Categorical Syllogisms

6.1 Standard-Form Categorical Syllogisms

We are now in a position to use categorical propositions in more extended reasoning. Arguments that rely on A, E, I, and O propositions commonly have two categorical propositions as premises and one categorical proposition as a conclusion. Such arguments are called syllogisms; a syllogism is, in general, a deductive argument in which a conclusion is inferred from two premises.

The syllogisms with which we are concerned here are called categorical because they are arguments based on the relations of classes, or categories—relations that are expressed by the categorical propositions with which we are familiar. More formally, we define a categorical syllogism as a deductive argument consisting of three categorical propositions that together contain exactly three terms, each of which occurs in exactly two of the constituent propositions.

Syllogisms are very common, very clear, and readily testable. The system of categorical syllogisms that we will explore is powerful and deep. The seventeenth-century philosopher and mathematician Gottfried Leibniz said, of the invention of the form of syllogisms, that it was “one of the most beautiful and also one of the most important made by the human mind.” Syllogisms are the workhorse arguments with which deductive logic, as traditionally practiced, has been made effective in writing and in controversy.

It will be convenient to have an example to use as we discuss the parts and features of the syllogism. Here is a valid standard-form categorical syllogism that we shall use as illustrative:

No heroes are cowards.
Some soldiers are cowards.
Therefore some soldiers are not heroes.
To analyze such an argument accurately, it needs to be in standard form. A categorical syllogism is said to be in standard form (as the above sample is) when two things are true of it: (1) its premises and its conclusion are all standard-form categorical propositions (A, E, I, or O); and (2) those propositions are arranged in a specified standard order. The importance of this standard form will become evident when we turn to the task of testing the validity of syllogisms.

To explain the order of the premises that is required to put any syllogism into standard form, we need the logical names of the premises of the syllogism, and the names of the terms of the syllogism, and we must understand why those names—very useful and very important—are assigned to them. This is the next essential step in our analysis of categorical syllogisms.*

A. TERMS OF THE SYLLOGISM: MAJOR, MINOR, AND MIDDLE

The three categorical propositions in our example argument above contain exactly three terms: heroes, soldiers, and cowards. To identify the terms by name, we look to the conclusion of the syllogism, which of course contains exactly two terms. The conclusion in our sample is an O proposition, “Some soldiers are not heroes.” The term that occurs as the predicate of the conclusion (“heroes,” in this case) is called the major term of the syllogism. The term that occurs as the subject of the conclusion (“soldiers” in this case) is called the minor term of the syllogism. The third term of the syllogism (“cowards” in this case), which never occurs in the conclusion but always appears in both premises, is called the middle term.

The premises of a syllogism also have names. Each premise is named after the term that appears in it. The major term and the minor term must each occur in a different premise. The premise containing the major term is called the major premise. In the example, “heroes” is the major term, so the premise containing “heroes”—“No heroes are cowards”—is the major premise. It is the major premise not because it appears first, but only because it is the premise that contains the major term; it would be the major premise no matter in what order the premises were written.

The premise containing the minor term is called the minor premise. In the example, “soldiers” is the minor term, so the premise containing “soldiers”—“Some soldiers are cowards”—is the minor premise. It is the minor premise not because of its position, but because it is the premise that contains the minor term.

*In this chapter, for the sake of brevity we will refer to categorical syllogisms simply as “syllogisms,” even though there are other kinds of syllogisms that will be discussed in later chapters.
A syllogism is in standard form, we said, when its premises are arranged in a specified standard order. Now we can state that order: In a standard-form syllogism, the major premise is always stated first, the minor premise second, and the conclusion last. The reason for the importance of this order will soon become clear.

B. THE MOOD OF THE SYLLOGISM

Every syllogism has a mood. The mood of a syllogism is determined by the types (A, E, I, or O) of standard-form categorical propositions it contains. The mood of the syllogism is therefore represented by three letters, and those three letters are always given in standard-form order. That is, the first letter names the type of the syllogism’s major premise; the second letter names the type of the syllogism’s minor premise; the third letter names the type of the syllogism’s conclusion. In our example syllogism, the major premise (“No heroes are cowards”) is an E proposition; the minor premise (“Some soldiers are cowards”) is an I proposition; the conclusion (“Some soldiers are not heroes”) is an O proposition. Therefore the mood of this syllogism is EIO.

C. THE FIGURE OF THE SYLLOGISM

The mood of a standard-form syllogism is not enough, by itself, to characterize its logical form. This can be shown by comparing two syllogisms, A and B, with the same mood, which are logically very different.

A. Major Term | Middle Term
---|---
All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.

B. Middle Term | Major Term
---|---
All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.
Both of these are of mood AII, but one of them is valid and the other is not. The difference in their forms can be shown most clearly if we display their logical “skeletons” by abbreviating the minor terms as S (subject of the conclusion), the major terms as P (predicate of the conclusion), and the middle terms as M. Using the three-dot symbol “∴” for “therefore,” we get these skeletons:

A. All P is M. Some S is M. ∴ Some S is P.
B. All M is P. Some M is S. ∴ Some S is P.

These are very different. In the one labeled A, the middle term, M, is the predicate term of both premises; but in the one labeled B, the middle term, M, is the subject term of both premises. Syllogism B will be seen to be a valid argument; syllogism A, on the other hand, is invalid.

These examples show that although the form of a syllogism is partially described by its mood (AII in both of these cases), syllogisms that have the same mood may differ importantly in their forms, depending on the relative positions of their middle terms. To describe the form of a syllogism completely we must state its mood (the three letters of its three propositions) and its figure—where by figure we mean the position of the middle term in its premises.

Syllogisms can have four—and only four—possible different figures:

1. The middle term may be the subject term of the major premise and the predicate term of the minor premise; or
2. The middle term may be the predicate term of both premises; or
3. The middle term may be the subject term of both premises; or
4. The middle term may be the predicate term of the major premise and the subject term of the minor premise.

These different possible positions of the middle term constitute the first, second, third, and fourth figures, respectively. Every syllogism must have one or another of these four figures. The characters of these figures may be visualized more readily when the figures are schematized as in the following array, in which reference to mood is suppressed and the quantifiers and copulas are not shown—but the relative positions of the terms of the syllogism are brought out:

\[
\begin{array}{cccc}
M \to P & P \to M & M \to P & P \to M \\
S \to M & S \to M & M \to S & M \to S \\
\therefore S \to P & \therefore S \to P & \therefore S \to P & \therefore S \to P \\
\text{First Figure} & \text{Second Figure} & \text{Third Figure} & \text{Fourth Figure}
\end{array}
\]
Any standard-form syllogism is completely described when we specify its mood and its figure. The syllogism we have been using as an example is in the second figure; “cowards,” the middle term, is the predicate term of both premises. Its mood, as we pointed out, is EIO. So it is completely described as being a syllogism of the form EIO–2. It is a valid syllogism, as we noted; every valid syllogistic form, as we shall see, has its own name. The name of this form, EIO–2, is Festino. We say of this syllogism that it is “in Festino.”

Here is another example:

No M is P
All S is M
\[ \therefore \text{No S is P} \]

This syllogism is in the first figure (its middle term is the subject of the major premise and the predicate of the minor premise); its mood is EAE. So we may characterize it completely as EAE–1, a form whose unique name is Celarent. Any syllogism of this form is “in Celarent,” just as any syllogism of the earlier form is “in Festino.” And because Celarent (EAE–1) and Festino (EIO–2) are known to be valid forms, we may conclude that whenever we encounter an argument in one of these forms, it too is valid.

With these analytical tools we can identify every possible categorical syllogism by mood and figure. If we were to list all the possible moods, beginning with AAA, AAE, AAI, AAO, AEA, AEE, . . ., and so on, continuing until every possibility had been named, we would eventually (upon reaching OOO) have enumerated sixty-four possible moods. Each mood can occur in each of the four figures; \( 4 \times 64 = 256 \). It is certain, therefore, that there are exactly 256 distinct forms that standard-form syllogisms may assume.

Of these 256 possible forms, as we shall see, only a few are valid forms. And each of those valid forms has a unique name, as will be explained.

**EXERCISES**

Rewrite each of the following syllogisms in standard form, and name its mood and figure. *(Procedure: first, identify the conclusion; second, note its predicate term, which is the major term of the syllogism; third, identify the major premise, which is the premise containing the major term; fourth, verify that the other premise is the minor premise by checking to see that it contains the minor term, which is the subject term of the conclusion; fifth, rewrite the argument in standard form—major premise first,*
minor premise second, conclusion last; sixth, name the mood and figure of the syllogism.)

EXAMPLE

1. No nuclear-powered submarines are commercial vessels, so no warships are commercial vessels, because all nuclear-powered submarines are warships.

SOLUTION
Step 1. The conclusion is “No warships are commercial vessels.”
Step 2. “Commercial vessels” is the predicate term of this conclusion and is therefore the major term of the syllogism.
Step 3. The major premise, the premise that contains this term, is “No nuclear-powered submarines are commercial vessels.”
Step 4. The remaining premise, “All nuclear-powered submarines are warships,” is indeed the minor premise, because it does contain the subject term of the conclusion, “warships.”
Step 5. In standard form this syllogism is written thus:
   No nuclear-powered submarines are commercial vessels.
   All nuclear-powered submarines are warships.
   Therefore no warships are commercial vessels.
Step 6. The three propositions in this syllogism are, in order, E, A, and E. The middle term, “nuclear-powered submarines,” is the subject term of both premises, so the syllogism is in the third figure. The mood and figure of the syllogism therefore are EAE–3.

