action
- How to use Bayes’ theorem to revise probabilities in light of sample information
- About the concept of utility
As the manager of The Reliable Fund, you are responsible for purchasing and selling stocks for the fund. The investors in this mutual fund expect a large return on their investment, and at the same time they want to minimize their risk. At the present time, you need to decide between two stocks to purchase. An economist for your company has evaluated the potential one-year returns for both stocks, under four economic conditions: recession, stability, moderate growth, and boom. She has also estimated the probability of each economic condition occurring. How can you use the information provided by the economist to determine which stock to choose in order to maximize return and minimize risk?
In Chapter 4, you studied various rules of probability and used Bayes’ theorem to revise probabilities. In Chapter 5, you learned about discrete probability distributions and how to compute the expected value. In this chapter, these probability rules and probability distributions are applied to a decision-making process for evaluating alternative courses of action. In this context, you can consider the four basic features of a decision-making situation:

- **Alternative courses of action** A decision maker must have two or more possible choices to evaluate prior to selecting one course of action from among the alternative courses of action. For example, as a manager of a mutual fund in the Using Statistics scenario, you must decide whether to purchase stock A or stock B.

- **Events** A decision maker must list the events, or states of the world that can occur and consider the probability of occurrence of each event. To aid in selecting which stock to purchase in the Using Statistics scenario, an economist for your company has listed four possible economic conditions and the probability of occurrence of each event in the next year.

- **Payoffs** In order to evaluate each course of action, a decision maker must associate a value or payoff with the result of each event. In business applications, this payoff is usually expressed in terms of profits or costs, although other payoffs, such as units of satisfaction or utility, are sometimes considered. In the Using Statistics scenario, the payoff is the return on investment.

- **Decision criteria** A decision maker must determine how to select the best course of action. Section 19.2 discusses five decision criteria: maximax payoff, maximin payoff, expected monetary value, expected opportunity loss, and return-to-risk ratio.

## 19.1 Payoff Tables and Decision Trees

In order to evaluate the alternative courses of action for a complete set of events, you need to develop a payoff table or construct a decision tree. A payoff table contains each possible event that can occur for each alternative course of action and a value or payoff for each combination of an event and course of action. Example 19.1 discusses a payoff table for a marketing manager trying to decide how to market organic salad dressings.

### Example 19.1

A Payoff Table for Deciding How to Market Organic Salad Dressings

You are a marketing manager for a food products company, considering the introduction of a new brand of organic salad dressings. You need to develop a marketing plan for the salad dressings in which you must decide whether you will have a gradual introduction of the salad dressings (with only a few different salad dressings introduced to the market) or a concentrated introduction of the salad dressings (in which a full line of salad dressings will be introduced to the market).

You estimate that if there is a low demand for the salad dressings, your first year’s profit will be $1 million for a gradual introduction and $-5 million (a loss of $5 million) for a concentrated introduction. If there is high demand, you estimate that your first year’s profit will be $4 million for a gradual introduction and $10 million for a concentrated introduction.

**Solution** Table 19.1 is a payoff table for the organic salad dressings marketing example.

<table>
<thead>
<tr>
<th>EVENT, ( E_i )</th>
<th>ALTERNATIVE COURSE OF ACTION</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 )</td>
<td>Gradual, ( A_1 )</td>
<td>Concentrated, ( A_2 )</td>
</tr>
<tr>
<td>Low demand</td>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td>High demand</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

Using a decision tree is another way of representing the events for each alternative course of action. A decision tree pictorially represents the events and courses of action through a set of branches and nodes. Example 19.2 illustrates a decision tree.
EXAMPLE 19.2  A Decision Tree for the Organic Salad Dressings Marketing Decision

Given the payoff table for the organic salad dressings example, construct a decision tree.

SOLUTION  Figure 19.1 is the decision tree for the payoff table shown in Table 19.1.

In Figure 19.1, the first set of branches relates to the two alternative courses of action: gradual introduction to the market and concentrated introduction to the market. The second set of branches represents the possible events of low demand and high demand. These events occur for each of the alternative courses of action on the decision tree.

The decision structure for the organic salad dressings marketing example contains only two possible alternative courses of action and two possible events. In general, there can be several alternative courses of action and events. As a manager of The Reliable Fund in the Using Statistics scenario, you need to decide between two stocks to purchase for a short-term investment of one year. An economist at the company has predicted returns for the two stocks under four economic conditions: recession, stability, moderate growth, and boom. Table 19.2 presents the predicted one-year return of a $1,000 investment in each stock under each economic condition. Figure 19.2 shows the decision tree for this payoff table. The decision (which stock to purchase) is the first branch of the tree, and the second set of branches represents the four events (the economic conditions).

<table>
<thead>
<tr>
<th>ECONOMIC CONDITION</th>
<th>STOCK A</th>
<th>STOCK B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>30</td>
<td>-50</td>
</tr>
<tr>
<td>Stable economy</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>Moderate growth</td>
<td>100</td>
<td>250</td>
</tr>
<tr>
<td>Boom</td>
<td>150</td>
<td>400</td>
</tr>
</tbody>
</table>

FIGURE 19.1  Decision tree for the organic salad dressings marketing example (in millions of dollars)

FIGURE 19.2  Decision tree for the stock selection payoff table
Using the payoff table from Example 19.1, construct an opportunity loss table.

### SOLUTION

For the event “low demand,” the maximum profit occurs when there is a gradual introduction to the market (+$1 million). The opportunity that is lost with a concentrated introduction to the market is the difference between $1 million and $5 million, which is $6 million. If there is high demand, the best action is to have a concentrated introduction to the market ($10 million profit). The opportunity that is lost by making the incorrect decision of having a gradual introduction to the market is $10 million − $4 million = $6 million. The opportunity loss is always a nonnegative number because it represents the difference between the profit under the best action and any other course of action that is taken for the particular event. Table 19.3 shows the complete opportunity loss table for the organic salad dressings marketing example.

**OPPORTUNITY LOSS**

The *opportunity loss* is the difference between the highest possible profit for an event and the actual profit for an action taken.

Example 19.3 illustrates the computation of opportunity loss.

### EXAMPLE 19.3

**Finding Opportunity Loss in the Organic Salad Dressings Marketing Example**

<table>
<thead>
<tr>
<th>Event</th>
<th>Optimum Action</th>
<th>Profit of Optimum Action</th>
<th>Alternative Course of Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low demand</td>
<td>Gradual</td>
<td>1</td>
<td>1 − 1 = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 − (−5) = 6</td>
</tr>
<tr>
<td>High demand</td>
<td>Concentrated</td>
<td>10</td>
<td>10 − 4 = 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10 − 10 = 0</td>
</tr>
</tbody>
</table>

You can develop an opportunity loss table for the stock selection problem in the Using Statistics scenario. Here, there are four possible events or economic conditions that will affect the one-year return for each of the two stocks. In a recession, stock A is best, providing a return
Problems for Section 19.1

LEARNING THE BASICS

19.1 For this problem, use the following payoff table:

<table>
<thead>
<tr>
<th>EVENT</th>
<th>ACTION</th>
<th>A ($)</th>
<th>B ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>200</td>
<td>125</td>
</tr>
</tbody>
</table>

a. Construct an opportunity loss table.

b. Construct a decision tree.

19.2 For this problem, use the following payoff table:

<table>
<thead>
<tr>
<th>EVENT</th>
<th>ACTION</th>
<th>A ($)</th>
<th>B ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>300</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>500</td>
<td>200</td>
</tr>
</tbody>
</table>

a. Construct an opportunity loss table.

b. Construct a decision tree.

APPLYING THE CONCEPTS

19.3 A manufacturer of designer jeans must decide whether to build a large factory or a small factory in a particular location. The profit per pair of jeans manufactured is estimated as $10. A small factory will incur an annual cost of $200,000, with a production capacity of 50,000 pairs of jeans per year. A large factory will incur an annual cost of $400,000, with a production capacity of 100,000 pairs of jeans per year. Four levels of manufacturing demand are considered likely: 10,000, 20,000, 50,000, and 100,000 pairs of jeans per year.

a. Determine the payoffs for the possible levels of production for a small factory.

b. Determine the payoffs for the possible levels of production for a large factory.

c. Based on the results of (a) and (b), construct a payoff table, indicating the events and alternative courses of action.

d. Construct a decision tree.

e. Construct an opportunity loss table.

19.4 An author is trying to choose between two publishing companies that are competing for the marketing rights to her new novel. Company A has offered the author $10,000 plus $2 per book sold. Company B has offered the author $2,000 plus $4 per book sold. The author believes that five levels of demand for the book are possible: 1,000, 2,000, 5,000, 10,000, and 50,000 books sold.

a. Compute the payoffs for each level of demand for company A and company B.

b. Construct a payoff table, indicating the events and alternative courses of action.

c. Construct a decision tree.

d. Construct an opportunity loss table.

