11.3 The Randomized Block Design

Section 11.1 discussed how to use the one-way ANOVA $F$ test to evaluate differences among the means of more than two independent groups. Section 10.2 discussed how to use the paired $t$ test to evaluate the difference between the means of two groups when you had repeated measurements or matched samples. The randomized block design evaluates differences among more than two groups that contain matched samples or repeated measures that have been placed in blocks. Blocks are heterogeneous sets of items or individuals that have been either matched or on whom repeated measurements have been taken. Blocking removes as much variability as possible from the random error so that the differences among the groups are more evident.

Although blocks are used in a randomized block design, the focus of the analysis is on the differences among the different groups. As is the case in completely randomized designs, groups are often different levels pertaining to a factor of interest. A randomized block design is often more efficient statistically than a completely randomized design and therefore produces more precise results (see references 5, 6, and 9). For example, if the factor of interest is advertising medium, three groups could be subject to the following different levels: television, radio, and newspaper. Different cities could be used as blocks. The variability among the different cities is therefore removed from the random error in order to better detect differences among the three advertising mediums.

To compare a completely randomized design with a randomized block design, return to the Perfect Parachutes scenario on page 381. Suppose that a completely randomized design is used with 12 parachutes woven during a 24-hour period. Any variability among the shifts of workers becomes part of the random error, and therefore differences among the four suppliers might be difficult to detect. To reduce the random error, a randomized block experiment is designed, in which three shifts of workers are used and four parachutes are woven during each shift (one parachute using fibers from Supplier 1, one parachute using fibers from Supplier 2, etc.). The three shifts are considered blocks, but the factor of interest is still the four suppliers. The advantage of the randomized block design is that the variability among the three shifts is removed from the random error. Therefore, this design should provide more precise results concerning differences among the four suppliers.

Testing for Factor and Block Effects

Recall from Figure 11.1 on page 382 that, in the completely randomized design, the total variation ($SST$) is subdivided into variation due to differences among the $c$ groups ($SSA$) and variation due to variation within the $c$ groups ($SSW$). Within-group variation is considered random variation, and among-group variation is due to differences from group to group.

To remove the effects of the blocking from the random variation component in the randomized block design, the within-group variation ($SSW$) is subdivided into variation due to differences among the blocks ($SSBL$) and random variation ($SSE$). Therefore, as presented in Figure 11.17, in a randomized block design, the total variation is the sum of three components: among-group variation ($SSA$), among-block variation ($SSBL$), and random variation ($SSE$).
The following definitions are needed to develop the ANOVA procedure for the randomized block design:

\[ r = \text{the number of blocks} \]
\[ c = \text{the number of groups} \]
\[ n = \text{the total number of values (where } n = rc) \]
\[ X_{ij} = \text{the value in the } i\text{th block for the } j\text{th group} \]
\[ \bar{X}_i = \text{the mean of all the values in block } i \]
\[ \bar{X}_j = \text{the mean of all the values for group } j \]

\[ \sum_{j=1}^{c} \sum_{i=1}^{r} X_{ij} = \text{the grand total} \]

The total variation, also called sum of squares total (SST), is a measure of the variation among all the values. You compute SST by summing the squared differences between each individual value and the grand mean, \( \bar{X} \), that is based on all \( n \) values. Equation (11.18) shows the computation for total variation.

**TOTAL VARIATION IN THE RANDOMIZED BLOCK DESIGN**

\[
SST = \sum_{j=1}^{c} \sum_{i=1}^{r} (X_{ij} - \bar{X})^2
\]  

(11.18)

where

\[ \bar{X} = \frac{\sum_{j=1}^{c} \sum_{i=1}^{r} X_{ij}}{rc} \] (i.e., the grand mean)

You compute the among-group variation, also called the sum of squares among groups (SSA), by summing the squared differences between the sample mean of each group, \( \bar{X}_j \), and the grand mean, \( \bar{X} \), weighted by the number of blocks, \( r \). Equation (11.19) shows the computation for the among-group variation.