2. Some evergreens are objects of worship, because all fir trees are evergreens, and some objects of worship are fir trees.

3. All artificial satellites are important scientific achievements; therefore some important scientific achievements are not U.S. inventions, inasmuch as some artificial satellites are not U.S. inventions.

4. No television stars are certified public accountants, but all certified public accountants are people of good business sense; it follows that no television stars are people of good business sense.

*5. Some conservatives are not advocates of high tariff rates, because all advocates of high tariff rates are Republicans, and some Republicans are not conservatives.
6. All CD players are delicate mechanisms, but no delicate mechanisms are suitable toys for children; consequently, no CD players are suitable toys for children.

7. All juvenile delinquents are maladjusted individuals, and some juvenile delinquents are products of broken homes; hence some maladjusted individuals are products of broken homes.

8. No stubborn individuals who never admit a mistake are good teachers, so, because some well-informed people are stubborn individuals who never admit a mistake, some good teachers are not well-informed people.

9. All proteins are organic compounds, hence all enzymes are proteins, as all enzymes are organic compounds.

*10. No sports cars are vehicles intended to be driven at moderate speeds, but all automobiles designed for family use are vehicles intended to be driven at moderate speeds, from which it follows that no sports cars are automobiles designed for family use.

6.2 The Formal Nature of Syllogistic Argument

In all deductive logic we aim to discriminate valid arguments from invalid ones; in classical logic this becomes the task of discriminating valid syllogisms from invalid ones. It is reasonable to assume that the constituent propositions of a syllogism are all contingent—that is, that no one of those propositions is necessarily true, or necessarily false. Under this assumption, the validity or invalidity of any syllogism depends entirely on its form. Validity and invalidity are completely independent of the specific content of the argument or its subject matter. Thus any syllogism of the form AAA–1

\[
\text{All } M \text{ is } P \\
\text{All } S \text{ is } M \\
\therefore \text{All } S \text{ is } P
\]

is valid, regardless of its subject matter. The name of this syllogism’s form is Barbara; no matter what terms are substituted for the letters S, P, and M, the resulting argument, “in Barbara,” will always be valid. If we substitute “Athenians” and “humans” for S and P, and “Greeks” for M, we obtain this valid argument:

All Greeks are humans.
All Athenians are Greeks.
\therefore \text{All Athenians are humans.}
And if we substitute the terms “soaps,” “water-soluble substances,” and “sodium salts” for the letters \(S, P,\) and \(M\) in the same form, we obtain

- All sodium salts are water-soluble substances.
- All soaps are sodium salts.
- Therefore all soaps are water-soluble substances.

which also is valid.

A valid syllogism is a formally valid argument—valid by virtue of its form alone. This implies that if a given syllogism is valid, \textit{any other syllogism of the same form will also be valid}. And if a syllogism is invalid, \textit{any other syllogism of the same form will also be invalid}. The common recognition of this fact is attested to by the frequent use of “logical analogies” in argumentation. Suppose that we are presented with the argument

- All liberals are proponents of national health insurance.
- Some members of the administration are proponents of national health insurance.
- Therefore some members of the administration are liberals.

and felt (justifiably) that, regardless of the truth or falsehood of its constituent propositions, the argument is invalid. The best way to expose its fallacious character is to construct another argument that has exactly the same form but whose invalidity is immediately apparent. We might seek to expose the given argument by replying: You might as well argue that

- All rabbits are very fast runners.
- Some horses are very fast runners.
- Therefore some horses are rabbits.

We might continue: You cannot seriously defend this argument, because here there is no question about the facts. The premises are known to be true and the conclusion is known to be false. Your argument is of the same

\[\text{We assume, as noted above, that the constituent propositions are themselves contingent, that is, neither logically true (e.g., “All easy chairs are chairs”) nor logically false (e.g., “Some easy chairs are not chairs”). The reason for the assumption is this: If it contained either a logically false premise or a logically true conclusion, then the argument would be valid regardless of its syllogistic form—valid in that it would be logically impossible for its premises to be true and its conclusion false. We also assume that the only logical relations among the terms of the syllogism are those asserted or entailed by its premises. The point of these restrictions is to limit our considerations in this chapter and the next to syllogistic arguments alone and to exclude other kinds of arguments whose validity turns on more complex logical considerations that are not appropriate to introduce at this place.}\]
pattern as this analogous one about horses and rabbits. This one is invalid—so your argument is invalid. This is an excellent method of arguing; the logical analogy is one of the most powerful weapons that can be used in debate.

Underlying the method of logical analogy is the fact that the validity or invalidity of such arguments as the categorical syllogism is a purely formal matter. Any fallacious argument can be proved to be invalid by finding a second argument that has exactly the same form and is known to be invalid by the fact that its premises are known to be true while its conclusion is known to be false. (It should be remembered that an invalid argument may very well have a true conclusion—that an argument is invalid simply means that its conclusion is not logically implied or necessitated by its premises.)

This method of testing the validity of arguments has serious limitations, however. Sometimes a logical analogy is difficult to “think up” on the spur of the moment. And there are far too many invalid forms of syllogistic argument (well over two hundred!) for us to prepare and remember refuting analogies of each of them in advance. Moreover, although being able to think of a logical analogy with true premises and false conclusion proves its form to be invalid, not being able to think of one does not prove the form valid, for it may merely reflect the limitations of our thinking. There may be an invalidating analogy even though we are not able to think of it. A more effective method of establishing the formal validity or invalidity of syllogisms is required. The explanation of effective methods of testing syllogisms is the object of the remaining sections of this chapter.

**EXERCISES**

Refute any of the following arguments that are invalid by the method of constructing logical analogies.

**EXAMPLE**

1. All business executives are active opponents of increased corporation taxes, for all active opponents of increased corporation taxes are members of the chamber of commerce, and all members of the chamber of commerce are business executives.

**SOLUTION**

One possible refuting analogy is this: All bipeds are astronauts, for all astronauts are humans and all humans are bipeds.
2. No medicines that can be purchased without a doctor’s prescription are habit-forming drugs, so some narcotics are not habit-forming drugs, because some narcotics are medicines that can be purchased without a doctor’s prescription.

3. No Republicans are Democrats, so some Democrats are wealthy stockbrokers, because some wealthy stockbrokers are not Republicans.

4. No college graduates are persons having an IQ of less than 70, but all persons who have an IQ of less than 70 are morons, so no college graduates are morons.

5. All fireproof buildings are structures that can be insured at special rates, so some structures that can be insured at special rates are not wooden houses, because no wooden houses are fireproof buildings.

6. All blue-chip securities are safe investments, so some stocks that pay a generous dividend are safe investments, because some blue-chip securities are stocks that pay a generous dividend.

7. Some pediatricians are not specialists in surgery, so some general practitioners are not pediatricians, because some general practitioners are not specialists in surgery.

8. No intellectuals are successful politicians, because no shy and retiring people are successful politicians, and some intellectuals are shy and retiring people.

9. All trade union executives are labor leaders, so some labor leaders are conservatives in politics, because some conservatives in politics are trade union executives.

10. All new automobiles are economical means of transportation, and all new automobiles are status symbols; therefore some economical means of transportation are status symbols.

6.3 Venn Diagram Technique for Testing Syllogisms

In Chapter 5 we explained the use of two-circle Venn diagrams to represent standard-form categorical propositions. In order to test a categorical syllogism using Venn diagrams, one must first represent both of its premises in one diagram. That requires drawing three overlapping circles, for the two premises of a standard-form syllogism contain three different terms—minor term, major term, and middle term—which we abbreviate as $S$, $P$, and $M$, respectively.
We first draw two circles, just as we did to diagram a single proposition, and then we draw a third circle beneath, overlapping both of the first two. We label the three circles $S$, $P$, and $M$, in that order. Just as one circle labeled $S$ diagrammed both the class $S$ and the class $\bar{S}$, and as two overlapping circles labeled $S$ and $P$ diagrammed four classes ($SP$, $SP\bar{P}$, $S\bar{P}$, and $S\bar{P}\bar{M}$), three overlapping circles, labeled $S$, $P$, and $M$, diagram eight classes: $SP\bar{M}$, $S\bar{P}M$, $SP\bar{M}$, $S\bar{P}M$, $SP\bar{M}$, $S\bar{P}M$, $SP\bar{M}$, and $S\bar{P}M$. These are represented by the eight parts into which the three circles divide the plane, as shown in Figure 6-1.

Figure 6-1 can be interpreted, for example, in terms of the various different classes determined by the class of all Swedes ($S$), the class of all peasants ($P$), and the class of all musicians ($M$). $SPM$ is the product of these three classes, which is the class of all Swedish peasant musicians. $S\bar{P}M$ is the product of the first two and the complement of the third, which is the class of all Swedish peasants who are not musicians. $S\bar{P}M$ is the product of the first and third and the complement of the second: the class of all Swedish musicians who are not peasants. $S\bar{P}M$ is the product of the first and the complement of the others: the class of all Swedes who are neither peasants nor musicians. Next, $S\bar{P}M$ is the product of the second and third classes with the complement of the first: the class of all peasant musicians who are not Swedes. $S\bar{P}M$ is the product of the second class with the complements of the other two: the class of all peasants who are neither Swedes nor musicians. $S\bar{P}M$ is the product of the third class and the complements of the first two: the class of all musicians who are neither Swedes nor peasants. Finally, $S\bar{P}M$ is the product of the complements of the three original classes: the class of all things that are neither Swedes nor peasants nor musicians.