19.5 The DellaVecchia Garden Center purchases and sells Christmas trees during the holiday season. It purchases the trees for $10 each and sells them for $20 each. Any trees not sold by Christmas day are sold for $2 each to a company that makes wood chips. The garden center estimates that four levels of demand are possible: 100, 200, 500, and 1,000 trees.

a. Compute the payoffs for purchasing 100, 200, 500, or 1,000 trees for each of the four levels of demand.

b. Construct a payoff table, indicating the events and alternative courses of action.

c. Construct a decision tree.

d. Construct an opportunity loss table.
19.2 Criteria for Decision Making

After you compute the profit and opportunity loss for each event under each alternative course of action, you need to determine the criteria for selecting the most desirable course of action. Some criteria involve the assignment of probabilities to each event, but others do not. This section introduces two criteria that do not use probabilities: the maximax payoff and the maximin payoff.

This section presents three decision criteria involving probabilities: expected monetary value, expected opportunity loss, and the return-to-risk ratio. For criteria in which a probability is assigned to each event, the probability is based on information available from past data, from the opinions of the decision maker, or from knowledge about the probability distribution that the event may follow. Using these probabilities, along with the payoffs or opportunity losses of each event–action combination, you select the best course of action according to a particular criterion.

Maximax Payoff

The maximax payoff criterion is an optimistic payoff criterion. Using this criterion, you do the following:

1. Find the maximum payoff for each action.
2. Choose the action that has the highest of these maximum payoffs.

Example 19.4 illustrates the application of the maximax criterion to the organic salad dressings marketing example.

Return to Table 19.1, the payoff table for deciding how to market organic salad dressings. Determine the best course of action according to the maximax criterion.

**SOLUTION**

First you find the maximum profit for each action. For a gradual introduction to the market, the maximum profit is $4 million. For a concentrated introduction to the market, the maximum profit is $10 million. Because the maximum of the maximum profits is $10 million, you choose the action that involves a concentrated introduction to the market. Table 19.5 summarizes the use of this criterion.

**TABLE 19.5**

Using the Maximax Criterion for the Organic Salad Dressings Marketing Example (in Millions of Dollars)

<table>
<thead>
<tr>
<th>EVENT, $E_i$</th>
<th>ALTERNATIVE COURSE OF ACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gradual, $A_1$</td>
</tr>
<tr>
<td>High demand, $E_1$</td>
<td>1</td>
</tr>
<tr>
<td>High demand, $E_2$</td>
<td>4</td>
</tr>
<tr>
<td>Maximum profit for each action</td>
<td>4</td>
</tr>
</tbody>
</table>

As a second application of the maximax payoff criterion, return to the Using Statistics scenario and the payoff table presented in Table 19.2. Table 19.6 summarizes the maximax payoff criterion for that example.

**TABLE 19.6**

Using the Maximax Criterion for the Predicted One-Year Return ($) on $1,000 Investment in Each of Two Stocks, Under Four Economic Conditions

<table>
<thead>
<tr>
<th>ECONOMIC CONDITION</th>
<th>STOCK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>Recession</td>
<td>30</td>
</tr>
<tr>
<td>Stable economy</td>
<td>70</td>
</tr>
<tr>
<td>Moderate growth</td>
<td>100</td>
</tr>
<tr>
<td>Boom</td>
<td>150</td>
</tr>
<tr>
<td>Maximum profit for each action</td>
<td>150</td>
</tr>
</tbody>
</table>
Maximin Payoff
The maximin payoff criterion is a pessimistic payoff criterion. Using this criterion, you do the following:

1. Find the minimum payoff for each action.
2. Choose the action that has the highest of these minimum payoffs.

Example 19.5 illustrates the application of the maximin criterion to the organic salad dressing marketing example.

EXAMPLE 19.5
Finding the Best Course of Action According to the Maximin Criterion for the Organic Salad Dressings Marketing Example

Return to Table 19.1, the payoff table for deciding how to market organic salad dressings. Determine the best course of action according to the maximin criterion.

SOLUTION First, you find the minimum profit for each action. For a gradual introduction to the market, the minimum profit is $1 million. For a concentrated introduction to the market, the minimum profit is $5 million. Because the maximum of the minimum profits is $1 million, you choose the action that involves a gradual introduction to the market. Table 19.7 summarizes the use of this criterion.

Maximin Payoff
The maximin payoff criterion is a pessimistic payoff criterion. Using this criterion, you do the following:

1. Find the minimum payoff for each action.
2. Choose the action that has the highest of these minimum payoffs.

Example 19.5 illustrates the application of the maximin criterion to the organic salad dressing marketing example.

Because the maximum of the minimum profits is $30, you choose stock A.

Expected Monetary Value
The expected value of a probability distribution was computed in Equation (5.1) on page 163. Now you use Equation (5.1) to compute the expected monetary value for each alternative course of action. The expected monetary value (EMV) for a course of action, j, is the payoff (Xij) for each combination of event i and action j multiplied by P_i, the probability of occurrence of event i, summed over all events [see Equation (19.1)].
Returning to the payoff table for deciding how to market organic salad dressings (Example 19.1), suppose that the probability is 0.60 that there will be low demand (so that the probability is 0.40 that there will be high demand). Compute the expected monetary value for each alternative course of action and determine how to market organic salad dressings.

**SOLUTION** You use Equation (19.1) to determine the expected monetary value for each alternative course of action. Table 19.9 summarizes these computations.

\[
EMV(j) = \sum_{i=1}^{N} X_{ij} P_i
\]  

(19.1)

where

- \( EMV(j) \) = expected monetary value of action \( j \)
- \( X_{ij} \) = payoff that occurs when course of action \( j \) is selected and event \( i \) occurs
- \( P_i \) = probability of occurrence of event \( i \)
- \( N \) = number of events

Criterion: Select the course of action with the largest \( EMV \).

Example 19.6 illustrates the application of expected monetary value to the organic salad dressings marketing example.

**TABLE 19.9**

<table>
<thead>
<tr>
<th>Event</th>
<th>( P_i )</th>
<th>Gradual, ( A_1 )</th>
<th>Concentrated, ( A_2 )</th>
<th>( X_{ij} P_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low demand, ( E_1 )</td>
<td>0.60</td>
<td>1</td>
<td>-5</td>
<td>-5(0.6) = -3.0</td>
</tr>
<tr>
<td>High demand, ( E_2 )</td>
<td>0.40</td>
<td>4</td>
<td>10</td>
<td>10(0.4) = 4.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( E_{MV}(A_1) = 2.2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( E_{MV}(A_2) = 1.0 )</td>
<td></td>
</tr>
</tbody>
</table>

The expected monetary value for a gradual introduction to the market is $2.2 million, and the expected monetary value for a concentrated introduction to the market is $1 million. Thus, if your objective is to choose the action that maximizes the expected monetary value, you would choose the action of a gradual introduction to the market because its \( EMV \) is highest.

As a second application of expected monetary value, return to the Using Statistics scenario and the payoff table presented in Table 19.2. Suppose the company economist assigns the following probabilities to the different economic conditions:

- \( P(\text{Recession}) = 0.10 \)
- \( P(\text{Stable economy}) = 0.40 \)
- \( P(\text{Moderate growth}) = 0.30 \)
- \( P(\text{Boom}) = 0.20 \)
19.2 Criteria for Decision Making

Thus, the expected monetary value, or profit, for stock A is $91, and the expected monetary value, or profit, for stock B is $162. Using these results, you should choose stock B because the expected monetary value for stock B is almost twice that for stock A. In terms of expected rate of return on the $1,000 investment, stock B is 16.2% compared to 9.1% for stock A.

**Expected Opportunity Loss**

In the previous examples, you learned how to use the expected monetary value criterion when making a decision. An equivalent criterion, based on opportunity losses, is introduced next. Payoffs and opportunity losses can be viewed as two sides of the same coin, depending on whether you wish to view the problem in terms of maximizing expected monetary value or minimizing expected opportunity loss. The expected opportunity loss (EOL) of action j is the loss, $L_{ij}$, for each combination of event i and action j multiplied by $P_i$, the probability of occurrence of the event i, summed over all events [see Equation (19.2)].

$$EOL(j) = \sum_{i=1}^{N} L_{ij}P_i \quad (19.2)$$

where

$L_{ij} =$ opportunity loss that occurs when course of action $j$ is selected and event $i$ occurs

$P_i =$ probability of occurrence of event $i$

Criterion: Select the course of action with the smallest $EOL$. Selecting the course of action with the smallest $EOL$ is equivalent to selecting the course of action with the largest $EMV$. See Equation (19.1)

Example 19.7 illustrates the application of expected opportunity loss for the organic salad dressings marketing example.