**AMONG-GROUP VARIATION IN THE RANDOMIZED BLOCK DESIGN**

\[
SSA = r \sum_{j=1}^{c} (\bar{X}_j - \bar{X})^2
\]  

(11.19)

where

\[ \bar{X}_j = \frac{\sum_{i=1}^{r} X_{ij}}{r} \]

You compute the among-block variation, also called the sum of squares among blocks (SSBL), by summing the squared differences between the mean of each block, \( \bar{X}_j \), and the grand mean, \( \bar{X} \), weighted by the number of groups, \( c \). Equation (11.20) shows the computation for the among-block variation.
11.3 The Randomized Block Design

You compute the random variation, also called the sum of squares error (SSE), by summing the squared differences among all the values after the effect of the groups and blocks have been accounted for. Equation (11.21) shows the computation for random variation.

**AMONG-BLOCK VARIATION IN THE RANDOMIZED BLOCK DESIGN**

\[
SSBL = c \sum_{i=1}^{r} (\bar{X}_i - \bar{X})^2
\]  

(11.20)

where

\[
\bar{X}_i = \frac{\sum_{j=1}^{c} X_{ij}}{c}
\]

You compute the random variation, also called the sum of squares error (SSE), by summing the squared differences among all the values after the effect of the groups and blocks have been accounted for. Equation (11.21) shows the computation for random variation.

**RANDOM VARIATION IN THE RANDOMIZED BLOCK DESIGN**

\[
SSE = \sum_{j=1}^{r} \sum_{i=1}^{c} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{X})^2
\]  

(11.21)

Because you are comparing \(c\) groups, there are \(c - 1\) degrees of freedom associated with the sum of squares among groups (SSA). Similarly, because there are \(r\) blocks, there are \(r - 1\) degrees of freedom associated with the sum of squares among blocks (SSBL). Moreover, there are \(n - 1\) degrees of freedom associated with the sum of squares total (SST) because you are comparing each value, \(X_{ij}\), to the grand mean, \(\bar{X}\), based on all \(n\) values. Therefore, because the degrees of freedom for each of the sources of variation must add to the degrees of freedom for the total variation, you compute the degrees of freedom for the sum of squares error (SSE) component by subtraction and algebraic manipulation. Thus, the degrees of freedom associated with the sum of squares error is \((r - 1)(c - 1)\).

If you divide each of the component sums of squares by its associated degrees of freedom, you have the three variances, or mean square terms (MSA, MSBL, and MSE). Equations (11.22a–c) give the mean square terms needed for the ANOVA table.

**THE MEAN SQUARES IN THE RANDOMIZED BLOCK DESIGN**

\[
MSA = \frac{SSA}{c - 1}
\]  

(11.22a)

\[
MSBL = \frac{SSBL}{r - 1}
\]  

(11.22b)

\[
MSE = \frac{SSE}{(r - 1)(c - 1)}
\]  

(11.22c)

The first step in analyzing a randomized block design is to test for a factor effect—that is, to test for any differences among the \(c\) group means. If the assumptions of the analysis of variance are valid, the null hypothesis of no differences in the \(c\) group means:

\[
H_0: \mu_1 = \mu_2 = \cdots = \mu_c
\]
is tested against the alternative that not all the $c$ group means are equal:

$$H_1: \text{Not all } \mu_j \text{ are equal (where } j = 1, 2, \ldots, c)$$

by computing the $F_{\text{STAT}}$ test statistic given in Equation (11.23).

\[
F_{\text{STAT}} = \frac{MSA}{MSE} \tag{11.23}
\]

The $F_{\text{STAT}}$ test statistic follows an $F$ distribution with $c - 1$ degrees of freedom for the $MSA$ term and $(r - 1)(c - 1)$ degrees of freedom for the $MSE$ term. For a given level of significance $\alpha$, you reject the null hypothesis if the computed $F_{\text{STAT}}$ test statistic is greater than the upper-tail critical value, $F_a$, from the $F$ distribution with $c - 1$ and $(r - 1)(c - 1)$ degrees of freedom (see Table E.5). The decision rule is:

- Reject $H_0$ if $F_{\text{STAT}} > F_a$;
- otherwise, do not reject $H_0$.