If we focus our attention on just the two circles labeled $P$ and $M$, it is clear that by shading out, or by inserting an $x$, we can diagram any standard-form categorical proposition whose two terms are $P$ and $M$, regardless of which is the subject term and which is the predicate. Thus, to diagram the proposition “All $M$ is $P$” ($M\bar{P} = 0$), we shade out all of $M$ that is not contained
in (or overlapped by) $P$. This area, it is seen, includes both the portions labeled $S\bar{P}M$ and $\bar{S}P\bar{M}$. The diagram then becomes Figure 6-2.

If we focus our attention on just the two circles $S$ and $M$, by shading out, or by inserting an $x$, we can diagram any standard-form categorical proposition whose terms are $S$ and $M$, regardless of the order in which they appear in it. To diagram the proposition “All $S$ is $M$” ($S\bar{M} = 0$), we shade out all of $S$ that is not contained in (or overlapped by) $M$. This area, it is seen, includes both the portions labeled $S\bar{P}M$ and $S\bar{M}$. The diagram for this proposition will appear as Figure 6-3.

The advantage of using three overlapping circles is that it allows us to diagram two propositions together—on the condition, of course, that only three different terms occur in them. Thus diagramming both “All $M$ is $P$” and “All $S$ is $M$” at the same time gives us Figure 6-4.
This is the diagram for both premises of the syllogism AAA–1:

- All $M$ is $P$.
- All $S$ is $M$.

$\therefore$ All $S$ is $P$.

This syllogism is valid if and only if the two premises imply or entail the conclusion—that is, if together they say what is said by the conclusion. Consequently, diagramming the premises of a valid argument should suffice to diagram its conclusion also, with no further marking of the circles needed. To diagram the conclusion “All $S$ is $P$” is to shade out both the portion labeled $SPM$ and the portion labeled $SM$. Inspecting the diagram that represents the two premises, we see that it also diagrams the conclusion. And from this we can conclude that AAA–1 is a valid syllogism.*

Let us now apply the Venn diagram test to an obviously invalid syllogism, one containing three A propositions in the second figure:

- All dogs are mammals.
- All cats are mammals.
- Therefore all cats are dogs.

Diagramming both premises gives us Figure 6-5.

In this diagram, where $S$ designates the class of all cats, $P$ the class of all dogs, and $M$ the class of all mammals, the portions $SPM$, $SPM$, and $SPM$, have been

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*The mood of this syllogism is AAA because it consists of three A propositions; it is in the first figure because its middle term is the subject of its major premise and the predicate of its minor premise. Any syllogism of this valid form, AAA–1, is called (as noted earlier) a syllogism in Barbara. The names of other valid syllogisms will be given in Section 6.5.
shaded out. But the conclusion has not been diagrammed, because the part $S\overline{P}M$ has been left unshaded, and to diagram the conclusion both $S\overline{P}M$ and $S\overline{P}M$ must be shaded. Thus we see that diagramming both the premises of a syllogism of form $\text{AAA–2}$ does not suffice to diagram its conclusion, which proves that the conclusion says something more than is said by the premises, which shows that the premises do not imply the conclusion. An argument whose premises do not imply its conclusion is invalid, so our diagram proves that the given syllogism is invalid. (It proves, in fact, that any syllogism of the form $\text{AAA–2}$ is invalid.)

When we use a Venn diagram to test a syllogism with one universal premise and one particular premise, it is important to diagram the universal premise first. Thus, in testing the $\text{AII–3}$ syllogism,

All artists are egotists.

Some artists are paupers.

Therefore some paupers are egotists.

we should diagram the universal premise, “All artists are egotists,” before inserting an $x$ to diagram the particular premise, “Some artists are paupers.” Properly diagrammed, the syllogism looks like Figure 6-6.

![Venn Diagram](image)

Figure 6-6

Had we tried to diagram the particular premise first, before the region $S\overline{P}M$ was shaded out along with $S\overline{P}M$ in diagramming the universal premise, we would not have known whether to insert an $x$ in $SPM$ or in $S\overline{P}M$ or in both. And had we put it in $S\overline{P}M$ or on the line separating it from $SPM$, the subsequent shading of $S\overline{P}M$ would have obscured the information the diagram was intended to exhibit. Now that the information contained in the premises has been inserted into the diagram, we can examine it to see whether the conclusion already has been diagrammed. If the conclusion, “Some paupers are egotists,” has been diagrammed, there will be an $x$ somewhere in the overlapping
part of the circles labeled “Paupers” and “Egotists.” This overlapping part consists of both of the regions \( SP\bar{M} \) and \( SPM \), which together constitute \( SP \). There is an \( x \) in the region \( SPM \), so there is an \( x \) in the overlapping part \( SP \).

What the conclusion of the syllogism says has already been diagrammed by the diagramming of its premises; therefore the syllogism is valid.

Let us consider still another example, the discussion of which will bring out another important point about the use of Venn diagrams. Let’s say we are testing the argument

- All great scientists are college graduates.
- Some professional athletes are college graduates.
- Therefore some professional athletes are great scientists.

After diagramming the universal premise first (Figure 6-7) by shading out both regions \( SP\bar{M} \) and \( \bar{S}PM \),

we may still be puzzled about where to put the \( x \) needed in order to diagram the particular premise. That premise is “Some professional athletes are college graduates,” so an \( x \) must be inserted somewhere in the overlapping part of the two circles labeled “Professional athletes” and “College graduates.” That overlapping part, however, contains two regions, \( SPM \) and \( \bar{S}PM \). In which of these should we put an \( x \)? The premises do not tell us, and if we make an arbitrary decision to place it in one rather than the other, we would be inserting more information into the diagram than the premises warrant—which would spoil the diagram’s use as a test for validity. Placing \( x \)'s in each of them would also go beyond what the premises assert. Yet by placing an \( x \) on the line that divides the overlapping region \( SM \) into the two parts \( SPM \) and \( \bar{S}PM \), we can diagram exactly what the second premise asserts without adding anything to it. Placing an \( x \) on the line between two regions indicates that there is something that belongs in one of them, but does not indicate which one. The completed diagram of both premises thus looks like Figure 6-8.
When we inspect this diagram of the premises to see whether the conclusion of the syllogism has already been diagrammed in it, we find that it has not. For the conclusion, “Some professional athletes are great scientists,” to be diagrammed, an \( x \) must appear in the overlapping part of the two upper circles, either in \( SPM \) or in \( SPM' \). The first of these is shaded out and certainly contains no \( x \). The diagram does not show an \( x \) in \( SPM \) either. True, there must be a member of either \( SPM \) or \( SPM' \), but the diagram does not tell us that it is in the former rather than the latter and so, for all the premises tell us, the conclusion may be false. We do not know that the conclusion is false, only that it is not asserted or implied by the premises. The latter is enough, however, to let us know that the argument is invalid. The diagram suffices to show not only that the given syllogism is invalid, but that all syllogisms of the form \( AII-2 \) are invalid.

The general technique of using Venn diagrams to test the validity of any standard-form syllogism may be summarized as follows. First, label the circles of a three-circle Venn diagram with the syllogism’s three terms. Next, diagram both premises, diagramming the universal one first if there is one universal and one particular, being careful in diagramming a particular proposition to put an \( x \) on a line if the premises do not determine on which side of the line it should go. Finally, inspect the diagram to see whether the diagram of the premises contains a diagram of the conclusion: If it does, the syllogism is valid; if it does not, the syllogism is invalid.

What is the theoretical rationale for using Venn diagrams to distinguish valid from invalid syllogisms? The answer to this question divides into two parts. The first part has to do with the formal nature of syllogistic argument as explained in Section 6.2. It was shown there that one legitimate test of the validity or invalidity of a syllogism is to establish the validity or invalidity of a different syllogism that has exactly the same form. This technique is basic to the use of Venn diagrams.

The explanation of how the diagrams serve this purpose constitutes the second part of the answer to our question. Ordinarily, a syllogism will be about classes of objects that are not all present, such as the class of all musicians,
Where do I place the \( x \) in a Venn diagram?

In the Venn diagram representing a categorical syllogism, the three terms of the syllogism (Subject term, Predicate term, and Middle term) are represented by three interlocking circles labeled \( S, P, \) and \( M \).

When one of the premises of a syllogism calls for an \( x \) to be placed on a line in such a Venn diagram, we may ask: Which line? And why? Answer: The \( x \) is always placed on the line of the circle designating the class not mentioned in that premise.