---

**TABLE 19.10**

Expected Monetary Value ($) for Each of Two Stocks Under Four Economic Conditions

<table>
<thead>
<tr>
<th>Event</th>
<th>$P_i$</th>
<th>A</th>
<th>$X_{ij}P_i$</th>
<th>B</th>
<th>$X_{ij}P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.10</td>
<td>30</td>
<td>30(0.1) = 3</td>
<td>-50</td>
<td>-50(0.1) = -5</td>
</tr>
<tr>
<td>Stable economy</td>
<td>0.40</td>
<td>70</td>
<td>70(0.4) = 28</td>
<td>30</td>
<td>30(0.4) = 12</td>
</tr>
<tr>
<td>Moderate growth</td>
<td>0.30</td>
<td>100</td>
<td>100(0.3) = 30</td>
<td>250</td>
<td>250(0.3) = 75</td>
</tr>
<tr>
<td>Boom</td>
<td>0.20</td>
<td>150</td>
<td>150(0.2) = 30</td>
<td>400</td>
<td>400(0.2) = 80</td>
</tr>
</tbody>
</table>

$EMV(A) = 91$

Thus, the expected monetary value, or profit, for stock $A$ is $91, and the expected monetary value, or profit, for stock $B$ is $162. Using these results, you should choose stock $B$ because the expected monetary value for stock $B$ is almost twice that for stock $A$. In terms of expected rate of return on the $1,000 investment, stock $B$ is 16.2% compared to 9.1% for stock $A$. 

**EXAMPLE 19.7**

Computing the $EOL$ for the Organic Salad Dressings Marketing Example

Referring to the opportunity loss table given in Table 19.3, and assuming that the probability is 0.60 that there will be low demand, compute the expected opportunity loss for each alternative course of action (see Table 19.11). Determine how to market the organic salad dressings.
CHAPTER 19 Decision Making

The expected opportunity loss from the best decision is called the expected value of perfect information (EVPI). Equation (19.3) defines the EVPI.

**Expected Value of Perfect Information**

The expected profit under certainty represents the expected profit that you could make if you had perfect information about which event will occur.

\[
EVPI = \text{expected profit under certainty} - \text{expected monetary value of the best alternative} \quad (19.3)
\]

Example 19.8 illustrates the expected value of perfect information.

**Example 19.8**

Computing the EVPI in the Organic Salad Dressings Marketing Example

Referring to the data in Example 19.6, compute the expected profit under certainty and the expected value of perfect information.

**SOLUTION**

As the marketing manager of the food products company, if you could always predict the future, a profit of $1 million would be made for the 60% of the time that there is low demand, and a profit of $10 million would be made for the 40% of the time that there is high demand. Thus,

\[
\text{Expected profit under certainty} = 0.60(1) + 0.40(10) = 0.60 - 4.00 = 4.60
\]

The $4.60 million represents the profit you could make if you knew with certainty what the demand would be for the organic salad dressings. You use the EMV calculations in Table 19.9 and Equation (19.3) to compute the expected value of perfect information:

\[
EVPI = \text{Expected profit under certainty} - \text{expected monetary value of the best alternative} = 4.60 - (2.2) = 2.4
\]

This EVPI value of $2.4 million represents the maximum amount that you should be willing to pay for perfect information. Of course, you can never have perfect information, and you should never pay the entire EVPI for more information. Rather, the EVPI provides a guideline for an upper bound on how much you might consider paying for better information. The EVPI is also the expected opportunity loss for a gradual introduction to the market, the best action according to the EMV criterion.
Return to the Using Statistics scenario and the opportunity loss table presented in Table 19.4. Table 19.12 presents the computations to determine the expected opportunity loss for stock A and stock B.

<table>
<thead>
<tr>
<th>Event</th>
<th>( P_i )</th>
<th>( A )</th>
<th>( L_i P_i )</th>
<th>( B )</th>
<th>( L_i P_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.10</td>
<td>0</td>
<td>0(0.1) = 0</td>
<td>80</td>
<td>80(0.1) = 8</td>
</tr>
<tr>
<td>Stable economy</td>
<td>0.40</td>
<td>0</td>
<td>0(0.4) = 0</td>
<td>40</td>
<td>40(0.4) = 16</td>
</tr>
<tr>
<td>Moderate growth</td>
<td>0.30</td>
<td>150</td>
<td>150(0.3) = 45</td>
<td>0</td>
<td>0(0.3) = 0</td>
</tr>
<tr>
<td>Boom</td>
<td>0.20</td>
<td>250</td>
<td>250(0.2) = 50</td>
<td>0</td>
<td>0(0.2) = 0</td>
</tr>
</tbody>
</table>

\[ EOL(A) = 95 \]
\[ EOL(B) = EVPI = 24 \]

The expected opportunity loss is lower for stock B than for stock A. Your optimal decision is to choose stock B, which is consistent with the decision made using expected monetary value. The expected value of perfect information is $24 (per $1,000 invested), meaning that you should be willing to pay up to $24 for perfect information.

**Return-to-Risk Ratio**

Unfortunately, neither the expected monetary value nor the expected opportunity loss criterion takes into account the variability of the payoffs for the alternative courses of action under different events. From Table 19.2, you see that the return for stock A varies from $30 in a recession to $150 in an economic boom, whereas the return for stock B (the one chosen according to the expected monetary value and expected opportunity loss criteria) varies from a loss of $50 in a recession to a profit of $400 in an economic boom.

To take into account the variability of the events (in this case, the different economic conditions), you can compute the variance and standard deviation of each stock, using Equations (5.2) and (5.3) on pages 163 and 164. Using the information presented in Table 19.10, for stock A, \( EMV(A) = \mu_A = $91 \), and the variance is

\[
\sigma_A^2 = \sum_{i=1}^{N} (X_i - \mu)^2 P(X_i) \\
= (30 - 91)^2(0.1) + (70 - 91)^2(0.4) + (100 - 91)^2(0.3) + (150 - 91)^2(0.2) \\
= 1,269 \\
\text{and } \sigma_A = \sqrt{1,269} = $35.62.
\]

For stock B, \( EMV(B) = \mu_B = $162 \), and the variance is

\[
\sigma_B^2 = \sum_{i=1}^{N} (X_i - \mu)^2 P(X_i) \\
= (-50 - 162)^2(0.1) + (30 - 162)^2(0.4) + (250 - 162)^2(0.3) \\
+ (400 - 162)^2(0.2) \\
= 25,116 \\
\text{and } \sigma_B = \sqrt{25,116} = $158.48.
\]

Because you are comparing two stocks with different means, you should evaluate the relative risk associated with each stock. Once you compute the standard deviation of the return from each stock, you compute the coefficient of variation discussed in Section 3.2.
14  CHAPTER 19  Decision Making

\( \sigma \) for \( S \) and \( EMV \) for \( \bar{X} \) in Equation (3.7) on page 93, you find that the coefficient of variation for stock \( A \) is equal to

\[
CV_A = \left( \frac{\sigma_A}{EMV_A} \right) 100\%
\]

\[
= \left( \frac{35.62}{91} \right) 100\% = 39.1\%
\]

whereas the coefficient of variation for stock \( B \) is equal to

\[
CV_B = \left( \frac{\sigma_B}{EMV_B} \right) 100\%
\]

\[
= \left( \frac{158.48}{162} \right) 100\% = 97.8\%
\]

Thus, there is much more variation in the return for stock \( B \) than for stock \( A \).

When there are large differences in the amount of variability in the different events, a criterion other than \( EMV \) or \( EOL \) is needed to express the relationship between the return (as expressed by the \( EMV \)) and the risk (as expressed by the standard deviation). Equation (19.4) defines the return-to-risk ratio (\( RTRR \)) as the expected monetary value of action \( j \) divided by the standard deviation of action \( j \).

**RETURN-TO-RISK RATIO**

\[
RTRR(j) = \frac{EMV(j)}{\sigma_j}
\]  \hspace{1cm} (19.4)

where

\[
EMV(j) = \text{expected monetary value for alternative course of action } j
\]

\[
\sigma_j = \text{standard deviation for alternative course of action } j
\]

Criterion: Select the course of action with the largest \( RTRR \).

For each of the two stocks discussed previously, you compute the return-to-risk ratio as follows. For stock \( A \), the return-to-risk ratio is equal to

\[
RTRR(A) = \frac{91}{35.62} = 2.55
\]

For stock \( B \), the return-to-risk ratio is equal to

\[
RTRR(B) = \frac{162}{158.48} = 1.02
\]

Thus, relative to the risk as expressed by the standard deviation, the expected return is much higher for stock \( A \) than for stock \( B \). Stock \( A \) has a smaller expected monetary value than stock \( B \) but also has a much smaller risk than stock \( B \). The return-to-risk ratio shows \( A \) to be preferable to \( B \). Figure 19.4 shows the worksheet results for this problem.
19.2 Criteria for Decision Making

**FIGURE 19.4**
Expected monetary value and standard deviation worksheet results for stock selection problem

Figure 19.4 displays the COMPUTE worksheet of the Expected Monetary Value workbook. Create this worksheet using the instructions in Section EG19.2.