To examine whether the randomized block design was advantageous to use, some statisticians suggest that you perform the $F$ test for block effects. The null hypothesis of no block effects:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_r$$

is tested against the alternative:

$$H_1: \text{Not all } \mu_i \text{ are equal (where } i = 1, 2, \ldots, r)$$

using the $F_{\text{STAT}}$ test statistic for block effect given in Equation (11.24).

\[
F_{\text{STAT}} = \frac{MSBL}{MSE} \tag{11.24}
\]

You reject the null hypothesis at the $\alpha$ level of significance if the computed $F_{\text{STAT}}$ test statistic is greater than the upper-tail critical value $F_a$ from the $F$ distribution with $r - 1$ and $(r - 1)(c - 1)$ degrees of freedom (see Table E.5). That is, the decision rule is:

- Reject $H_0$ if $F_{\text{STAT}} > F_a$;
- otherwise, do not reject $H_0$.

The results of the analysis-of-variance procedure are usually displayed in an ANOVA summary table, as shown in Table 11.10.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square (Variance)</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Among groups ($A$)</td>
<td>$c - 1$</td>
<td>SSA</td>
<td>$MSA = \frac{SSA}{c - 1}$</td>
<td>$F_{\text{STAT}} = \frac{MSA}{MSE}$</td>
</tr>
<tr>
<td>Among blocks ($BL$)</td>
<td>$r - 1$</td>
<td>SSBL</td>
<td>$MSBL = \frac{SSBL}{r - 1}$</td>
<td>$F_{\text{STAT}} = \frac{MSBL}{MSE}$</td>
</tr>
<tr>
<td>Error</td>
<td>$(r - 1)(c - 1)$</td>
<td>SSE</td>
<td>$MSE = \frac{SSE}{(r - 1)(c - 1)}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$rc - 1$</td>
<td>SST</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 11.10**

Analysis-of-Variance Table for the Randomized Block Design
To illustrate the randomized block design, suppose that a fast-food chain wants to evaluate the service at four restaurants. The customer service director for the chain hires six evaluators with varied experiences in food-service evaluations to act as raters. To reduce the effect of the variability from rater to rater, you use a randomized block design, with raters serving as the blocks. The four restaurants are the groups of interest.

The six raters evaluate the service at each of the four restaurants in a random order. A rating scale from 0 (low) to 100 (high) is used. Table 11.11 summarizes the results (stored in FChain), along with the group totals, group means, block totals, block means, grand total, and grand mean.

<table>
<thead>
<tr>
<th>Raters</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Totals</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>61</td>
<td>82</td>
<td>74</td>
<td>287</td>
<td>71.75</td>
</tr>
<tr>
<td>2</td>
<td>77</td>
<td>75</td>
<td>88</td>
<td>76</td>
<td>316</td>
<td>79.00</td>
</tr>
<tr>
<td>3</td>
<td>76</td>
<td>67</td>
<td>90</td>
<td>80</td>
<td>313</td>
<td>78.25</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>63</td>
<td>96</td>
<td>76</td>
<td>315</td>
<td>78.75</td>
</tr>
<tr>
<td>5</td>
<td>84</td>
<td>66</td>
<td>92</td>
<td>84</td>
<td>326</td>
<td>81.50</td>
</tr>
<tr>
<td>6</td>
<td>78</td>
<td>68</td>
<td>98</td>
<td>86</td>
<td>330</td>
<td>82.50</td>
</tr>
<tr>
<td>Totals</td>
<td>465</td>
<td>400</td>
<td>546</td>
<td>476</td>
<td>1,887</td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>77.50</td>
<td>66.67</td>
<td>91.00</td>
<td>79.33</td>
<td>78.625</td>
<td></td>
</tr>
</tbody>
</table>

