EXERCISES

A. If we assume that the first proposition in each of the following sets is true, what can we affirm about the truth or falsehood of the remaining propositions in each set? B. If we assume that the first proposition in each set is false, what can we affirm?

*1. a. All successful executives are intelligent people.
   b. No successful executives are intelligent people.
   c. Some successful executives are intelligent people.
   d. Some successful executives are not intelligent people.

2. a. No animals with horns are carnivores.
   b. Some animals with horns are carnivores.
   c. Some animals with horns are not carnivores.
   d. All animals with horns are carnivores.

3. a. Some uranium isotopes are highly unstable substances.
   b. Some uranium isotopes are not highly unstable substances.
   c. All uranium isotopes are highly unstable substances.
   d. No uranium isotopes are highly unstable substances.

4. a. Some college professors are not entertaining lecturers.
   b. All college professors are entertaining lecturers.
   c. No college professors are entertaining lecturers.
   d. Some college professors are entertaining lecturers.

5.6 Further Immediate Inferences

Three other important kinds of immediate inference are not associated directly with the square of opposition: Conversion, Obversion, and Contraposition. These we now explain:

A. CONVERSION

Conversion is an inference that proceeds by interchanging the subject and predicate terms of the proposition. “No men are angels” converts to “No angels are men,” and these propositions may be validly inferred from one another. Similarly, “Some women are writers” and “Some writers are women” are logically equivalent, and by conversion either can be validly inferred from the other. Conversion is perfectly valid for all E propositions and for all I propositions. One standard-form categorical proposition is therefore said to be the converse of another when we derive it by simply interchanging the subject and predicate terms of that other proposition. The proposition from which it is derived is called the convertend. Thus, “No idealists are politicians” is the converse of “No politicians are idealists,” which is its convertend.
The conversion of an $O$ proposition is not, in general, valid. The $O$ proposition, “Some animals are not dogs,” is plainly true; its converse is the proposition, “Some dogs are not animals,” which is plainly false. An $O$ proposition and its converse are not, in general, logically equivalent.

The $A$ proposition presents a special problem here. Of course, the converse of an $A$ proposition does not in general follow from its convertend. From “All dogs are animals” we certainly may not infer that “All animals are dogs.” Traditional logic recognized this, of course, but asserted, nevertheless, that something like conversion was valid for $A$ propositions. On the traditional square of opposition, one could validly infer from the $A$ proposition, “All dogs are animals,” its subaltern $I$ proposition, “Some dogs are animals.” The $A$ proposition says something about all members of the subject class (dogs), the $I$ proposition makes a more limited claim, about only some of the members of that class. In general, it was held that one could infer “Some $S$ is $P$” from “All $S$ is $P$.” And, as we saw earlier, an $I$ proposition may be converted validly; if some dogs are animals, then some animals are dogs.

So, if we are given the $A$ proposition that “All dogs are animals,” we first infer that “Some dogs are animals” by subalternation, and from that subaltern we can by conversion validly infer that “Some animals are dogs.” Hence, by a combination of subalternation and conversion, we advance validly from “All $S$ is $P$” to “Some $P$ is $S$.” This pattern of inference, called conversion by limitation (or conversion per accidens) proceeds by interchanging subject and predicate terms and changing the quantity of the proposition from universal to particular. This type of conversion will be considered further in the next section.

In all conversions, the converse of a given proposition contains exactly the same subject and predicate terms as the convertend, their order being reversed, and always has the same quality (of affirmation or denial). A complete picture of this immediate inference as traditionally understood is given by the following table:

<table>
<thead>
<tr>
<th>Convertend</th>
<th>Converse</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$: All $S$ is $P$.</td>
<td>$I$: Some $P$ is $S$. (by limitation)</td>
</tr>
<tr>
<td>$E$: No $S$ is $P$.</td>
<td>$E$: No $P$ is $S.$</td>
</tr>
<tr>
<td>$I$: Some $S$ is $P$.</td>
<td>$I$: Some $P$ is $S.$</td>
</tr>
<tr>
<td>$O$: Some $S$ is not $P$.</td>
<td>(conversion not valid)</td>
</tr>
</tbody>
</table>
B. CLASSES AND CLASS COMPLEMENTS

To explain other types of immediate inference we must examine more closely the concept of a “class” and explain what is meant by the complement of a class. Any class, we have said, is the collection of all objects that have a certain common attribute, which we may refer to as the “class-defining characteristic.” The class of all humans is the collection of all things that have the characteristic of being human; its class-defining characteristic is the attribute of being human. The class-defining characteristic need not be a “simple” attribute; any attribute may determine a class. For example, the complex attribute of being left-handed and red-headed and a student determines a class—the class of all left-handed, red-headed students.