**Example:** Suppose you are given as premise, “Some \( S \) is \( M \).” You may not be able to determine whether the \( x \) representing that “some” is a \( P \) or is not a \( P \)—so the \( x \) goes on the line of the \( P \) circle, thus:

**Another example:** Suppose you are given as premise, “Some \( M \) is not \( P \).” You may not be able to determine whether the \( M \) that is not \( P \) is an \( S \) or is not an \( S \)—so the \( x \) goes on the line of the \( S \) circle, thus:
or great scientists, or sodium salts. The relations of inclusion or exclusion among such classes may be reasoned about and may be empirically discoverable in the course of scientific investigation. But they certainly are not open to direct inspection, because not all members of the classes involved are ever present at one time to be inspected. We can, however, examine situations of our own making, in which the only classes concerned contain by their very definitions only things that are present and open to direct inspection. And we can argue syllogistically about such situations of our own making. Venn diagrams are devices for expressing standard-form categorical propositions, but they also are situations of our own making, patterns of graphite or ink on paper, or lines of chalk on blackboards. And the propositions they express can be interpreted as referring to the diagrams themselves. An example can help to make this clear. Suppose we have a particular syllogism whose terms denote various kinds of people who are successful, interested in their work, and able to concentrate, and who may be scattered widely over all parts of the world:

All successful people are people who are keenly interested in their work.
No people who are keenly interested in their work are people whose attention is easily distracted when they are working.
Therefore no people whose attention is easily distracted when they are working are successful people.

Its form is AEE–4, and it may be schematized as

All $P$ is $M$.
No $M$ is $S$.
∴ No $S$ is $P$.

We may test it by constructing the Venn diagram shown in Figure 6-9, in which regions $SP\overline{M}$ and $\overline{S}M$ are shaded out to express the first premise, and $\overline{S}P$ and $SP\overline{M}$ are shaded out to express the second premise.

![Figure 6-9](image-url)
Examining Figure 6-9, we find that SP (which consists of the regions SPM and SPM) has been shaded out, so the syllogism’s conclusion has already been diagrammed. How does this tell us that the given syllogism is valid? This syllogism concerns large classes of remote objects: There are many people whose attention is easily distracted while they are working, and they are scattered far and wide. However, we can construct a syllogism of the same form that involves objects that are immediately present and directly available for our inspection. These objects are the points within the unshaded portions of the circles labeled S, P, and M in our Venn diagram. Here is the new syllogism:

All points within the unshaded part of the circle labeled P are points within the unshaded part of the circle labeled M.

No points within the unshaded part of the circle labeled M are points within the unshaded part of the circle labeled S.

Therefore no points within the unshaded part of the circle labeled S are points within the unshaded part of the circle labeled P.

This new syllogism refers to nothing remote; it is about the parts of a situation we ourselves have created: the Venn diagram we have drawn. All the parts and all the possibilities of inclusion and exclusion among these classes are immediately present to us and directly open to inspection. We can literally see all the possibilities here, and know that because all the points of P are also points of M, and because M and S have no points in common, S and P cannot possibly have any points in common. Because the new syllogism refers only to classes of points in the diagram, it can be literally seen to be valid by looking at the things it talks about. The original syllogism about classes of people has exactly the same form as this second one, so we are assured by the formal nature of syllogistic argument that the original syllogism is also valid. The explanation is exactly the same for Venn diagram proofs of the invalidity of invalid syllogisms; there, too, we test the original syllogism indirectly by testing directly a second syllogism that has exactly the same form and referring to the diagram that exhibits that form.

EXERCISES

A. Write out each of the following syllogistic forms, using S and P as the subject and predicate terms of the conclusion, and M as the middle term. (Refer to the chart of the four syllogistic figures, if necessary, on p. 227.) Then test the validity of each syllogistic form using a Venn diagram.

EXAMPLE

1. AEE–1
SOLUTION

We are told that this syllogism is in the first figure, and therefore the middle term, \( M \), is the subject term of the major premise and the predicate term of the minor premise. (See chart on p. 228.) The conclusion of the syllogism is an \( E \) proposition and therefore reads: No \( S \) is \( P \). The first (major) premise (which contains the predicate term of the conclusion) is an \( A \) proposition, and therefore reads: All \( M \) is \( P \). The second (minor) premise (which contains the subject term of the conclusion) is an \( E \) proposition and therefore reads: No \( S \) is \( M \). This syllogism therefore reads as follows:

\[
\begin{align*}
\text{All } M & \text{ is } P. \\
\text{No } S & \text{ is } M. \\
\text{Therefore no } S & \text{ is } P.
\end{align*}
\]

Tested by means of a Venn diagram, as in Figure 6-10, this syllogism is shown to be invalid.

![Figure 6-10](image)

2. EIO–2
3. OAO–3
4. AOO–4
5. EIO–4
6. OAO–2
7. AOO–1
8. EAE–3
9. EIO–3
*10. IAI–4
11. AOO–3
12. EAE–1
13. IAI–1
14. OAO–4
*15. EIO–1

B. Put each of the following syllogisms into standard form, name its mood and figure, and test its validity using a Venn diagram.

*1. Some reformers are fanatics, so some idealists are fanatics, because all reformers are idealists.

2. Some philosophers are mathematicians; hence some scientists are philosophers, because all scientists are mathematicians.
3. Some mammals are not horses, for no horses are centaurs, and all centaurs are mammals.

4. Some neurotics are not parasites, but all criminals are parasites; it follows that some neurotics are not criminals.

5. All underwater craft are submarines; therefore no submarines are pleasure vessels, because no pleasure vessels are underwater craft.

6. No criminals were pioneers, for all criminals are unsavory persons, and no pioneers were unsavory persons.

7. No musicians are astronauts; all musicians are baseball fans; consequently, no astronauts are baseball fans.

8. Some Christians are not Methodists, for some Christians are not Protestants, and some Protestants are not Methodists.

9. No people whose primary interest is in winning elections are true liberals, and all active politicians are people whose primary interest is in winning elections, which entails that no true liberals are active politicians.

10. No weaklings are labor leaders, because no weaklings are true liberals, and all labor leaders are true liberals.

6.4 Syllogistic Rules and Syllogistic Fallacies

A syllogism may fail to establish its conclusion in many different ways. To help avoid common errors we set forth rules—six of them—to guide the reasoner; any given standard-form syllogism can be evaluated by observing whether any one of these rules has been violated. Mastering the rules by which syllogisms may be evaluated also enriches our understanding of the syllogism itself; it helps us to see how syllogisms work, and to see why they fail to work if the rules are broken.

A violation of any one of these rules is a mistake, and it renders the syllogism invalid. Because it is a mistake of that special kind, we call it a fallacy; and because it is a mistake in the form of the argument, we call it a formal fallacy (to be contrasted with the informal fallacies described in Chapter 4). In reasoning with syllogisms, one must scrupulously avoid the fallacies that violations of the rules invariably yield. Each of these formal fallacies has a traditional name, explained below.

Rule 1. Avoid four terms.

A valid standard-form categorical syllogism must contain exactly three terms, each of which is used in the same sense throughout the argument.

In every categorical syllogism, the conclusion asserts a relationship between two terms, the subject (minor term) and the predicate (major term). Such a conclusion
can be justified only if the premises assert the relationship of each of those two
terms to the same third term (middle term). If the premises fail to do this consist-
tently, the needed connection of the two terms in the conclusion cannot be estab-
lished, and the argument fails. So every valid categorical syllogism must involve
three terms—no more and no less. If more than three terms are involved, the syl-
logism is invalid. The fallacy thus committed is called the **fallacy of four terms.**

The mistake that commonly underlies this fallacy is equivocation, using
one word or phrase with two different meanings. Most often it is the middle
term whose meaning is thus shifted, in one direction to connect it with the
minor term, in a different direction to connect it with the major term. In doing
this the two terms of the conclusion are connected with two different terms
(rather than with the same middle term), and so the relationship asserted by
the conclusion is not established.∗

When the expression “*categorical syllogism*” was defined at the beginning of
this chapter, we noted that by its nature every syllogism must have three and only
three terms.† So this rule (“Avoid four terms”) may be regarded as a reminder to
make sure that the argument being appraised really is a categorical syllogism.

**Rule 2. Distribute the middle term in at least one premise.**

* *A term is “distributed” in a proposition when (as was explained in Section 5.4)
the proposition refers to all members of the class designated by that term. If the
middle term is not distributed in at least one premise, the connection required by
the conclusion cannot be made.*

Historian Barbara Tuchman (in *The Proud Tower*, New York: Macmillan,
1966) observed that many early critics of anarchism relied on the following
“unconscious syllogism”:

*All Russians were revolutionists.  
All anarchists were revolutionists.  
Therefore, all anarchists were Russians.*

This syllogism is plainly invalid. Its mistake is that it asserts a connection be-
tween anarchists and Russians by relying on the links between each of those
classes and the class of revolutionists—but revolutionists is an *undistributed*
term in both of the premises. The first premise does not refer to all revolutionists,

∗Because it is the middle term that is most often manipulated, this fallacy is sometimes
called “the fallacy of the ambiguous middle.” However, this name is not generally applic-
cable, because one (or more) of the other terms may have its meaning shifted as well.
Ambiguities may result in as many as five or six different terms being involved, but the
mistake retains its traditional name: the fallacy of four terms.

†The term *syllogism* is sometimes defined more broadly than it has been in this book. The
informal fallacy of equivocation, explained and warned against in Chapter 4, may arise in
many different argumentative contexts, of course.
and neither does the second. Revolutionists is the middle term in this argument, and if the middle term is not distributed in at least one premise of a syllogism, that syllogism cannot be valid. The fallacy this syllogism commits is called the **fallacy of the undistributed middle**.