In addition, from Table 11.11,

\[ r = 6 \quad c = 4 \quad n = rc = 24 \]

and

\[
\bar{X} = \frac{\sum_{j=1}^{c} \sum_{i=1}^{r} X_{ij}}{rc} = \frac{1,887}{24} = 78.625
\]

Figure 11.18 shows a worksheet solution for this randomized block design.
Using the 0.05 level of significance to test for differences among the restaurants, you reject the null hypothesis \((H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4)\) if the computed \(F_{\text{STAT}}\) test statistic is greater than 3.29, the upper-tail critical value from the \(F\) distribution with 3 and 15 degrees of freedom in the numerator and denominator, respectively (see Figure 11.19).

Because \(F_{\text{STAT}} = 39.7581 > F_a = 3.29\), or because the \(p\)-value \(= 0.000 < 0.05\), you reject \(H_0\) and conclude that there is evidence of a difference in the mean ratings among the different restaurants. The extremely small \(p\)-value indicates that if the means from the four restaurants are equal, there is virtually no chance that you will get differences as large or larger among the sample means, as observed in this study. Thus, there is little degree of belief in the null hypothesis. You conclude that the alternative hypothesis is correct: The mean ratings among the four restaurants are different.

As a check on the effectiveness of blocking, you can test for a difference among the raters. The decision rule, using the 0.05 level of significance, is to reject the null hypothesis \((H_0: \mu_1 = \mu_2 = \cdots = \mu_6)\) if the computed \(F_{\text{STAT}}\) test statistic is greater than 2.90, the upper-tail critical value from the \(F\) distribution with 5 and 15 degrees of freedom (see Figure 11.20). Because \(F_{\text{STAT}} = 3.7818 > F_a = 2.90\) or because the \(p\)-value \(= 0.0205 < 0.05\), you reject \(H_0\) and conclude that there is evidence of a difference among the raters. Thus, you conclude that the blocking has been advantageous in reducing the random error.

The assumptions of the one-way analysis of variance (randomness and independence, normality, and homogeneity of variance) also apply to the randomized block design. If the normality assumption is violated, you can use the Friedman rank test (see references 2 and 3). In addition, you need to assume that there is no interacting effect between the groups and the blocks. In other words, you need to assume that any differences between the groups (the restaurants) are consistent across the entire set of blocks (the raters). The concept of interaction is discussed further in Section 11.2.

Did the blocking result in an increase in precision in comparing the different groups? To answer this question, use Equation (11.25) to calculate the estimated relative efficiency (\(RE\)) of the randomized block design as compared with the completely randomized design.

\[
RE = \frac{(r - 1)MSBL + r(c - 1)MSE}{(rc - 1)MSE}
\] (11.25)
11.3 The Randomized Block Design

Using Figure 11.18,

\[ RE = \frac{(5)(56.675) + (6)(3)(14.986)}{(23)(14.986)} = 1.60 \]

This value for relative efficiency means that it would take 1.6 times as many observations in a one-way ANOVA design as compared to the randomized block design in order to have the same precision in comparing the restaurants.

**Multiple Comparisons: The Tukey Procedure**

As in the case of the completely randomized design, once you reject the null hypothesis of no differences between the groups, you need to determine which groups are significantly different from the others. For the randomized block design, you can use a procedure developed by Tukey (see reference 9). Equation (11.26) gives the critical range for the Tukey multiple comparisons procedure for randomized block designs.

\[
\text{Critical range} = Q_a \sqrt{\frac{MSE}{r}} \quad (11.15)
\]

where \( Q_a \) is the upper-tail critical value from a Studentized range distribution having \( c \) degrees of freedom in the numerator and \( (r - 1)(c - 1) \) degrees of freedom in the denominator. Values for the Studentized range distribution are found in Table E.7.