---

**Problems for Section 19.2**

**LEARNING THE BASICS**

**19.6** For the following payoff table, the probability of event 1 is 0.5, and the probability of event 2 is also 0.5:

| EVENT | ACTION
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A ($)</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
</tr>
</tbody>
</table>

a. Determine the optimal action based on the maximax criterion.
b. Determine the optimal action based on the maximin criterion.
c. Compute the expected monetary value ($EMV$) for actions $A$ and $B$.
d. Compute the expected opportunity loss ($EOL$) for actions $A$ and $B$.
e. Explain the meaning of the expected value of perfect information ($EVPI$) in this problem.
f. Based on the results of (c) or (d), which action would you choose? Why?
g. Compute the coefficient of variation for each action.
h. Compute the return-to-risk ratio ($RTRR$) for each action.
i. Based on (g) and (h), what action would you choose? Why?
j. Compare the results of (f) and (i) and explain any differences.

**19.7** For the following payoff table, the probability of event 1 is 0.8, the probability of event 2 is 0.1, and the probability of event 3 is 0.1:

| EVENT | ACTION
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A ($)</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
</tr>
</tbody>
</table>

a. Determine the optimal action based on the maximax criterion.
b. Determine the optimal action based on the maximin criterion.
c. Compute the expected monetary value (EMV) for actions A and B.
d. Compute the expected opportunity loss (EOL) for actions A and B.
e. Explain the meaning of the expected value of perfect information (EVPI) in this problem.
f. Based on the results of (c) or (d), which action would you choose? Why?
g. Compute the coefficient of variation for each action.
h. Compute the return-to-risk ratio (RTRR) for each action.
i. Based on (g) and (h), what action would you choose? Why?
j. Compare the results of (f) and (i) and explain any differences.
k. Would your answers to (f) and (i) be different if the probabilities for the three events were 0.1, 0.1, and 0.8, respectively? Discuss.

19.8 For a potential investment of $1,000, if a stock has an EMV of $100 and a standard deviation of $25, what is the
a. rate of return?
b. coefficient of variation?
c. return-to-risk ratio?

19.9 A stock has the following predicted returns under the following economic conditions:

<table>
<thead>
<tr>
<th>Economic Condition</th>
<th>Probability</th>
<th>Return ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.30</td>
<td>50</td>
</tr>
<tr>
<td>Stable economy</td>
<td>0.30</td>
<td>100</td>
</tr>
<tr>
<td>Moderate growth</td>
<td>0.30</td>
<td>120</td>
</tr>
<tr>
<td>Boom</td>
<td>0.10</td>
<td>200</td>
</tr>
</tbody>
</table>

Compute the
a. expected monetary value.
b. standard deviation.
c. coefficient of variation.
d. return-to-risk ratio.

19.10 The following are the returns ($) for two stocks:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected monetary value</td>
<td>90</td>
<td>60</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Which stock would you choose and why?

19.11 The following are the returns ($) for two stocks:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected monetary value</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

Which stock would you choose and why?

**APPLYING THE CONCEPTS**

19.12 A vendor at a local baseball stadium must determine whether to sell ice cream or soft drinks at today’s game. The vendor believes that the profit made will depend on the weather. The payoff table (in $) is as follows:

<table>
<thead>
<tr>
<th>ACTION</th>
<th>Sell Soft Drinks</th>
<th>Sell Ice Cream</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cool weather</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>Warm weather</td>
<td>60</td>
<td>90</td>
</tr>
</tbody>
</table>

Based on her past experience at this time of year, the vendor estimates the probability of warm weather as 0.60.

a. Determine the optimal action based on the maximax criterion.
b. Determine the optimal action based on the maximin criterion.
c. Compute the expected monetary value (EMV) for selling soft drinks and selling ice cream.
d. Compute the expected opportunity loss (EOL) for selling soft drinks and selling ice cream.
e. Explain the meaning of the expected value of perfect information (EVPI) in this problem.
f. Based on the results of (c) or (d), which would you choose to sell, soft drinks or ice cream? Why?
g. Compute the coefficient of variation for selling soft drinks and selling ice cream.
h. Compute the return-to-risk ratio (RTRR) for selling soft drinks and selling ice cream.
i. Based on (g) and (h), what would you choose to sell, soft drinks or ice cream? Why?
j. Compare the results of (f) and (i) and explain any differences.

19.13 The Islander Fishing Company purchases clams for $1.50 per pound from fishermen and sells them to various restaurants for $2.50 per pound. Any clams not sold to the restaurants by the end of the week can be sold to a local soup company for $0.50 per pound. The company can purchase 500, 1,000, or 2,000 pounds. The probabilities of various levels of demand are as follows:

<table>
<thead>
<tr>
<th>Demand (Pounds)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.2</td>
</tr>
<tr>
<td>1,000</td>
<td>0.4</td>
</tr>
<tr>
<td>2,000</td>
<td>0.4</td>
</tr>
</tbody>
</table>

a. For each possible purchase level (500, 1,000, or 2,000 pounds), compute the profit (or loss) for each level of demand.
b. Determine the optimal action based on the maximax criterion.
c. Determine the optimal action based on the maximin criterion.
d. Using the expected monetary value (EMV) criterion, determine the optimal number of pounds of clams the company should purchase from the fishermen. Discuss.
e. Compute the standard deviation for each possible purchase level.
f. Compute the expected opportunity loss (EOL) for purchasing 500, 1,000, and 2,000 pounds of clams.
g. Compute the coefficient of variation for purchasing 500, 1,000, and 2,000 pounds of clams. Discuss.
h. Compute the return-to-risk ratio (RTTR) for purchasing 500, 1,000, and 2,000 pounds of clams. Discuss.
i. Based on (g) and (h), which investment would you choose? Why?
j. Compare the results of (f) and (i) and explain any differences.
k. Suppose that clams can be sold to restaurants for $3 per pound. Repeat (a) through (j) with this selling price for clams and compare the results with those in (k).
l. What would be the effect on the results in (a) through (k) if the probability of the demand for 500, 1,000, and 2,000 clams were 0.4, 0.4, and 0.2, respectively?

19.14 An investor has a certain amount of money available to invest now. Three alternative investments are available. The estimated profits ($) of each investment under each economic condition are indicated in the following payoff table:

<table>
<thead>
<tr>
<th>EVENT</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy declines</td>
<td>500</td>
<td>-2,000</td>
<td>-7,000</td>
</tr>
<tr>
<td>No change</td>
<td>1,000</td>
<td>2,000</td>
<td>-1,000</td>
</tr>
<tr>
<td>Economy expands</td>
<td>2,000</td>
<td>5,000</td>
<td>20,000</td>
</tr>
</tbody>
</table>

Based on his own past experience, the investor assigns the following probabilities to each economic condition:

\[ P(\text{Economy declines}) = 0.30 \]
\[ P(\text{No change}) = 0.50 \]
\[ P(\text{Economy expands}) = 0.20 \]

a. Determine the optimal action based on the maximax criterion.
b. Determine the optimal action based on the maximin criterion.
c. Compute the expected monetary value (EMV) for building a small factory and building a large factory.
d. Compute the expected opportunity loss (EOL) for building a small factory and building a large factory.
e. Explain the meaning of the expected value of perfect information (EVPI) in this problem.
f. Based on the results of (c) or (d), would you choose to build a small factory or a large factory? Why?
g. Compute the coefficient of variation for building a small factory and building a large factory.
h. Compute the return-to-risk ratio (RTTR) for building a small factory and building a large factory.
i. Based on (g) and (h), would you choose to build a small factory or a large factory? Why?
j. Compare the results of (f) and (i) and explain any differences.
k. Suppose that the probabilities of demand are 0.4, 0.2, 0.2, and 0.2, respectively. Repeat (c) through (j) with these probabilities and compare the results with those in (e)–(j).

19.15 In Problem 19.3, you developed a payoff table for building a small factory or a large factory for manufacturing designer jeans. Given the results of that problem, suppose that the probabilities of the demand are as follows:

1. 0.1, 0.6, and 0.3
2. 0.1, 0.3, and 0.6
3. 0.4, 0.4, and 0.2
4. 0.6, 0.3, and 0.1

Repeat (c) through (j) with each of these sets of probabilities and compare the results with those originally computed in (c)–(j). Discuss.

19.16 In Problem 19.4, you developed a payoff table to assist an author in choosing between signing with company A or with company B. Given the results computed in that
Before determining whether to use a gradual or concentrated introduction to the market, the marketing research department conducts an extensive study and releases a report, either that there will be low demand or high demand. In the past, when there was low demand, 30% of the time the market research department stated that there would be high demand. When there was high demand, 80% of the time the market research department stated that there would be high demand. For the organic salad dressings, the marketing research department has stated that there will be high demand. Compute the expected monetary value of each alternative course of action, given this information.