What underlies this rule is the need to *link* the minor and the major terms. If they are to be linked by the middle term, either the subject or the predicate of the conclusion must be related to the *whole* of the class designated by the middle term. If that is not so, it is possible that each of the terms in the conclusion may be connected to a different part of the middle term, and not necessarily connected with each other.

This is precisely what happens in the syllogism given in the preceding example. The Russians are included in a *part* of the class of revolutionists (by the first premise), and the anarchists are included in a *part* of the class of revolutionists (by the second premise)—but different parts of this class (the middle term of the syllogism) may be involved, and so the middle term does not successfully link the minor and major terms of the syllogism. In a valid syllogism, the middle term must be distributed in at least one premise.

**Rule 3.** Any term distributed in the conclusion must be distributed in the premises.

*To refer to all members of a class is to say more about that class than is said when only some of its members are referred to. Therefore, when the conclusion of a syllogism distributes a term that was undistributed in the premises, it says more about that term than the premises did. But a valid argument is one whose premises logically entail its conclusion, and for that to be true the conclusion must not assert any more than is asserted in the premises. A term that is distributed in the conclusion but is not distributed in the premises is therefore a sure mark that the conclusion has gone beyond its premises and has reached too far. The fallacy is that of illicit process.*

The conclusion may overreach with respect to either the minor term (its subject), or the major term (its predicate). So there are two different forms of illicit process, and different names have been given to the two formal fallacies involved. They are

Illicit process of the major term (an **illicit major**).

Illicit process of the minor term (an **illicit minor**).

To illustrate an illicit process of the major term, consider this syllogism:

All dogs are mammals.

No cats are dogs.

Therefore no cats are mammals.
The reasoning is obviously bad, but where is the mistake? The mistake is in the conclusion’s assertion about *all* mammals, saying that all of them fall outside the class of cats. Bear in mind that an A proposition distributes its subject term but does not distribute its predicate term. Hence the premises make no assertion about *all* mammals—so the conclusion illicitly goes beyond what the premises assert. Because “mammals” is the major term in this syllogism, the fallacy here is that of an illicit major.

To illustrate illicit process of the minor term, consider this syllogism:

- All traditionally religious people are fundamentalists.
- All traditionally religious people are opponents of abortion.
- Therefore all opponents of abortion are fundamentalists.

Again we sense quickly that something is wrong with this argument, and what is wrong is this: The conclusion makes an assertion about *all* opponents of abortion, but the premises make no such assertion; they say nothing about *all* abortion opponents. So the conclusion here goes illicitly beyond what the premises warrant. And in this case “opponents of abortion” is the minor term, so the fallacy is that of an illicit minor.

**Rule 4. Avoid two negative premises.**

*Any negative proposition (E or O) denies class inclusion; it asserts that some or all of one class is excluded from the whole of the other class. Two premises asserting such exclusion cannot yield the linkage that the conclusion asserts, and therefore cannot yield a valid argument. The mistake is named the fallacy of exclusive premises.*

Understanding the mistake identified here requires some reflection. Suppose we label the minor, major, and middle terms of the syllogism *S, P, and M* respectively. What can two negative premises tell us about the relations of these three terms? They can tell us that *S* (the subject of the conclusion) is wholly or partially excluded from all or part of *M* (the middle term), and that *P* (the predicate of the conclusion) is wholly or partially excluded from all or part of *M*. However, any one of these relations may very well be established no matter how *S and P* are related. The negative premises cannot tell us that *S and P* are related by inclusion or by exclusion, partial or complete. Two negative premises (where *M* is a term in each) simply cannot justify the assertion of *any* relationship whatever between *S and P*. Therefore, if both premises of a syllogism are negative, the argument must be invalid.

**Rule 5. If either premise is negative, the conclusion must be negative.**

*If the conclusion is affirmative—that is, if it asserts that one of the two classes, *S* or *P*, is wholly or partly contained in the other—it can only be inferred from premises that assert the existence of a third class that contains the first and*
is itself contained in the second. However, class inclusion can be stated only by affirmative propositions. Therefore, an affirmative conclusion can follow validly only from two affirmative premises. The mistake here is called the fallacy of drawing an affirmative conclusion from a negative premise.

If an affirmative conclusion requires two affirmative premises, as has just been shown, we can know with certainty that if either of the premises is negative, the conclusion must also be negative, or the argument is not valid.

Unlike some of the fallacies identified here, this fallacy is not common, because any argument that draws an affirmative conclusion from negative premises will be instantly recognized as highly implausible. Even an illustration of the mistake will appear strained:

No poets are accountants.
Some artists are poets.

Therefore some artists are accountants.

Immediately it will be seen that the exclusion of poets and accountants, asserted by the first premise of this syllogism, cannot justify any valid inference regarding the inclusion of artists and accountants.

Rule 6. From two universal premises no particular conclusion may be drawn.
In the Boolean interpretation of categorical propositions (explained in Section 5.7), universal propositions (A and E) have no existential import, but particular propositions (I and O) do have such import. Wherever the Boolean interpretation is supposed, as in this book, a rule is needed that precludes passage from premises that have no existential import to a conclusion that does have such import.

This final rule is not needed in the traditional or Aristotelian account of the categorical syllogism, because that traditional account paid no attention to the problem of existential import. However, when existential import is carefully considered, it will be clear that if the premises of an argument do not assert the existence of anything at all, the conclusion will be unwarranted when, from it, the existence of some thing may be inferred. The mistake is called the existential fallacy.

Here is an example of a syllogism that commits this fallacy:

All household pets are domestic animals.
No unicorns are domestic animals.

Therefore some unicorns are not household pets.

If the conclusion of this argument were the universal proposition, “No unicorns are household pets,” the syllogism would be perfectly valid for all. And because, under the traditional interpretation, existential import may be inferred from universal as well as from particular propositions, it would not be problematic.
(in that traditional view) to say that the conclusion in the example given here is simply a “weaker” version of the conclusion we all agree is validly drawn.

In our Boolean view, however, the conclusion of the example (“Some unicorns are not household pets”), because it is a particular proposition, is not just “weaker,” it is very different. It is an O proposition, a particular proposition, and thus has an existential import that the E proposition (“No unicorns are household pets”) cannot have. Reasoning that is acceptable under the traditional view is therefore unacceptable under the Boolean view because, from the Boolean perspective, that reasoning commits the existential fallacy—a mistake that cannot be made under the traditional interpretation.*

The six rules given here are intended to apply only to standard-form categorical syllogisms. In this realm they provide an adequate test for the validity of any argument. If a standard-form categorical syllogism violates any one of these rules, it is invalid; if it conforms to all of these rules, it is valid.

**OVERVIEW**

<table>
<thead>
<tr>
<th>Syllogistic Rules and Fallacies</th>
</tr>
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<tbody>
<tr>
<td>Rule</td>
</tr>
<tr>
<td>1. Avoid four terms.</td>
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<tr>
<td>2. Distribute the middle term</td>
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<td>in at least one premise.</td>
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<tr>
<td>3. Any term distributed in the</td>
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<tr>
<td>conclusion must be distributed</td>
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<tr>
<td>in the premises.</td>
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<tr>
<td>4. Avoid two negative premises.</td>
</tr>
<tr>
<td>5. If either premise is negative,</td>
</tr>
<tr>
<td>the conclusion must be negative.</td>
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<tr>
<td>6. No particular conclusion may be drawn from two universal premises.</td>
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</tbody>
</table>

*Another interesting consequence of the difference between the traditional and the Boolean interpretation of categorical propositions is this: In the traditional view there is a need for a rule that states the converse of Rule 5 (“If either premise is negative, the conclusion must be negative”). The converse states simply that “If the conclusion of a valid syllogism is negative, at least one premise must be negative.” And that is indisputable, because if the conclusion is negative, it denies inclusion. But affirmative premises assert inclusion. Therefore affirmative premises cannot entail a negative conclusion. This corollary is unnecessary in the Boolean interpretation because the rule precluding the existential fallacy (Rule 6) will, in the presence of the other rules, suffice to invalidate any syllogism with affirmative premises and a negative conclusion.
FLOWCHART FOR APPLYING THE SIX SYLLOGISTIC RULES

The following chart captures the process for working through the six rules of validity for categorical syllogisms.

1. Identify the premises and the conclusion.
2. Does the argument have exactly three terms used consistently throughout?
   - NO → Fallacy of Four Terms.
   - YES → Is the middle term distributed at least once?
     - NO → Fallacy of Undistributed Middle
     - YES → Is the major term distributed in the conclusion?
       - YES → Is the major term distributed in the major premise?
         - NO → Fallacy of Illicit Major
         - YES → Is the minor term distributed in the conclusion?
           - YES → Is the minor term distributed in the minor premise?
             - NO → Fallacy of Illicit Minor
             - YES → No other fallacy can be committed. The argument is INVALID.
6.4 Syllogistic Rules and Syllogistic Fallacies

Are there two negative premises?

YES → Fallacy of Exclusive Premises

NO

Is there an affirmative conclusion?

YES → Fallacy of Affirmative Conclusion from a Negative Premise

NO

Is there a negative premise?

YES

Is there a particular conclusion?

YES

Has a fallacy been committed?

