8.4 Argument Forms and Refutation by Logical Analogy

The central task of deductive logic, we have said, is discriminating valid arguments from invalid ones. If the premises of a valid argument are true (we explained in the very first chapter), its conclusion must be true. If the conclusion of a valid argument is false, at least one of the premises must be false. In short, the premises of a valid argument give incontrovertible proof of the conclusion drawn.

This informal account of validity must now be made more precise. To do this we introduce the concept of an argument form. Consider the following two arguments, which plainly have the same logical form. Suppose we are presented with the first of these arguments:

If Bacon wrote the plays attributed to Shakespeare, then Bacon was a great writer.
Bacon was a great writer.
Therefore Bacon wrote the plays attributed to Shakespeare.

We may agree with the premises but disagree with the conclusion, judging the argument to be invalid. One way of proving invalidity is by the method of logical analogy. “You might as well argue,” we could retort, “that
If Washington was assassinated, then Washington is dead.
Washington is dead.
Therefore Washington was assassinated.

And you cannot seriously defend this argument,” we would continue, “because here the premises are known to be true and the conclusion is known to be false. This argument is obviously invalid; your argument is of the same form, so yours is also invalid.” This type of refutation is very effective.

This method of refutation by logical analogy points the way to an excellent general technique for testing arguments. To prove the invalidity of an argument, it suffices to formulate another argument that (1) has exactly the same form as the first and (2) has true premises and a false conclusion. This method is based on the fact that validity and invalidity are purely formal characteristics of arguments, which is to say that any two arguments that have exactly the same form are either both valid or both invalid, regardless of any differences in the subject matter with which they are concerned.*

*Here we assume that the simple statements involved are neither logically true (e.g., “All chairs are chairs”) nor logically false (e.g., “Some chairs are nonchairs”). We also assume that the only logical relations among the simple statements involved are those asserted or entailed by the premises. The point of these restrictions is to limit our considerations, in this chapter and the next, to truth-functional arguments alone, and to exclude other kinds of arguments whose validity turns on more complex logical considerations that are not appropriately introduced at this point.
A given argument exhibits its form very clearly when the simple statements that appear in it are abbreviated by capital letters. Thus we may abbreviate the statements, “Bacon wrote the plays attributed to Shakespeare,” “Bacon was a great writer,” “Washington was assassinated,” and “Washington is dead,” by the letters $B, G, A$, and $D$, respectively, and using the familiar three-dot symbol “∴” for “therefore,” we may symbolize the two preceding arguments as

\[
B \supset G \quad A \supset D
\]

\[
B \quad \text{and} \quad D \\
\therefore B \quad \therefore A
\]

So written, their common form is easily seen.

To discuss forms of arguments rather than particular arguments having those forms, we need some method of symbolizing argument forms themselves. To achieve such a method, we introduce the notion of a variable. In the preceding sections we used capital letters to symbolize particular simple statements. To avoid confusion, we use small, or lowercase, letters from the middle part of the alphabet, $p, q, r, s, \ldots$, as statement variables. A statement variable, as we shall use the term, is simply a letter for which, or in place of which, a statement may be substituted. Compound statements as well as simple statements may be substituted for statement variables.

We define an argument form as any array of symbols containing statement variables but no statements, such that when statements are substituted for the statement variables—the same statement being substituted for the same statement variable throughout—the result is an argument. For definiteness, we establish the convention that in any argument form, $p$ shall be the first statement variable that occurs in it, $q$ shall be the second, $r$ the third, and so on. Thus the expression

\[
p \supset q \quad q \quad \therefore p
\]

is an argument form, for when the statements $B$ and $G$ are substituted for the statement variables $p$ and $q$, respectively, the result is the first argument in this section. If the statements $A$ and $D$ are substituted for the variables $p$ and $q$, the result is the second argument. Any argument that results from the substitution of statements for statement variables in an argument form is called a substitution instance of that argument form. It is clear that any substitution instance of an argument form may be said to have that form, and that any argument that has a certain form is a substitution instance of that form.
For any argument there are usually several argument forms that have the given argument as a substitution instance. For example, the first argument of this section,

\[ B \supset G \]

\[ G \]

\[ \therefore B \]

is a substitution instance of each of the four argument forms

\[ p \supset q \quad p \supset q \quad p \supset q \quad p \]

\[ q \quad r \quad r \quad q \]

\[ \therefore p \quad \therefore p \quad \therefore s \quad \therefore r \]

Thus we obtain the given argument by substituting \( B \) for \( p \) and \( G \) for \( q \) in the first argument form; by substituting \( B \) for \( p \) and \( G \) for both \( q \) and \( r \) in the second; \( B \) for both \( p \) and \( s \) and \( G \) for both \( q \) and \( r \) in the third; and \( B \supset G \) for \( p \), \( G \) for \( q \), and \( B \) for \( r \) in the fourth. Of these four argument forms, the first corresponds more closely to the structure of the given argument than do the others. It does so because the given argument results from the first argument form by substituting a different simple statement for each different statement variable in it. We call the first argument form the specific form of the given argument. Our definition of the specific form of a given argument is the following: If an argument is produced by substituting consistently a different simple statement for each different statement variable in an argument form, that argument form is the specific form of that argument. For any given argument, there is a unique argument form that is the specific form of that argument.