Every class has, associated with it, a complementary class, or complement, which is the collection of all things that do not belong to the original class. The complement of the class of all people is the class of all things that are not people. The class-defining characteristic of that complementary class is the (negative) attribute of not being a person. The complement of the class of all people contains no people, but it contains everything else: shoes and ships and sealing wax and cabbages—but no kings, because kings are people. It is often convenient to speak of the complement of the class of all persons as the “class of all nonpersons.” The complement of the class designated by the term $S$ is then designated by the term $\text{non}-S$; we may speak of the term $\text{non}-S$ as being the complement of the term $S$.

The word complement is thus used in two senses. In one sense it is the complement of a class, in the other it is the complement of a term. These are different but very closely connected. One term is the (term) complement of another just in case the first term designates the (class) complement of the class designated by the second term.

Note that a class is the (class) complement of its own complement. Likewise, a term is the (term) complement of its own complement. A sort of “double negative” rule is involved here, to avoid strings of “non’s” prefixed to a term. Thus, the complement of the term “voter” is “nonvoter,” but the complement of “nonvoter” should be written simply as “voter” rather than as “nonnonvoter.”

One must be careful not to mistake contrary terms for complementary terms. “Coward” and “hero” are contraries, because no person can be both a coward and a hero. But we must not identify “cowards” with “nonheroes” because not everyone, and certainly not everything, need be one or the other.

*Sometimes we reason using what is called the relative complement of a class, its complement within some other class. For example, within the class of “children of mine” there is a subclass, “daughters of mine,” whose relative complement is another subclass, “children of mine who are not daughters,” or “sons of mine.” But obversions, and other immediate inferences, rely on the absolute complement of classes, as defined above.
Likewise, the complement of the term “winner” is not “loser” but “nonwinner,” for although not everything, or even everyone, is either a winner or a loser, absolutely everything is either a winner or a nonwinner.

C. OBVERSION

Obversion is an immediate inference that is easy to explain once the concept of a term complement is understood. To obvert a proposition, we change its quality (affirmative to negative or negative to affirmative) and replace the predicate term with its complement. However, the subject term remains unchanged, and so does the quantity of the proposition being obverted. For example, the A proposition, “All residents are voters,” has as its obverse the E proposition, “No residents are nonvoters.” These two are logically equivalent propositions, and either may be validly inferred from the other.

Obversion is a valid immediate inference when applied to any standard-form categorical proposition:

- The E proposition, “No umpires are partisans,” has as its obverse the logically equivalent A proposition, “All umpires are nonpartisans.”
- The I proposition, “Some metals are conductors,” has as its obverse the O proposition, “Some metals are not nonconductors.”
- The O proposition, “Some nations were not belligerents,” has as its obverse the I proposition, “Some nations were nonbelligerents.”

The proposition serving as premise for the obversion is called the obvertend; the conclusion of the inference is called the obverse. Every standard-form categorical proposition is logically equivalent to its obverse, so obversion is a valid form of immediate inference for all standard-form categorical propositions. To obtain the obverse of any proposition, we leave the quantity (universal or particular) and the subject term unchanged; we change the quality of the proposition and replace the predicate term with its complement. The following table gives a complete picture of all valid obversions:

### OVERVIEW

<table>
<thead>
<tr>
<th>Obvertend</th>
<th>Obverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: All S is P.</td>
<td>E: No S is non-P.</td>
</tr>
<tr>
<td>E: No S is P.</td>
<td>A: All S is non-P.</td>
</tr>
<tr>
<td>I: Some S is P.</td>
<td>O: Some S is not non-P.</td>
</tr>
<tr>
<td>O: Some S is not P.</td>
<td>I: Some S is non-P.</td>
</tr>
</tbody>
</table>
D. CONTRAPOSITION

A third type of immediate inference, contraposition, can be reduced to the first two, conversion and obversion. To form the contrapositive of a given proposition, we replace its subject term with the complement of its predicate term, and we replace its predicate term with the complement of its subject term. Neither the quality nor the quantity of the original proposition is changed, so the contrapositive of an A proposition is an A proposition, the contrapositive of an O proposition is an O proposition, and so forth.