YES → The argument is INVALID

NO → The argument is VALID

NO

Is there a particular premise?

NO → Existential Fallacy

YES

EXERCISES

A. Identify the rule that is broken by invalid syllogisms of the following forms, and name the fallacy that each commits.

EXAMPLE

1. AAA–2

SOLUTION

Any syllogism in the second figure has the middle term as predicate of both the major and the minor premise. Thus any syllogism consisting of three A propositions, in the second figure, must read: All P is M; all S is M; therefore all S is P. M is not distributed in either of the premises in that form, and therefore it cannot validly be inferred from such premises that all S is P. Thus every syllogism of the form AAA–2 violates the rule that the middle term must be distributed in at least one premise, thereby committing the fallacy of the undistributed middle.

2. EAA–1
3. IAO–3
4. OEO–4
*5. AAA–3
6. IAI–2
7. OAA–3
8. EAO–4
9. OAI–3
*10. IEO–1
11. EAO–3
12. AII–2
13. EEE–1
14. OAO–2
*15. IAA–3

B. Identify the rule that is broken by any of the following syllogisms that are invalid, and name the fallacy that is committed.
EXAMPLE

1. All textbooks are books intended for careful study.
   Some reference books are books intended for careful study.
   Therefore some reference books are textbooks.

SOLUTION

In this syllogism, “textbooks” is the major term (the predicate of the conclusion) and “reference books” is the minor term (the subject of the conclusion). “Books intended for careful study” is therefore the middle term, and it appears as the predicate of both premises. In neither of the premises is this middle term distributed, so the syllogism violates the rule that the middle term must be distributed in at least one premise, thereby committing the fallacy of the undistributed middle.

2. All criminal actions are wicked deeds.
   All prosecutions for murder are criminal actions.
   Therefore all prosecutions for murder are wicked deeds.

3. No tragic actors are idiots.
   Some comedians are not idiots.
   Therefore some comedians are not tragic actors.

4. Some parrots are not pests.
   All parrots are pets.
   Therefore no pets are pests.

5. All perpetual motion devices are 100 percent efficient machines.
   All 100 percent efficient machines are machines with frictionless bearings.
   Therefore some machines with frictionless bearings are perpetual motion devices.

6. Some good actors are not powerful athletes.
   All professional wrestlers are powerful athletes.
   Therefore all professional wrestlers are good actors.

7. Some diamonds are precious stones.
   Some carbon compounds are not diamonds.
   Therefore some carbon compounds are not precious stones.
8. Some diamonds are not precious stones.
   Some carbon compounds are diamonds.
   Therefore some carbon compounds are not precious stones.

9. All people who are most hungry are people who eat most.
   All people who eat least are people who are most hungry.
   Therefore all people who eat least are people who eat most.

*10. Some spaniels are not good hunters.
   All spaniels are gentle dogs.
   Therefore no gentle dogs are good hunters.

C. Identify the rule that is broken by any of the following syllogisms that are invalid, and name the fallacy that is committed.

**EXAMPLE**

1. All chocolate eclairs are fattening foods, because all chocolate eclairs are rich desserts, and some fattening foods are not rich desserts.

**SOLUTION**

In this syllogism the conclusion is affirmative (“all chocolate eclairs are fattening foods”), while one of the premises is negative (“some fattening foods are not rich desserts”). The syllogism therefore is invalid, violating the rule that if either premise is negative the conclusion must also be negative, thereby committing the fallacy of affirmative conclusion from a negative premise.

2. All inventors are people who see new patterns in familiar things, so all inventors are eccentrics, because all eccentrics are people who see new patterns in familiar things.

3. Some snakes are not dangerous animals, but all snakes are reptiles, therefore some dangerous animals are not reptiles.

4. Some foods that contain iron are toxic substances, for all fish containing mercury are foods that contain iron, and all fish containing mercury are toxic substances.

*5. All opponents of basic economic and political changes are outspoken critics of the liberal leaders of Congress, and all right-wing extremists are opponents of basic economic and political changes. It follows that all outspoken critics of the liberal leaders of Congress are right-wing extremists.

6. No writers of lewd and sensational articles are honest and decent citizens, but some journalists are not writers of lewd and sensational articles; consequently, some journalists are honest and decent citizens.
7. All supporters of popular government are democrats, so all supporters of popular government are opponents of the Republican Party, inasmuch as all Democrats are opponents of the Republican Party.

8. No coal-tar derivatives are nourishing foods, because all artificial dyes are coal-tar derivatives, and no artificial dyes are nourishing foods.

9. No coal-tar derivatives are nourishing foods, because no coal-tar derivatives are natural grain products, and all natural grain products are nourishing foods.

*10. All people who live in London are people who drink tea, and all people who drink tea are people who like it. We may conclude, then, that all people who live in London are people who like it.

6.5  Exposition of the Fifteen Valid Forms of the Categorical Syllogism

The mood of a syllogism is its character as determined by the forms (A, E, I, or O) of the three propositions it contains. There are sixty-four possible moods of the categorical syllogism; that is, sixty-four possible sets of three propositions: AAA, AAI, AAE, and so on, to . . . EOO, OOO.

The figure of a syllogism is its logical shape, as determined by the position of the middle term in its premises. So there are four possible figures, which can be most clearly grasped if one has in mind a chart, or iconic representation, of the four possibilities, as exhibited in the Overview table:

### Overview

**The Four Figures**

<table>
<thead>
<tr>
<th>First Figure</th>
<th>Second Figure</th>
<th>Third Figure</th>
<th>Fourth Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>M → P</td>
<td>P → M</td>
<td>M → P</td>
<td>P → M</td>
</tr>
<tr>
<td>S → M</td>
<td>S → M</td>
<td>M → S</td>
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<tr>
<td>S → P</td>
<td>S → P</td>
<td>S → P</td>
<td>S → P</td>
</tr>
</tbody>
</table>

**Schematic Representation**

- M → P: The middle term is the subject of the major premise and the predicate of the minor premise.
- P → M: The middle term is the predicate of both major and minor premises.
- M → S: The middle term is the subject of both the major and minor premises.
- S → P: The middle term is the predicate of the major premise and the subject of the minor premise.
It will be seen that:

- In the first figure the middle term is the subject of the major premise and the predicate of the minor premise;
- In the second figure the middle term is the predicate of both premises;
- In the third figure the middle term is the subject of both premises;
- In the fourth figure the middle term is the predicate of the major premise and the subject of the minor premise.

Each of the sixty-four moods can appear in each of the four figures. The mood and figure of a given syllogism, taken together, uniquely determine the logical form of that syllogism. Therefore there are (as noted earlier) exactly 256 \((64 \times 4)\) possible forms of the standard-form categorical syllogism.

The vast majority of these forms are not valid. We can eliminate every form that violates one or more of the syllogistic rules set forth in the preceding section. The forms that remain after this elimination are the only valid forms of the categorical syllogism. Of the 256 possible forms, there are exactly fifteen forms that cannot be eliminated and thus are valid.*

To advance the mastery of syllogistics, classical logicians gave a unique name to every valid syllogism, each characterized completely by mood and figure. Understanding this small set of valid forms, and knowing the name of each, is very useful when putting syllogistic reasoning to work. Each name, carefully devised, contained three vowels representing (in standard-form order: major premise, minor premise, conclusion) the mood of the syllogism named. Where there are valid syllogisms of a given mood but in different figures, a unique name was assigned to each. Thus, for example, a syllogism of the mood \(\text{EAE}\) in the first figure was named \(\text{Celarent}\), whereas

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*It should be borne in mind that we adopt here the Boolean interpretation of categorical propositions, according to which universal propositions (\(A\) and \(E\) propositions) do not have existential import. The classical interpretation of categorical propositions, according to which all the classes to which propositions refer do have members, makes acceptable some inferences that are found here to be invalid. Under that older interpretation, for example, it is plausible to infer the subaltern from its corresponding superaltern—to infer an \(I\) proposition from its corresponding \(A\) proposition, and an \(O\) proposition from its corresponding \(E\) proposition. This makes plausible the claim that there are other valid syllogisms (so-called weakened syllogisms) that are not considered valid here. Compelling reasons for the rejection of that older interpretation (and hence the justification of our stricter standards for valid syllogisms) were given at some length in Section 5.7.
a syllogism of the mood EAE in the second figure, also valid, was named Cesare.*

These names had (and still have) a very practical purpose: If one knows that only certain combinations of mood and figure are valid, and can recognize by name those valid arguments, the merit of any syllogism in a given figure, or of a given mood, can be determined almost immediately. For example, the mood AOO is valid only in the second figure. That unique form (AOO–2) is known as Baroko.† One who is familiar with Baroko and able to discern it readily may be confident that a syllogism of this mood presented in any other figure may be rejected as invalid.