**EXERCISES**

Here follows a group of arguments (Group A, lettered a–o) and a group of argument forms (Group B, numbered 1–24). For each of the arguments (in Group A), indicate which of the argument forms (in Group B), if any, have the given argument as a substitution instance. In addition, for each given argument (in Group A), indicate which of the argument forms (in Group B), if any, is the specific form of that argument.

**EXAMPLES**

Argument a in Group A: Examining all the argument forms in Group B, we find that the only one of which Argument a is a substitution instance is Number 3. Number 3 is also the specific form of Argument a.
Argument j in Group A: Examining all the argument forms in Group B, we find that Argument j is a substitution instance of both Number 6 and Number 23. But only Number 23 is the specific form of Argument j.

Argument m in Group A: Examining all the argument forms in Group B, we find that Argument m is a substitution instance of both Number 3 and Number 24. But there is no argument form in Group B that is the specific form of Argument m.

Group A—Arguments

a. \[A \cdot B\] \[\therefore A\]
b. \[C \cdot D\] \[\therefore C \cdot (C \cdot D)\]
c. \[E\] \[\therefore E \lor F\]
d. \[G \lor H\] \[\therefore \sim H\]
   \[\therefore \sim G\]
e. \[I\] \[\therefore I \cdot J\]
f. \[(K \cdot L) \cdot (M \cdot N)\] \[\therefore K \lor M\]
   \[\therefore L \lor N\]
g. \[O \lor P\] \[\therefore \sim O\]
   \[\therefore \sim P\]
h. \[Q \lor R\] \[\therefore Q \cdot S\]
   \[\therefore R \lor S\]
i. \[(T \cdot U)\] \[\therefore U \lor V\]
   \[\therefore V \lor T\]
j. \[(W \cdot X) \lor (Y \cdot Z)\] \[\therefore (W \cdot X) \lor [(W \cdot X) \cdot (Y \cdot Z)]\]
l. \[(D \lor E) \cdot \sim F\] \[\therefore D \lor E\]
m. \[G \lor (G \cdot H)\] \[\therefore G \lor (G \cdot H)\]
   \[\therefore G \lor (G \cdot H)\]
o. \[(K \cdot L) \cdot (M \cdot N)\] \[\therefore K \lor L\]

Group B—Argument Forms

*1. \[p \lor q\] \[\therefore \sim q \lor \sim p\]

2. \[p \lor q\] \[\therefore \sim p \lor \sim q\]

3. \[p \cdot q\] \[\therefore p\]

4. \[p\] \[\therefore p \lor q\]

*5. \[p\] \[\therefore p \lor q\]

6. \[p \lor q\] \[\therefore p \lor (p \cdot q)\]

7. \[(p \cdot q) \lor (p \cdot q)\] \[\therefore (p \lor q) \lor (q \lor p)\]

8. \[p \lor q\] \[\therefore \sim p \lor \sim q\]

10. \[p\] \[\therefore p \lor q\]

11. \[p \lor q\] \[\therefore p \lor r\]
   \[\therefore q \lor r\]

12. \[p \lor q\] \[\therefore q \lor r\]
   \[\therefore r \lor p\]

13. \[p \lor (q \lor r)\] \[\therefore p \lor q\]
   \[\therefore p \lor r\]

14. \[p \lor (q \lor r)\] \[\therefore p \lor (q \lor r)\]
   \[\therefore \sim p \lor \sim q \lor \sim r\]
   \[\therefore \sim p \lor \sim q \lor \sim r\]
8.5 The Precise Meaning of “Invalid” and “Valid”

We are now in a position to address with precision the central questions of deductive logic:

1. *What precisely is meant* by saying that an argument form is invalid, or valid?

2. *How do we decide* whether a deductive argument form is invalid, or valid?

The first of these questions is answered in this section, the second in the following section.

We proceed by using the technique of refutation by logical analogy.* If the specific form of a given argument has any substitution instance whose premises are true and whose conclusion is false, then the given argument is invalid. We may define the term invalid as applied to argument forms as follows: An argument form is invalid if and only if it has at least one substitution instance with true premises and a false conclusion. Refutation by logical analogy is based on the fact that any argument whose specific form is an invalid argument form is an invalid argument. Any argument form that is not invalid must be valid. Hence an argument form is valid if and only if it has no substitution instances with true premises and a false conclusion. And because validity is a formal notion, an argument is valid if and only if the specific form of that argument is a valid argument form.

A given argument is proved invalid if a refuting analogy can be found for it, but “thinking up” such refuting analogies may not always be easy. Happily, it is not necessary, because for arguments of this type there is a simpler, purely

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*Just as in analyzing the categorical syllogism; we discuss refutation by logical analogy in Section 6.2.