To perform the multiple comparisons, you do the following:

1. Compute the absolute mean differences, \( |\bar{X}_j - \bar{X}_{j'}| \), (where \( j \neq j' \)), among all \( c(c - 1)/2 \) pairs of sample means.
2. Compute the critical range for the Tukey procedure using Equation (11.26).
3. Compare each of the \( c(c - 1)/2 \) pairs against the critical range. If the absolute difference in a specific pair of sample means, say \( |\bar{X}_j - \bar{X}_{j'}| \), is greater than the critical range, then group \( j \) and group \( j' \) is significantly different.
4. Interpret the results.

To apply the Tukey procedure, return to the fast-food chain study. Because there are four restaurants, there are \( 4(4 - 1)/2 = 6 \) possible pairwise comparisons. From Figure 11.18, the absolute mean differences are

1. \( |\bar{X}_1 - \bar{X}_2| = |77.50 - 66.67| = 10.83 \)
2. \( |\bar{X}_1 - \bar{X}_3| = |77.50 - 91.00| = 13.50 \)
3. \( |\bar{X}_1 - \bar{X}_4| = |77.50 - 79.33| = 1.83 \)
4. \( |\bar{X}_2 - \bar{X}_3| = |66.67 - 91.00| = 24.33 \)
5. \( |\bar{X}_2 - \bar{X}_4| = |66.67 - 79.33| = 12.66 \)
6. \( |\bar{X}_3 - \bar{X}_4| = |91.00 - 79.33| = 11.67 \)

Locate \( MSE = 14.986 \) and \( r = 6 \) in Figure 11.18 to determine the critical range. From Table E.7 [for \( \alpha = .05, c = 4, \) and \( (r - 1)(c - 1) = 15 \), \( Q_a \)], the upper-tail critical value of the test statistic with 4 and 15 degrees of freedom, is 4.08. Using Equation (11.26),

\[
\text{Critical range} = 4.08 \sqrt{\frac{14.986}{6}} = 6.448
\]
All pairwise comparisons except $|\bar{X}_1 - \bar{X}_4|$ are greater than the critical range. Therefore, you conclude with 95% confidence that there is evidence of a significant difference in the mean rating between all pairs of restaurant branches except for branches $A$ and $D$. In addition, branch $C$ has the highest ratings (i.e., is most preferred) and branch $B$ has the lowest (i.e., is least preferred).

Problems for Section 11.3

**LEARNING THE BASICS**

11.50 Given a randomized block experiment with five groups and seven blocks, answer the following:
   a. How many degrees of freedom are there in determining the among-group variation?
   b. How many degrees of freedom are there in determining the among-block variation?
   c. How many degrees of freedom are there in determining the random variation?
   d. How many degrees of freedom are there in determining the total variation?

11.51 From Problem 11.50,
   a. if $SSA = 60$, $SSBL = 75$, and $SST = 210$, what is $SSE$?
   b. what are $MSA$, $MSBL$, and $MSE$?
   c. what is the value of the $F_{STAT}$ test statistic for the factor effect?
   d. what is the value of the $F_{STAT}$ test statistic for the block effect?

11.52 From Problems 11.50 and 11.51,
   a. construct the ANOVA summary table and fill in all values in the body of the table.
   b. at the 0.05 level of significance, is there evidence of a difference in the group means?
   c. at the 0.05 level of significance, is there evidence of a difference due to blocks?

11.53 From Problems 11.50, 11.51, and 11.52,
   a. to perform the Tukey procedure, how many degrees of freedom are there in the numerator, and how many degrees of freedom are there in the denominator of the Studentized range distribution?
   b. at the 0.05 level of significance, what is the upper-tail critical value from the Studentized range distribution?
   c. to perform the Tukey procedure, what is the critical range?