The standard form of the categorical syllogism is the key to the system. A neat and efficient method of identifying the few valid syllogisms from among the many possible syllogisms is at hand, but it depends on the assumption that the propositions of the syllogism in question either are in (or can be put into) standard order—major premise, minor premise, then conclusion. The unique identification of each valid syllogism relies on the specification of its mood, and its mood is determined by the letters characterizing its three constituent propositions in that standard order. If the premises of a valid syllogism were to be set forth in a different order, then that syllogism would remain valid, of course; the Venn diagram technique can prove this. But much would be lost. Our ability to identify syllogisms uniquely, and with that identification our ability to comprehend the forms

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*The principles that governed the construction of those traditional names, the selection and placement of consonants as well as vowels, were quite sophisticated. Some of these conventions relate to the place of the weakened syllogisms noted just above and are therefore not acceptable in the Boolean interpretation we adopt. Some other conventions remain acceptable. For example, the letter s that follows the vowel e indicates that when that E proposition is converted simpliciter, or simply (as all E propositions will convert), then that syllogism reduces to, or is transformed into, another syllogism of the same mood in the first figure, which is viewed as the most basic figure. To illustrate, Festino, in the second figure, reduces, when its major premise is converted simply, to Ferio; and Cesare, in the second figure, reduces to Celarent, and so on. The possibility of these and other reductions explains why the names of groups of syllogisms begin with the same consonant. The intricate details of the classical naming system need not be fully recounted here.

†Here is an example of Baroko:

All good mathematicians have creative intellects.
Some scholars do not have creative intellects.
Therefore some scholars are not good mathematicians.

With practice one comes to recognize the cadence of the different valid forms.
of those syllogisms fully and to test their validity crisply, all rely on their being in standard form.*

Classical logicians studied these forms closely, and they became fully familiar with their structure and their logical “feel.” This elegant system, finely honed, enabled reasoners confronting syllogisms in speech or in texts to recognize immediately those that were valid, and to detect with confidence those that were not. For centuries it was common practice to defend the solidity of reasoning in progress by giving the names of the forms of the valid syllogisms being relied on. The ability to provide these identifications even in the midst of heated oral disputes was considered a mark of learning and acumen, and it gave evidence that the chain of deductive reasoning being relied on was indeed unbroken. Once the theory of the syllogism has been fully mastered, this practical skill can be developed with profit and pleasure.

Syllogistic reasoning was so very widely employed, and so highly regarded as the most essential tool of scholarly argument, that the logical treatises of its original and greatest master, Aristotle, were venerated for more than a thousand years. His analytical account of the syllogism still carries the simple name that conveys respect and awe: the *Organon*, the *Instrument*.†

As students of this remarkable logical system, our proficiency in syllogistics may be only moderate—but we will nevertheless find it useful to have before us a synoptic account of all the valid syllogisms. These fifteen valid syllogisms (under the Boolean interpretation) may be divided by figure into four groups:‡

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†Valid syllogisms are powerful weapons in controversy, but the effectiveness of those weapons depends, of course, on the truth of the premises. A great theologian, defiant in battling scholars who resisted his reform of the Church, wrote: “They may attack me with an army of six hundred syllogisms...” (Erasmus, *The Praise of Folly*, 1511).
‡In the older tradition, in which reasoning from universal premises to particular conclusions was believed to be correct, the number of valid syllogisms (each uniquely named) was of course more than fifteen. To illustrate, if an *I* proposition may be inferred from its corresponding *A* proposition (as we think mistaken), the valid syllogism known as *Barbara* (*AAA*–1) will have a putatively valid “weakened” sister, *Barbari* (*AAI*–1); and if an *O* proposition may be inferred from its corresponding *E* proposition (as we think mistaken), the valid syllogism known as *Camestres* (*AEE*–2) will have a putatively valid “weakened” brother, *Camestrop* (*AEO*–2).
EXERCISES

A. At the conclusion of Section 6.3, in exercise group B (on pp. 243–244), ten syllogisms were to be tested using Venn diagrams. Of these ten syllogisms, numbers 1, 4, 6, 9, and 10 are valid. What is the name of each of these five valid syllogisms?

EXAMPLE

Number 1 is IAI–3 (Disamis).
APPENDIX

Deduction of the Fifteen Valid Forms of the Categorical Syllogism

In Section 6.5 the fifteen valid forms of the categorical syllogism were identified and precisely characterized. The unique name of each syllogism is also given there—a name assigned in view of its unique combination of mood and figure. The summary account of these fifteen syllogisms appears in the Overview immediately preceding.

It is possible to prove that these, and only these, are the valid forms of the categorical syllogism. This proof—the deduction of the valid forms of the categorical syllogism—is presented as an appendix, rather than in the body of the chapter, because mastering it is not essential for the student of logic. However, understanding it can give one a deeper appreciation of the system of syllogistics. And for those who derive satisfaction from the intricacies of analytical syllogistics, thinking through this deduction will be a pleasing, if somewhat arduous challenge.

We emphasize that if the chief aims of study are to recognize, understand, and apply the valid forms of the syllogism, as exhibited in Section 6.5, this appendix may be bypassed.

The deduction of the fifteen valid syllogisms is not easy to follow. Those who pursue it must keep two things very clearly in mind: (1) The rules of the syllogism, six basic rules set forth in Section 6.4, are the essential tools of the deduction; and (2) The four figures of the syllogism, as depicted in the Overview in Section 6.5 (p. 255) are referred to repeatedly as the rules are invoked.

We have seen that there are 256 possible forms of the syllogism, sixty-four moods (or combinations of the four categorical propositions) in each of the four figures. The deduction of the fifteen valid syllogisms proceeds by eliminating the syllogisms that violate one of the basic rules and that thus cannot be valid.

The conclusion of every syllogism is a categorical proposition, either A, or E, or I, or O. We begin by dividing all the possible syllogistic forms into four groups, each group having a conclusion with a different form (A, E, I, or O). Every syllogism must of course fall into one of these four groups. Taking each of the four groups in turn, we ask what characteristics a valid syllogism with such a conclusion must possess. That is, we ask what forms are excluded by one or more of the syllogistic rules if the conclusion is an A proposition, and if the conclusion is an E proposition, and so on.

After excluding all those invalid syllogisms, only the valid syllogisms remain. To assist in visualization, we note in the margin as we proceed the moods and figures, and the names, of the fifteen valid categorical syllogisms.
Case 1: If the conclusion of the syllogism is an A proposition
In this case, neither premise can be an E or an O proposition, because if either premise is negative, the conclusion must be negative (Rule 5). Therefore the two premises must be I or A propositions. The minor premise cannot be an I proposition because the minor term (the subject of the conclusion, which is an A) is distributed in the conclusion, and therefore if the minor premise were an I proposition, a term would be distributed in the conclusion that is not distributed in the premises, violating Rule 3. The two premises, major and minor, cannot be I and A, because if they were, either the distributed subject of the conclusion would not be distributed in the premise, violating Rule 3, or the middle term of the syllogism would not be distributed in either premise, violating Rule 2. So the two premises (if the conclusion is an A) must both be A as well, which means that the only possible valid mood is AAA. But in the second figure AAA again results in the middle term being distributed in neither premise; and in both the third figure and the fourth figure AAA results in a term being distributed in the conclusion that is not distributed in the premise in which it appears. Therefore, if the conclusion of the syllogism is an A proposition, the only valid form it can take is AAA in the first figure. This valid form, AAA–1, is the syllogism traditionally given the name Barbara.

Summary of Case 1: If the syllogism has an A conclusion, there is only one possibly valid form: AAA–1—Barbara.

Case 2: If the conclusion of the syllogism is an E proposition
Both the subject and the predicate of an E proposition are distributed, and therefore all three terms in the premises of a syllogism having such a conclusion must be distributed, and this is possible only if one of the premises is also an E. Both premises cannot be E propositions, because two negative premises are never allowed (Rule 4), and the other premise cannot be an O proposition because then both premises would also be negative. Nor can the other premise be an I proposition, for if it were, a term distributed in the conclusion would then not be distributed in the premise, violating Rule 3. So the other premise must be an A, and the two premises must be either AE or EA. The only possible moods (if the conclusion of the syllogism is an E proposition) are therefore AEE and EAE.

If the mood is AEE, it cannot be either in the first figure or in the third figure, because in either of those cases a term distributed in the conclusion would then not be distributed in the premises. Therefore, the mood AEE is possibly valid only in the second figure, AEE–2 (traditionally called Camestres), or in the fourth figure, AEE–4 (traditionally called Camenes). And if the mood is EAE, it cannot be in the third figure or in the fourth figure, because again that would mean that a term distributed in the conclusion would not be distributed in the premises, which leaves as valid only the first...
figure, \textit{EAE–1} (traditionally called \textit{Celarent}), and the second figure, \textit{EAE–2} (traditionally called \textit{Cesare}).

\textbf{Summary of Case 2:} If the syllogism has an \textit{E} conclusion, there are only four possibly valid forms: \textit{AEE–2}, \textit{AEE–4}, \textit{EAE–1}, and \textit{EAE–2}—\textit{Camestres}, \textit{Camenes}, \textit{Celarent}, and \textit{Cesare}, respectively.

\textbf{Case 3:} If the conclusion is an \textit{I} proposition

In this case, neither premise can be an \textit{E} or an \textit{O}, because if either premise is negative, the conclusion must be negative. The two premises cannot both be \textit{A}, because a syllogism with a particular conclusion cannot have two universal premises (Rule 6). Neither can both premises be \textit{I}, because the middle term must be distributed in at least one premise (Rule 2). So the premises must be either \textit{AI} or \textit{IA}, and therefore the only possible moods with an \textit{I} conclusion are \textit{AII} and \textit{IAI}.