### Example 19.9
**Decision Making Using Sample Information for the Organic Salad Dressings Marketing Example**

180155
**SOLUTION** You need to use Bayes’ theorem (see Section 4.3) to revise the probabilities. To use Equation (4.9) on page 149 for the organic salad dressings marketing example, let

- event $D =$ low demand
- event $D'$ = high demand
- event $M =$ market research predicts low demand
- event $M'$ = market research predicts high demand

and

$$P(D) = 0.60 \quad P(M' | D) = 0.30$$
$$P(D') = 0.40 \quad P(M' | D') = 0.80$$

Then, using Equation (4.9),

$$P(D' | M') = \frac{P(M' | D')P(D')}{P(M' | D)P(D) + P(M' | D')P(D')}$$
$$= \frac{(0.80)(0.40)}{(0.30)(0.60) + (0.80)(0.40)} = \frac{0.32}{0.18 + 0.32} = \frac{0.32}{0.50} = 0.64$$

The probability of high demand, given that the market research department predicted high demand, is 0.64. Thus, the probability of low demand, given that the market research department predicted high demand, is $1 - 0.64 = 0.36$. Table 19.13 summarizes the computation of the probabilities.

| Event, $S_i$ | Prior Probability, $P(D_i)$ | Conditional Probability, $P(M' | D_i)$ | Joint Probability, $P(M' | D_i)P(D_i)$ | Revised Probability, $P(D_i | M')$ |
|-------------|-----------------------------|--------------------------------------|----------------------------------------|-----------------------------------|
| $D =$ low demand | 0.60                         | 0.30                                 | 0.18                                   | $P(D_i | M') = 0.18/0.50 = 0.36$    |
| $D' =$ high demand | 0.40                        | 0.80                                 | 0.32                                   | $P(D' | M') = 0.32/0.50 = 0.64$     |

You need to use the revised probabilities, not the original subjective probabilities, to compute the expected monetary value of each alternative. Table 19.14 illustrates the computations.

<table>
<thead>
<tr>
<th>Alternative Course of Action</th>
<th>Gradual, $A_1$</th>
<th>Concentrated, $A_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event</td>
<td>$X_iP_i$</td>
<td>$X_iP_i$</td>
</tr>
<tr>
<td>Low demand</td>
<td>1 $(0.36) = 0.36$</td>
<td>$-5$</td>
</tr>
<tr>
<td>High demand</td>
<td>4 $(0.64) = 2.56$</td>
<td>$10$ $(0.64) = 6.4$</td>
</tr>
</tbody>
</table>

$EMV(A_1) = 2.92$  $EMV(A_2) = 4.6$

In this case, the optimal decision is to use a concentrated introduction to the market because a profit of $4.6$ million is expected as compared to a profit of $2.92$ million if the organic salad dressings have a gradual introduction to the market. This decision is different from the one considered optimal prior to the collection of the sample information in the form of the market research report (see Example 19.6). The favorable recommendation contained in the report greatly increases the probability that there will be high demand for the organic salad dressings.
CHAPTER 19 Decision Making

Because the relative desirability of the two stocks under consideration in the Using Statistics scenario is directly affected by economic conditions, you should use a forecast of the economic conditions in the upcoming year. You can then use Bayes’ theorem, introduced in Section 4.3, to revise the probabilities associated with the different economic conditions. Suppose that such a forecast can predict either an expanding economy ($F_1$) or a declining or stagnant economy ($F_2$). Past experience indicates that, with a recession, prior forecasts predicted an expanding economy 20% of the time. With a stable economy, prior forecasts predicted an expanding economy 40% of the time. With moderate growth, prior forecasts predicted an expanding economy 70% of the time. Finally, with a boom economy, prior forecasts predicted an expanding economy 90% of the time.

If the forecast is for an expanding economy, you can revise the probabilities of economic conditions by using Bayes’ theorem, Equation (4.9) on page 149. Let

- event $E_1$ = recession
- event $F_1$ = expanding economy is predicted
- event $E_2$ = stable economy
- event $F_2$ = declining or stagnant economy is predicted
- event $E_3$ = moderate growth
- event $E_4$ = boom economy

and

$$P(E_1) = 0.10 \quad P(F_1 | E_1) = 0.20$$
$$P(E_2) = 0.40 \quad P(F_1 | E_2) = 0.40$$
$$P(E_3) = 0.30 \quad P(F_1 | E_3) = 0.70$$
$$P(E_4) = 0.20 \quad P(F_1 | E_4) = 0.90$$

Then, using Bayes’ theorem,

$$P(E_1 | F_1) = \frac{P(F_1 | E_1)P(E_1)}{P(F_1 | E_1)P(E_1) + P(F_1 | E_2)P(E_2) + P(F_1 | E_3)P(E_3) + P(F_1 | E_4)P(E_4)}$$
$$= \frac{(0.20)(0.10)}{(0.20)(0.10) + (0.40)(0.40) + (0.70)(0.30) + (0.90)(0.20)}$$
$$= \frac{0.02}{0.57} = 0.035$$

$$P(E_2 | F_1) = \frac{P(F_1 | E_2)P(E_2)}{P(F_1 | E_1)P(E_1) + P(F_1 | E_2)P(E_2) + P(F_1 | E_3)P(E_3) + P(F_1 | E_4)P(E_4)}$$
$$= \frac{(0.40)(0.40)}{(0.20)(0.10) + (0.40)(0.40) + (0.70)(0.30) + (0.90)(0.20)}$$
$$= \frac{0.16}{0.57} = 0.281$$

$$P(E_3 | F_1) = \frac{P(F_1 | E_3)P(E_3)}{P(F_1 | E_1)P(E_1) + P(F_1 | E_2)P(E_2) + P(F_1 | E_3)P(E_3) + P(F_1 | E_4)P(E_4)}$$
$$= \frac{(0.70)(0.30)}{(0.20)(0.10) + (0.40)(0.40) + (0.70)(0.30) + (0.90)(0.20)}$$
$$= \frac{0.21}{0.57} = 0.368$$

$$P(E_4 | F_1) = \frac{P(F_1 | E_4)P(E_4)}{P(F_1 | E_1)P(E_1) + P(F_1 | E_2)P(E_2) + P(F_1 | E_3)P(E_3) + P(F_1 | E_4)P(E_4)}$$
$$= \frac{(0.90)(0.20)}{(0.20)(0.10) + (0.40)(0.40) + (0.70)(0.30) + (0.90)(0.20)}$$
$$= \frac{0.18}{0.57} = 0.316$$
19.3 Decision Making with Sample Information

Table 19.15 summarizes the computation of these probabilities. Figure 19.5 displays the joint probabilities in a decision tree. You need to use the revised probabilities, not the original subjective probabilities, to compute the expected monetary value. Table 19.16 shows these computations.

**Table 19.15**
Bayes’ Theorem Calculations for the Stock Selection Example

| Event, $E_i$ | Prior Probability, $P(E_i)$ | Conditional Probability, $P(F_j | E_i)$ | Joint Probability, $P(F_j | E_i)P(E_i)$ | Revised Probability, $P(E_i | F_j)$ |
|-------------|-----------------------------|--------------------------------------|--------------------------------------|----------------------------------|
| Recession, $E_1$ | 0.10 | 0.20 | 0.02 | $0.02/0.57 = 0.035$ |
| Stable economy, $E_2$ | 0.40 | 0.40 | 0.16 | $0.16/0.57 = 0.281$ |
| Moderate growth, $E_3$ | 0.30 | 0.70 | 0.21 | $0.21/0.57 = 0.368$ |
| Boom, $E_4$ | 0.20 | 0.90 | 0.18 | $0.18/0.57 = 0.316$ |

**Figure 19.5**
Decision tree with joint probabilities for the stock selection example

**Table 19.16**
Expected Monetary Value, Using Revised Probabilities, for Each of Two Stocks Under Four Economic Conditions

<table>
<thead>
<tr>
<th>Event</th>
<th>$P_i$</th>
<th>$A$</th>
<th>$X_iP_i$</th>
<th>$B$</th>
<th>$X_iP_i$</th>
<th>$EMV(A)$</th>
<th>$EMV(B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.035</td>
<td>30</td>
<td>30(0.035)</td>
<td>-50</td>
<td>-50(0.035)</td>
<td>-1.75</td>
<td></td>
</tr>
<tr>
<td>Stable economy</td>
<td>0.281</td>
<td>70</td>
<td>70(0.281)</td>
<td>30</td>
<td>30(0.281)</td>
<td>8.43</td>
<td></td>
</tr>
<tr>
<td>Moderate growth</td>
<td>0.368</td>
<td>100</td>
<td>100(0.368)</td>
<td>250</td>
<td>250(0.368)</td>
<td>92.00</td>
<td></td>
</tr>
<tr>
<td>Boom</td>
<td>0.316</td>
<td>150</td>
<td>150(0.316)</td>
<td>400</td>
<td>400(0.316)</td>
<td>126.40</td>
<td></td>
</tr>
</tbody>
</table>