11.54 Given a randomized block experiment with three groups and seven blocks,
   a. how many degrees of freedom are there in determining the among-group variation?
   b. how many degrees of freedom are there in determining the among-block variation?
   c. how many degrees of freedom are there in determining the random variation?
   d. how many degrees of freedom are there in determining the total variation?

11.55 From Problem 11.54, if $SSA = 36$ and the randomized block $F_{STAT}$ statistic is 6.0,
   a. what are $MSE$ and $SSE$?
   b. what is $SSBL$ if the $F_{STAT}$ test statistic for block effect is 4.0?
   c. what is $SST$?
   d. at the 0.01 level of significance, is there evidence of an effect due to groups, and is there evidence of an effect due to blocks?

11.56 Given a randomized block experiment with four groups and eight blocks, in the following ANOVA summary table, fill in all the missing results.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square (Variance)</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Among groups</td>
<td>$c - 1$ = ?</td>
<td>$SSA$ = ?</td>
<td>$MSA = 80$ $F_{STAT} = ?$</td>
<td></td>
</tr>
<tr>
<td>Among blocks</td>
<td>$r - 1$ = ?</td>
<td>$SSBL = 540$</td>
<td>$MSBL = ? F_{STAT} = 5.0$</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>$(r - 1)(c - 1)$ = ?</td>
<td>$SSE = ?$</td>
<td>$MSE = ?$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$rc - 1$ = ?</td>
<td>$SST = ?$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11.57 From Problem 11.56,
   a. at the 0.05 level of significance, is there evidence of a difference among the four group means?
   b. at the 0.05 level of significance, is there evidence of an effect due to blocks?

**APPLYING THE CONCEPTS**

11.58 Nine experts rated four brands of Colombian coffee in a taste-testing experiment. A rating on a 7-point scale ($1 = \text{extremely unpleasing}$, $7 = \text{extremely pleasing}$) is given for each of four characteristics: taste, aroma, richness, and acidity. The following data (stored in Coffee) display the summated ratings, accumulated over all four characteristics.
11.3 The Randomized Block Design

a. At the 0.05 level of significance, determine whether there is evidence of a difference in the mean prices for these kitchen staples at the four supermarkets.

b. What assumptions are necessary to perform this test?

c. If appropriate, use the Tukey procedure to determine which supermarkets differ.

d. Do you think that there was a significant block effect in this experiment? Explain.

11.61 An article (J. Graham, “Prices Going Up, but It’s Not Gas; It’s Online Music,” USA Today, May 13, 2004, p. 1B) states that the cost of a legitimate music download is increasing. The following data (stored in ) represent the prices of five albums at five digital music services.

<table>
<thead>
<tr>
<th>Album/Artist</th>
<th>iTunes</th>
<th>Wal-Mart</th>
<th>Music Now</th>
<th>Music Match</th>
<th>Napster</th>
</tr>
</thead>
</table>

Source: Extracted from J. Graham, “Prices Going Up, but It’s Not Gas; It’s Online Music,” USA Today, May 13, 2004, p. 1B.

a. At the 0.05 level of significance, determine whether there is evidence of a difference in the mean prices for albums at the five digital music services.

b. What assumptions are necessary to perform this test?

c. If appropriate, use the Tukey procedure to determine which digital music services differ. (Use $\alpha = 0.05$.)

d. Do you think that there was a significant block effect in this experiment? Explain.

11.62 Philips Semiconductors is a leading European manufacturer of integrated circuits. Integrated circuits are...
produced on silicon wafers, which are ground to target thickness early in the production process. The wafers are positioned in different locations on a grinder and kept in place using vacuum decompression. One of the goals of process improvement is to reduce the variability in the thickness of the wafers in different positions and in different batches. Data were collected from a sample of 30 batches. In each batch, the thickness of the wafers on positions 1 and 2 (outer circle), 18 and 19 (middle circle), and 28 (inner circle) was measured and stored in Circuits. At the 0.01 level of significance, completely analyze the data to determine whether there is evidence of a difference in the mean thickness of the wafers for the five positions and, if so, which of the positions are different. What can you conclude?