\textit{AII} is not possibly valid in the second figure or in the fourth figure because the middle term must be distributed in at least one premise. The only valid forms remaining for the mood \textit{AII}, therefore, are \textit{AII–1} (traditionally called \textit{Darii}) and \textit{AII–3} (traditionally called \textit{Datisi}). If the mood is \textit{IAI}, it cannot be \textit{IAI–1} or \textit{IAI–2}, because they also would violate the rule that requires the middle term to be distributed in at least one premise. This leaves as valid only \textit{IAI–3} (traditionally called \textit{Disamis}), and \textit{IAI–4} (traditionally called \textit{Dimaris}).

\textbf{Summary of Case 3:} If the syllogism has an \textit{I} conclusion, there are only four possibly valid forms: \textit{AII–1}, \textit{AII–3}, \textit{IAI–3}, and \textit{IAI–4}—\textit{Darii}, \textit{Datisi}, \textit{Disamis}, and \textit{Dimaris}, respectively.

\textbf{Case 4:} If the conclusion is an \textit{O} proposition

In this case, the major premise cannot be an \textit{I} proposition, because any term distributed in the conclusion must be distributed in the premises. So the major premise must be either an \textit{A} or an \textit{E} or an \textit{O} proposition.

Suppose the major premise is an \textit{A}. In that case, the minor premise cannot be either an \textit{A} or an \textit{E}, because two universal premises are not permitted when the conclusion (an \textit{O}) is particular. Neither can the minor premise then be an \textit{I}, because if it were, either the middle term would not be distributed at all (a violation of Rule 2), or a term distributed in the conclusion would not be distributed in the premises. So, if the major premise is an \textit{A}, the minor premise has to be an \textit{O}, yielding the mood \textit{AOO}. In the fourth figure, \textit{AOO} cannot possibly be valid, because in that case the middle term would not be distributed, and in the first figure and the third figure \textit{AOO} cannot possibly be valid either, because that would result in terms being distributed in the conclusion.
that were not distributed in the premises. For the mood AOO, the only possibly valid form remaining, if the major premise is an A, is therefore in the second figure, AOO–2 (traditionally called Baroko).

Now suppose (if the conclusion is an O) that the major premise is an E. In that case, the minor premise cannot be either an E or an O, because two negative premises are not permitted. Nor can the minor premise be an A, because two universal premises are precluded if the conclusion is particular (Rule 6). This leaves only the mood EIO—and this mood is valid in all four figures, traditionally known as Ferio (EIO–1), Festino (EIO–2), Ferison (EIO–3), and Fresison (EIO–4).

Finally, suppose (if the conclusion is an O) that the major premise is also an O proposition. Then, again, the minor premise cannot be an E or an O, because two negative premises are forbidden. And the minor premise cannot be an I, because then the middle term would not be distributed, or a term that is distributed in the conclusion would not be distributed in the premises. Therefore, if the major premise is an O, the minor premise must be an A, and the mood must be OAO. But OAO–1 is eliminated, because in that case the middle term would not be distributed. OAO–2 and OAO–4 are also eliminated, because in both a term distributed in the conclusion would then not be distributed in the premises. This leaves as valid only OAO–3 (traditionally known as Bokardo).

Summary of Case 4: If the syllogism has an O conclusion, there are only six possibly valid forms: AOO–2, EIO–1, EIO–2, EIO–3, EIO–4, and OAO–3—Baroko, Ferio, Festino, Ferison, Fresison, and Bokardo.

This analysis has demonstrated, by elimination, that there are exactly fifteen valid forms of the categorical syllogism: one if the conclusion is an A proposition, four if the conclusion is an E proposition, four if the conclusion is an I proposition, and six if the conclusion is an O proposition. Of these fifteen valid forms, four are in the first figure, four are in the second figure, four are in the third figure, and three are in the fourth figure. This completes the deduction of the fifteen valid forms of the standard-form categorical syllogism.

**EXERCISES**

For students who enjoy the complexities of analytical syllogistics, here follow some theoretical questions whose answers can all be derived from the systematic application of the six rules of the syllogism set forth in Section 6.4. Answering these questions will be much easier if you have fully grasped the deduction of the fifteen valid syllogistic forms presented in this appendix.
EXAMPLE

1. Can any standard-form categorical syllogism be valid that contains exactly three terms, each of which is distributed in both of its occurrences?

SOLUTION

No, such a syllogism cannot be valid. If each of the three terms were distributed in both of its occurrences, all three of its propositions would have to be E propositions, and the mood of the syllogism would thus be EEE, which violates Rule 4, which forbids two negative premises.

2. In what mood or moods, if any, can a first-figure standard-form categorical syllogism with a particular conclusion be valid?

3. In what figure or figures, if any, can the premises of a valid standard-form categorical syllogism distribute both major and minor terms?

4. In what figure or figures, if any, can a valid standard-form categorical syllogism have two particular premises?

*5. In what figure or figures, if any, can a valid standard-form categorical syllogism have only one term distributed, and that one only once?

6. In what mood or moods, if any, can a valid standard-form categorical syllogism have just two terms distributed, each one twice?

7. In what mood or moods, if any, can a valid standard-form categorical syllogism have two affirmative premises and a negative conclusion?

8. In what figure or figures, if any, can a valid standard-form categorical syllogism have a particular premise and a universal conclusion?

9. In what mood or moods, if any, can a second figure standard-form categorical syllogism with a universal conclusion be valid?

*10. In what figure or figures, if any, can a valid standard-form categorical syllogism have its middle term distributed in both premises?

11. Can a valid standard-form categorical syllogism have a term distributed in a premise that appears undistributed in the conclusion?

SUMMARY

In this chapter we have examined the standard-form categorical syllogism: its elements, its forms, its validity, and the rules governing its proper use.
In Section 6.1, the major, minor, and middle terms of a syllogism were identified:

- **Major term:** the predicate of the conclusion
- **Minor term:** the subject of the conclusion
- **Middle term:** the third term appearing in both premises but not in the conclusion.

We identified major and minor premises as those containing the major and minor terms, respectively. We specified that a categorical syllogism is in standard form when its propositions appear in precisely this order: major premise first, minor premise second, and conclusion last.

We also explained in Section 6.1 how the mood and figure of a syllogism are determined.

The mood of a syllogism is determined by the three letters identifying the types of its three propositions, **A, E, I,** or **O.** There are sixty-four possible different moods.

The figure of a syllogism is determined by the position of the middle term in its premises. The four possible figures are described and named thus:

- **First figure:** The middle term is the subject term of the major premise and the predicate term of the minor premise. Schematically: \( M\rightarrow P, S\rightarrow M \), therefore \( S\rightarrow P \).
- **Second figure:** The middle term is the predicate term of both premises. Schematically: \( P\rightarrow M, S\rightarrow M \), therefore \( S\rightarrow P \).
- **Third figure:** The middle term is the subject term of both premises. Schematically: \( M\rightarrow P, M\rightarrow S \), therefore \( S\rightarrow P \).
- **Fourth figure:** The middle term is the predicate term of the major premise and the subject term of the minor premise. Schematically: \( P\rightarrow M, M\rightarrow S \), therefore \( S\rightarrow P \).

In Section 6.2, we explained how the mood and figure of a standard-form categorical syllogism jointly determine its logical form. Because each of the sixty-four moods may appear in all four figures, there are exactly 256 standard-form categorical syllogisms, of which only a few are valid.

In Section 6.3, we explained the Venn diagram technique for testing the validity of syllogisms, using overlapping circles appropriately marked or shaded to exhibit the meaning of the premises.

In Section 6.4, we explained the six essential rules for standard-form syllogisms and named the fallacy that results when each of these rules is broken:

- **Rule 1.** A standard-form categorical syllogism must contain exactly three terms, each of which is used in the same sense throughout the argument. Violation: Fallacy of four terms.
Rule 2. In a valid standard-form categorical syllogism, the middle term must be distributed in at least one premise. Violation: Fallacy of undistributed middle.

Rule 3. In a valid standard-form categorical syllogism, if either term is distributed in the conclusion, then it must be distributed in the premises. Violation: Fallacy of the illicit major, or fallacy of the illicit minor.

Rule 4. No standard-form categorical syllogism having two negative premises is valid. Violation: Fallacy of exclusive premises.

Rule 5. If either premise of a valid standard-form categorical syllogism is negative, the conclusion must be negative. Violation: Fallacy of drawing an affirmative conclusion from a negative premise.

Rule 6. No valid standard-form categorical syllogism with a particular conclusion can have two universal premises. Violation: Existential fallacy.

In Section 6.5, we presented an exposition of the fifteen valid forms of the categorical syllogism, identifying their moods and figures, and explaining their traditional Latin names:

AAA–1 (Barbara); EAE–1 (Celarent); AII–1 (Darii); EIO–1 (Ferio); AEE–2 (Camestres); EAE–2 (Cesare); AOO–2 (Baroko); EIO–2 (Festino); AII–3 (Datisi); IAI–3 (Disamis); EIO–3 (Ferison); OAO–3 (Bokardo); AEE–4 (Camenes); IAI–4 (Dimaris); EIO–4 (Fresison).

In the Appendix to Chapter 6 (which may be omitted), we presented the deduction of the fifteen valid forms of the categorical syllogism, demonstrating, through a process of elimination, that only those fifteen forms can avoid all violations of the six basic rules of the syllogism.