Thus, the expected monetary value, or profit, for stock $A$ is $104.92$, and the expected monetary value, or profit, for stock $B$ is $225.08$. Using this criterion, you should once again choose stock $B$ because the expected monetary value is much higher for this stock. However, you should reexamine the return-to-risk ratios in light of these revised probabilities. Using
Equations (5.2) and (5.3) on pages 163 and 164, for stock \( A \) because \( EMV(A) = \mu_A = \$104.92 \),

\[
\sigma^2_A = \sum_{i=1}^{N} (X_i - \mu)^2 P(X_i)
\]

\[
= (30 - 104.92)^2(0.035) + (70 - 104.92)^2(0.281) + (100 - 104.92)^2(0.368) + (150 - 104.92)^2(0.316)
\]

\[
= 1,190.194
\]

\[
\sigma_A = \sqrt{1,190.194} = \$34.50.
\]

For stock \( B \), because \( \mu_B = \$225.08 \),

\[
\sigma^2_B = \sum_{i=1}^{N} (X_i - \mu)^2 P(X_i)
\]

\[
= (-50 - 225.08)^2(0.035) + (30 - 225.08)^2(0.281) + (250 - 225.08)^2(0.368) + (400 - 225.08)^2(0.316)
\]

\[
= 23,239.39
\]

\[
\sigma_B = \sqrt{23,239.39} = \$152.445.
\]

To compute the coefficient of variation, substitute \( \sigma \) for \( S \) and \( EMV \) for \( \bar{X} \) in Equation (3.8) on page 96,

\[
CV_A = \left( \frac{\sigma_A}{EMV_A} \right)100\%
\]

\[
= \left( \frac{34.50}{104.92} \right)100\% = 32.88\%
\]

and

\[
CV_B = \left( \frac{\sigma_B}{EMV_B} \right)100\%
\]

\[
= \left( \frac{152.445}{225.08} \right)100\% = 67.73\%
\]

Thus, there is still much more variation in the returns from stock \( B \) than from stock \( A \). For each of these two stocks, you calculate the return-to-risk ratios as follows. For stock \( A \), the return-to-risk ratio is equal to

\[
RTRR(A) = \frac{104.92}{34.50} = 3.041
\]

For stock \( B \), the return-to-risk ratio is equal to

\[
RTRR(B) = \frac{225.08}{152.445} = 1.476
\]

Thus, using the return-to-risk ratio, you should select stock \( A \). This decision is different from the one you reached when using expected monetary value (or the equivalent expected opportunity loss). What stock should you buy? Your final decision will depend on whether you believe it is more important to maximize the expected return on investment (select stock \( B \)) or to control the relative risk (select stock \( A \)).
19.3 Decision Making with Sample Information

**Problems for Section 19.3**

**LEARNING THE BASICS**

19.18 Consider the following payoff table:

<table>
<thead>
<tr>
<th>EVENT</th>
<th>ACTION A ($)</th>
<th>ACTION B ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>125</td>
</tr>
</tbody>
</table>

For this problem, \( P(E_1) = 0.5, P(E_2) = 0.5, P(F | E_1) = 0.6, \) and \( P(F | E_2) = 0.4. \) Suppose that you are informed that event \( F \) occurs.

- **a.** Revise the probabilities \( P(E_1) \) and \( P(E_2) \) now that you know that event \( F \) has occurred. Based on these revised probabilities, answer (b) through (i).
- **b.** Compute the expected monetary value of action \( A \) and action \( B \).
- **c.** Compute the expected opportunity loss of action \( A \) and action \( B \).
- **d.** Explain the meaning of the expected value of perfect information \( (EVPI) \) in this problem.
- **e.** On the basis of (b) and (c), which action should you choose? Why?
- **f.** Compute the coefficient of variation for each action.
- **g.** Compute the return-to-risk ratio \( (RTRR) \) for each action.
- **h.** On the basis of (f) and (g), which action should you choose? Why?
- **i.** Compare the results of (e) and (h) and explain any differences.

**APPLYING THE CONCEPTS**

19.20 In Problem 19.12, a vendor at a baseball stadium is deciding whether to sell ice cream or soft drinks at today’s game. Prior to making her decision, she decides to listen to the local weather forecast. In the past, when it has been cool, the weather reporter has forecast cool weather 80% of the time. When it has been warm, the weather reporter has forecast warm weather 70% of the time. The local weather forecast is for cool weather.

- **a.** Revise the prior probabilities now that you know that the weather forecast is for cool weather.
- **b.** Use these revised probabilities to repeat Problem 19.12.
- **c.** Compare the results in (b) to those in Problem 19.12.

19.21 In Problem 19.14, an investor is trying to determine the optimal investment decision among three investment opportunities. Prior to making his investment decision, the investor decides to consult with his financial adviser. In the past, when the economy has declined, the financial adviser has given a rosy forecast 20% of the time (with a gloomy forecast 80% of the time). When there has been no change in the economy, the financial adviser has given a rosy forecast 40% of the time. When there has been an expanding economy, the financial adviser has given a rosy forecast 70% of the time. The financial adviser in this case gives a gloomy forecast for the economy.

- **a.** Revise the probabilities of the investor based on this economic forecast by the financial adviser.
- **b.** Use these revised probabilities to repeat Problem 19.14.
- **c.** Compare the results in (b) to those in Problem 19.14.

19.22 In Problem 19.16, an author is deciding which of two competing publishing companies to select to publish her new novel. Prior to making a final decision, the author decides to have an experienced reviewer examine her novel. This reviewer has an outstanding reputation for predicting the success of a novel. In the past, for novels that sold 1,000 copies, only 1% received favorable reviews. Of novels that sold 5,000 copies, 25% received favorable reviews.
Of novels that sold 10,000 copies, 60% received favorable reviews. Of novels that sold 50,000 copies, 99% received favorable reviews. After examining the author’s novel, the reviewer gives it an unfavorable review.

**a.** Revise the probabilities of the number of books sold in light of the reviewer’s unfavorable review.

**b.** Use these revised probabilities to repeat Problem 19.16.

**c.** Compare the results in (b) to those in Problem 19.16.

### 19.4 Utility

The methods used in Sections 19.1 through 19.3 assume that each *incremental* amount of profit or loss has the same value as the previous amounts of profits attained or losses incurred. In fact, under many circumstances in the business world, this assumption of incremental changes is not valid. Most companies, as well as most individuals, make special efforts to avoid large losses. At the same time, many companies, as well as most individuals, place less value on extremely large profits than on initial profits. Such differential evaluation of incremental profits or losses is referred to as **utility**, a concept first discussed by Daniel Bernoulli in the eighteenth century (see reference 1). To illustrate this concept, suppose that you are faced with the following two choices:

**Choice 1:** A fair coin is to be tossed. If it lands on heads, you will receive $0.60; if it lands on tails, you will pay $0.40.

**Choice 2:** Do not play the game.

What decision should you choose? The expected value of playing this game is 

$$\frac{(0.60)(0.50)}{2} + \frac{(-0.40)(0.50)}{2} = +\$0.10,$$

and the expected value of not playing the game is 0.

Most people will decide to play the game because the expected value is positive, and only small amounts of money are involved. Suppose, however, that the game is formulated with a payoff of $600,000 when the coin lands on heads and a loss of $400,000 when the coin lands on tails. The expected value of playing the game is now +$100,000. With these payoffs, even though the expected value is positive, most individuals will not play the game because of the severe negative consequences of losing $400,000. Each additional dollar amount of either profit or loss does not have the same utility as the previous amount. Large negative amounts for most individuals have severely negative utility. Conversely, the extra value of each incremental dollar of profit decreases when high enough profit levels are reached. (In other words, the difference between 0 and $100,000 is much more than the difference between $1,000,000 and $1,100,000.)

An important part of the decision-making process, which is beyond the scope of this text (see references 2 and 4), is to develop a utility curve for a decision maker that represents the utility of each specified dollar amount. Figure 19.6 illustrates three types of utility curves: those of the risk averter, the risk seeker, and the risk-neutral person.

**FIGURE 19.6**

Three types of utility curves

The **risk averter’s curve** shows a rapid increase in utility for initial amounts of money followed by a gradual leveling off for increasing dollar amounts. This curve is appropriate for most individuals or businesses because the value of each additional dollar is not as great after large amounts of money have already been earned.
The risk seeker’s curve represents the utility of someone who enjoys taking risks. The utility is greater for large dollar amounts. This curve represents an individual who is interested only in “striking it rich” and is willing to take large risks for the opportunity of making large profits.

The risk-neutral curve represents the expected monetary value approach. Each additional dollar of profit has the same value as the previous dollar.