11.63 The data in Concrete3 represent the compressive strength in thousands of pounds per square inch of 40 samples of concrete taken 2, 7, and 28 days after pouring.


a. At the 0.05 level of significance, is there evidence of a difference in the mean compressive strength after 2, 7, and 28 days?
b. If appropriate, use the Tukey procedure to determine the days that differ in mean compressive strength. (Use \( \alpha = 0.05 \).)
c. Determine the relative efficiency of the randomized block design as compared with the completely randomized (one-way ANOVA) design.
d. Construct boxplots of the compressive strength for the different time periods.
e. Based on the results of (a), (b), and (d), is there a pattern in the compressive strength over the three time periods?

EG11.3 THE RANDOMIZED BLOCK DESIGN EXCEL GUIDE

In-Depth Excel Use the FINV, FDIST, and DEVSQ functions to help perform the two-way ANOVA needed for a randomized block design. (These functions appear in the ANOVA summary table area of the worksheet.) Enter \( \text{FINV} \) (level of significance, degrees of freedom for source, Error degrees of freedom) to compute the F critical value for the among-groups (A) and among-blocks (BL) sources of variation (see Table 11.9). Enter \( \text{FDIST} \) (F test statistic for source, degrees of freedom for rows, Error degrees of freedom within groups) to calculate the p-value for the two sources of variation.

Enter \( \text{DEVSQ} \) (cell range of all data) to compute \( \text{SSA} \). Enter a formula that subtracts the list of DEVSQs for each block from \( \text{SST} \) to compute \( \text{SSBL} \). Enter a formula that subtracts the list of DEVSQs for each group from \( \text{SST} \) to compute \( \text{SSA} \). To compute \( \text{SSE} \), subtract \( \text{SSA} \) and \( \text{SSBL} \) from \( \text{SST} \).

Use the COMPUTE worksheet of the Randomized Block workbook, shown in Figure 11.18, as a model for analyzing randomized block designs. The worksheet uses the data for the fast-food chain study example of Section 11.3 that is in the Data worksheet. In the ANOVA summary table of this worksheet, the source labeled Among groups (A) in Table 11.10 is labeled Columns, and the source Among blocks (BL) is labeled Rows.

Open to the COMPUTE_FORMULAS worksheet to examine the details of other formulas used in the COMPUTE worksheet. Modifying the COMPUTE worksheet workbook for use with other problems is both challenging and discouraged. To attempt a modification, study and then modify the Section EG11.2 In-Depth Excel instructions or, better, use the Analysis ToolPak instructions to create the worksheet results.

Analysis ToolPak Use the Anova: Two-Factor Without Replication procedure to analyze a randomized block design. This procedure requires that the labels that identify blocks appear stacked in column A and that group names appear in row 1, starting with cell B1.

For example, to create a worksheet similar to Figure 11.18 that analyzes the randomized block design for the fast-food chain study example of Section 11.3, open to the DATA worksheet of the FFChain workbook:

1. Select Data ➔ Data Analysis (Excel 2007) or Tools ➔ Data Analysis (Excel 2003).
2. In the Data Analysis dialog box, select Anova: Two-Factor Without Replication from the Analysis Tools list and then click OK.

In the procedure’s dialog box (see below):

3. Enter A1:E7 as the Input Range.
4. Check Labels and enter 0.05 as Alpha.
5. Click New Worksheet Ply.
6. Click OK to create the worksheet.

The Analysis ToolPak creates a worksheet that is visually similar to Figure 11.18 but contains only values and does not include any cell formulas. The ToolPak worksheet also does not contain the level of significance in row 34.