After a utility curve is developed in a specific situation, you convert the dollar amounts to utilities. Then you compute the utility of each alternative course of action and apply the decision criteria of expected utility value, expected opportunity loss, and return-to-risk ratio to make a decision.

Problems for Section 19.4

APPLYING THE CONCEPTS

19.23 Do you consider yourself a risk seeker, a risk averter, or a risk-neutral person? Explain.

19.24 Refer to Problems 19.3–19.5 and 19.12–19.14, respectively. In which problems do you think the expected monetary value (risk-neutral) criterion is inappropriate? Why?

THINK ABOUT THIS

Risky Business

When you make investment decisions, you need to think about your own personal tolerance for risk. When given the chance to make large sums of money, some people can shrug off a 20% loss, while others feel ruined when they lose even a small amount such as $20. What about you? Are you willing to risk losing a lot of money for a chance to strike it rich? Or are you more comfortable with a less risky scenario, even though your chance to strike it rich will be smaller?

How can you identify stocks, mutual funds, or other types of investments that fit your own personal tolerance for risk? One way to begin is to examine the standard deviation of the investments you are considering during recent periods of time. You will also want to assess your own willingness to tolerate different losses. What would you do if you lost 10%, 30%, 50%, or 90% of your investment? Would you sell? Would you buy more in the hope that the investment would go back up? Also, think about what you would do if your investment went up 10%, 50%, 100%, or 200%. Would you sell the entire investment? Part of the investment? Or would you buy more? You might want to think about the volatility of your investments—that is, any patterns of extreme increases and decreases in the value over short periods of time—that can add to the risk.

Everyone is different, but knowing how you would answer these questions before you are faced with making a decision will help you make better investment decisions.

USING STATISTICS

@ The Reliable Fund Revisited

In the Using Statistics scenario, you learned how the manager of The Reliable Fund could use various decision-making criteria to decide whether to purchase stock A or stock B. You also saw how sample information could be used to revise probabilities and possibly change the decision reached. You found that stock B had a higher expected monetary value, a lower expected opportunity loss, but a lower return-to-risk ratio.

SUMMARY

In this chapter, you learned how to develop payoff tables and decision trees, to use various criteria to choose between alternative courses of action, and to revise probabilities, using Bayes’ theorem, in light of sample information.
CHAPTER 19 Decision Making

KEY TERMS

- alternative courses of action 4
- decision criteria 4
- decision tree 4
- events, or states of the world 4
- expected monetary value (EMV) 9
- expected opportunity loss (EOL) 11
- expected profit under certainty 12
- expected value of perfect information (EVPI) 12
- maximax payoff criterion 8
- maximin payoff criterion 9
- opportunity loss 6
- payoff 4
- payoff table 4
- risk averter’s curve 24
- risk-neutral curve 25
- risk seeker’s curve 25
- utility 24
- return-to-risk ratio (RTRR) 14

KEY EQUATIONS

**Expected Monetary Value**

\[
EMV(j) = \sum_{i=1}^{N} X_{ij} P_i \quad (19.1)
\]

**Expected Opportunity Loss**

\[
EOL(j) = \sum_{i=1}^{N} L_{ij} P_i \quad (19.2)
\]

**Expected Value of Perfect Information**

\[
EVPI = \text{expected profit under certainty} - \text{expected monetary value of the best alternative} \quad (19.3)
\]

**Return-to-Risk Ratio**

\[
RTRR(j) = \frac{EMV(j)}{\sigma_j} \quad (19.4)
\]

CHAPTER REVIEW PROBLEMS

CHECKING YOUR UNDERSTANDING

19.25 What is the difference between an event and an alternative course of action?

19.26 What are the advantages and disadvantages of a payoff table as compared to a decision tree?

19.27 How are opportunity losses computed from payoffs?

19.28 Why can’t an opportunity loss be negative?

19.29 How does expected monetary value (EMV) differ from expected opportunity loss (EOL)?

19.30 What is the meaning of the expected value of perfect information (EVPI)?

19.31 How does the expected value of perfect information differ from the expected profit under certainty?

19.32 What are the advantages and disadvantages of using expected monetary value (EMV) as compared to the return-to-risk ratio (RTRR)?

19.33 How is Bayes’ theorem used to revise probabilities in light of sample information?

19.34 What is the difference between a risk averter and a risk seeker?

19.35 Why should you use utilities instead of payoffs in certain circumstances?

APPLYING THE CONCEPTS

19.36 A supermarket chain purchases large quantities of white bread for sale during a week. The stores purchase the bread for $0.75 per loaf and sell it for $1.10 per loaf. Any loaves not sold by the end of the week can be sold to a local thrift shop for $0.40. Based on past demand, the probability of various levels of demand is as follows:

<table>
<thead>
<tr>
<th>Demand (Loaves)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,000</td>
<td>0.10</td>
</tr>
<tr>
<td>8,000</td>
<td>0.50</td>
</tr>
<tr>
<td>10,000</td>
<td>0.30</td>
</tr>
<tr>
<td>12,000</td>
<td>0.10</td>
</tr>
</tbody>
</table>

a. Construct a payoff table, indicating the events and alternative courses of action.

b. Construct a decision tree.

c. Compute the expected monetary value (EMV) for purchasing 6,000, 8,000, 10,000, and 12,000 loaves.

d. Compute the expected opportunity loss (EOL) for purchasing 6,000, 8,000, 10,000, and 12,000 loaves.
e. Explain the meaning of the expected value of perfect information (EVPI) in this problem.

f. Based on the results of (c) or (d), how many loaves would you purchase? Why?

g. Compute the coefficient of variation for each purchase level.

h. Compute the return-to-risk ratio (RTRR) for each purchase level.

i. Based on (g) and (h), what action would you choose? Why?

j. Compare the results of (f) and (i) and explain any differences.

k. Suppose that new information changes the probabilities associated with the demand level. Use the following probabilities to repeat (c) through (j):

<table>
<thead>
<tr>
<th>Demand (Loaves)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,000</td>
<td>0.30</td>
</tr>
<tr>
<td>8,000</td>
<td>0.40</td>
</tr>
<tr>
<td>10,000</td>
<td>0.20</td>
</tr>
<tr>
<td>12,000</td>
<td>0.10</td>
</tr>
</tbody>
</table>

19.37 The owner of a company that supplies home heating oil would like to determine whether to offer a solar heating installation service to its customers. The owner of the company has determined that a startup cost of $150,000 would be necessary, but a profit of $2,000 can be made on each solar heating system installed. The owner estimates the probability of various demand levels as follows:

<table>
<thead>
<tr>
<th>Number of Units Installed</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.40</td>
</tr>
<tr>
<td>100</td>
<td>0.30</td>
</tr>
<tr>
<td>200</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Based on past experience, the product manager assigns the following probabilities to the different levels of national response:

- $P$(Weak national response) = 0.30
- $P$(Moderate national response) = 0.60
- $P$(Strong national response) = 0.10

a. Construct a decision tree.

b. Construct an opportunity loss table.

c. Compute the expected monetary value (EMV) for offering this new product package.

d. Compute the expected opportunity loss (EOL) for offering this new product package.

e. Explain the meaning of the expected value of perfect information (EVPI) in this problem.

f. Compute the return-to-risk ratio (RTRR) for offering this new product package.

g. Based on the results of (c), (d), and (f), should the company offer this new product package? Why?

h. What are your answers to parts (c) through (g) if the probabilities are 0.6, 0.3, and 0.1, respectively?

i. What are your answers to parts (c) through (g) if the probabilities are 0.1, 0.3, and 0.6, respectively?

j. If sales in the test city stayed the same, revise the original probabilities in light of this new information.

k. Use the revised probabilities in (j) to repeat (c) through (g).
1. If sales in the test city decreased, revise the original
probabilities in light of this new information.
m. Use the revised probabilities in (l) to repeat (c) through (g).

19.39 An entrepreneur wants to determine whether it
would be profitable to establish a gardening service in a
local suburb. The entrepreneur believes that there are four
possible levels of demand for this gardening service:

- Very low demand—1% of the households would use the
  service.
- Low demand—5% of the households would use the service.
- Moderate demand—10% of the households would use
  the service.
- High demand—25% of the households would use the service.

Based on past experiences in other suburbs, the entrepreneur
assigns the following probabilities to the various demand levels:

\[
\begin{align*}
P(\text{Very low demand}) & = 0.20 \\
P(\text{Low demand}) & = 0.50 \\
P(\text{Moderate demand}) & = 0.20 \\
P(\text{High demand}) & = 0.10 \\
\end{align*}
\]

The entrepreneur has calculated the following profits or
losses ($) of this garden service for each demand level (over
a period of one year):

<table>
<thead>
<tr>
<th>ACTION</th>
<th>DEMAND</th>
<th>Provide Garden Service</th>
<th>Do Not Provide Garden Service</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low</td>
<td>-50,000</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>60,000</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Moderate</td>
<td>130,000</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>300,000</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

a. Construct a decision tree.
b. Construct an opportunity loss table.
c. Compute the expected monetary value (EMV) for offering
   this garden service.
d. Compute the expected opportunity loss (EOL) for offering
   this garden service.
e. Explain the meaning of the expected value of perfect
   information (EVPI) in this problem.
f. Compute the return-to-risk ratio (RTRR) for offering this
   garden service.
g. Based on the results of (c), (d), and (f), should the entre-
   preneur offer this garden service? Why?

Before making a final decision, the entrepreneur conducts a
survey to determine demand for the gardening service. A
random sample of 20 households is selected, and 3 indicate
that they would use this gardening service.

h. Revise the prior probabilities in light of this sample
   information. (Hint: Use the binomial distribution to determine
   the probability of the outcome that occurred, given a particu-
   lar level of demand.)
i. Use the revised probabilities in (h) to repeat (c) through (g).

19.40 A manufacturer of a brand of inexpensive felt-tip
pens maintains a production process that produces 10,000
pens per day. In order to maintain the highest quality of
this product, the manufacturer guarantees free replacement
of any defective pen sold. Each defective pen produced
costs 20 cents for the manufacturer to replace. Based on
past experience, four rates of producing defective pens are
possible:

- Very low—1% of the pens manufactured will be
defective.
- Low—5% of the pens manufactured will be defective.
- Moderate—10% of the pens manufactured will be
defective.
- High—20% of the pens manufactured will be defective.

The manufacturer can reduce the rate of defective pens pro-
duced by having a mechanic fix the machines at the end of
each day. This mechanic can reduce the rate to 1%, but his
services will cost $80.

A payoff table based on the daily production of 10,000
pens, indicating the replacement costs ($) for each of the two
alternatives (calling in the mechanic and not calling in the
mechanic), is as follows:

<table>
<thead>
<tr>
<th>DEFECTIVE RATE</th>
<th>Action</th>
<th>Cost for Not Calling Mechanic</th>
<th>Cost for Calling Mechanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low (1%)</td>
<td>20</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Low (5%)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Moderate (10%)</td>
<td>200</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>High (20%)</td>
<td>400</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Based on past experience, each defective rate is assumed to
be equally likely to occur.

a. Construct a decision tree.
b. Construct an opportunity loss table.
c. Compute the expected monetary value (EMV) for calling
   and for not calling the mechanic.
d. Compute the expected opportunity loss (EOL) for calling
   and for not calling the mechanic.
e. Explain the meaning of the expected value of perfect
   information (EVPI) in this problem.
f. Compute the return-to-risk ratio (RTRR) for calling and
   not calling the mechanic.
g. Based on the results of (c), (d), and (f), should the com-
   pany call the mechanic? Why?
h. At the end of a day’s production, a sample of 15 pens is
   selected, and 2 are defective. Revise the prior probabili-
   ties in light of this sample information. (Hint: Use the
   binomial distribution to determine the probability of the
   outcome that occurred, given a particular defective rate.)
i. Use the revised probabilities in (h) to repeat (c) through (g).
WEB CASE

Apply your knowledge of decision-making techniques in this Web Case, which extends the EndRun Web Case from earlier chapters.

StraightArrow Banking & Investments is a Tri-Cities competitor of EndRun that is currently advertising its StraightArrow StraightDeal fund. Using a Web browser, open to the Web page for the Chapter 19 Web Case (or open SA_Home.htm if you have downloaded the Web Case files) to visit the StraightArrow Web home page. Examine the claims and supporting data for the fund. Compare those claims and data to the supporting data found on the EndRun Happy Bull and Worried Bear Funds Web page by opening to the Web page for the Chapter 5 Web Case (or ER_BullsandBears.htm if you have downloaded the Web Case files). Then answer the following:

1. Is the StraightArrow StraightDeal fund a better investment than either of the EndRun funds? Support your answer by performing an appropriate analysis and summarizing your results.

2. Before making a decision about which fund makes a better investment, you decide that you need a reliable forecast for the direction of the economy in the next year. After further investigation, you find that the consensus of leading economists is that the economy will be expanding in the next year. You also find out that in the past, when there has been a recession, leading economists predicted an expanding economy 10% of the time. When there was a stable economy, they predicted an expanding economy 50% of the time, and when there was an expanding economy, they predicted an expanding economy 75% of the time. When there was a rapidly expanding economy, they predicted an expanding economy 90% of the time. Does this information change your answer to Question 1? Why or why not?

REFERENCES

EG19.1 PAYOFF TABLES and DECISION TREES

PHStat2 Use the Opportunity Loss procedure to create a payoff table and an opportunity loss table. For example, to create the payoff and opportunity loss tables for the Example 19.3 marketing problem on page 6, select PHStat ➔ Decision-Making ➔ Opportunity Loss. In the procedure’s dialog box (shown below):

1. Enter 2 as the Number of Events.
2. Enter 2 as the Number of Alternative Actions.
3. Enter a Title and click OK.

In the opportunity loss analysis worksheet that the procedure creates, enter the payoff data and event and alternative action labels from Table 19.1 in the tinted table cells that begin in row 4. (The #DIV/0! messages that may appear in several cells will disappear after you enter your data. This is not an error.)

In-Depth Excel Enter payoff data as a simple table and use the INDEX, MATCH, and MAX functions to help perform an opportunity loss analysis. Enter \( \text{INDEX}(\text{cell range of the alternative action labels}, 1, \text{optimum action for event}) \) to display the correct alternative course of action label. Enter \( \text{MATCH}(\text{cell that contains maximum event payoff}, \text{cell range of payoffs for event}, 0) \) to determine the optimum action for event used by the INDEX function. Enter \( \text{MAX}(\text{cell range of payoffs for event}) \) to display the optimum profit for an event.

Use the COMPUTE worksheet of the Opportunity Loss workbook, shown in Figure 19.3, as a template for creating payoff tables and performing opportunity loss analysis. This worksheet performs the analysis for the Example 19.3 marketing problem that uses the Table 19.1 payoff data on page 4. The MATCH function in the column B formulas matches the optimum profit found in column C with one of two payoff values and returns either the value 1 or 2, depending on whether the first or second payoff value matched the optimum profit. In turn, INDEX uses the returned value (1 or 2) as the column number to decide which of the two alternative courses of action labels will be displayed in the column B cell. (Open to the COMPUTE_FORMULAS worksheet to examine the details of other formulas used in the COMPUTE worksheet.)

To perform opportunity loss analysis for other problems with two events and two alternative courses of action, enter the new payoff data and labels in the Payoff Table area in rows 4 through 6 in the COMPUTE worksheet. Use the COMPUTE_THREE worksheet for problems that contain three events and two alternative courses of action, and use the COMPUTE_FOUR worksheets for problems that contain four events and two alternative courses of action.

EG19.2 CRITERIA for DECISION MAKING

PHStat2 Use the Expected Monetary Value procedure to create expected monetary value, expected opportunity loss, and return-to-risk ratio decision criteria. For example, to create the three decision criteria for the Section 19.2 stock selection problem, select PHStat ➔ Decision-Making ➔ Expected Monetary Value. In the procedure’s dialog box (shown below):

1. Enter 4 as the Number of Events.
2. Enter 2 as the Number of Alternative Actions.
3. Check Expected Opportunity Loss.
4. Check Measures of Variation.
5. Enter a Title and click OK.
In the worksheet that the procedure creates, enter the probabilities, payoff data, and event and alternative action labels from Table 19.10 in the tinted table cells that begin in row 4. (The #DIV/0! messages that may appear in several cells will disappear after you enter your data. This is not an error.)

**In-Depth Excel** Use the SUMPRODUCT function to compute the expected monetary value and variance and use the INDEX, MATCH, and MAX functions (discussed in the Section EG19.1 *In-Depth Excel* instructions) to help perform an opportunity loss analysis. Enter \( \text{SUMPRODUCT(cell range of the probabilities of the events, cell range of the payoffs for one alternative course of action)} \) to compute the expected monetary value and enter \( \text{SUMPRODUCT(cell range of the probabilities of the events, cell range of squared differences for one alternative course of action)} \) to compute the variance.

Use the COMPUTE worksheet of the *Expected Monetary Value* workbook, shown in Figure 19.4, as a template for creating expected monetary value, expected opportunity loss, and the return-to-risk ratio decision criteria. This worksheet creates the decision criteria for the Section 19.2 stock selection problem. Open to the COMPUTE_FORMULAS worksheet to examine the details of all formulas used in the COMPUTE worksheet.

To create decision criteria for other problems involving four events and two alternative courses of action, make new entries in the Probabilities & Payoff Table area. Use the COMPUTE_THREE worksheet for problems that contain three events and two alternative courses of action.