For example, the contrapositive of the A proposition, “All members are voters,” is the A proposition, “All nonvoters are nonmembers.” These are logically equivalent propositions, as will be evident on reflection. Contraposition is plainly a valid form of immediate inference when applied to A propositions. It really introduces nothing new, because we can get from any A proposition to its contrapositive by first obverting it, next applying conversion, and then applying obversion again. Beginning with “All S is P,” we obvert it to obtain “No S is non-P,” which converts validly to “No non-P is S,” whose obverse is “All non-P is non-S.” The contrapositive of any A proposition is the obverse of the converse of the obverse of that proposition.

Contraposition is a valid form of immediate inference when applied to O propositions also, although its conclusion may be awkward to express. The contrapositive of the O proposition, “Some students are not idealists,” is the somewhat cumbersome O proposition, “Some nonidealists are not nonstudents,” which is logically equivalent to its premise. This also can be shown to be the outcome of first obverting, then converting, then obverting again. “Some S is not P” obverts to “Some S is non-P,” which converts to “Some non-P is S,” which obverts to “Some non-P is not non-S.”

For I propositions, however, contraposition is not, in general, a valid form of inference. The true I proposition, “Some citizens are nonlegislators,” has as its contrapositive the false proposition, “Some legislators are noncitizens.” The reason for this invalidity becomes evident when we try to derive the contrapositive of the I proposition by successively obverting, converting, and obverting. The obverse of the original I proposition, “Some S is P,” is the O proposition, “Some S is not non-P,” but (as we saw earlier) the converse of an O proposition does not generally follow validly from it.

In the case of E propositions, the contrapositive does not follow validly from the original, as can be seen when, if we begin with the true proposition, “No wrestlers are weaklings,” we get, as its contrapositive, the obviously false proposition, “No nonweaklings are nonwrestlers.” The reason for this invalidity we will see, again, if we attempt to derive it by successive obversion, conversion, and obversion. If we begin with the E proposition, “No S is P,” and obvert it, we obtain the A proposition, “All S is non-P”—which in general
cannot be validly converted except by limitation. If we do then convert it by limitation to obtain “Some non-P is S,” we can obvert this to obtain “Some non-P is not non-S.” This outcome we may call the contrapositive by limitation—and this too we will consider further in the next section.

Contraposition by limitation, in which we infer an O proposition from an E proposition (for example, we infer “Some non-P is not non-S” from “No S is P”), has the same peculiarity as conversion by limitation, on which it depends. Because a particular proposition is inferred from a universal proposition, the resulting contrapositive cannot have the same meaning, and cannot be logically equivalent to the proposition that was the original premise. On the other hand, the contrapositive of an A proposition is an A proposition, and the contrapositive of an O proposition is an O proposition, and in each of these cases the contrapositive and the premise from which it is derived are equivalent.

Contraposition is thus seen to be valid only when applied to A and O propositions. It is not valid at all for I propositions, and it is valid for E propositions only by limitation. The complete picture is exhibited in the following table:

<table>
<thead>
<tr>
<th>Premise</th>
<th>Contrapositive</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: All S is P.</td>
<td>A: All non-P is non-S.</td>
</tr>
<tr>
<td>E: No S is P.</td>
<td>O: Some non-P is not non-S. (by limitation)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(contraposition not valid)</td>
</tr>
<tr>
<td>I: Some S is P.</td>
<td>O: Some non-P is not non-S.</td>
</tr>
<tr>
<td>O: Some S is not-P.</td>
<td></td>
</tr>
</tbody>
</table>

Questions about the relations between propositions can often be answered by exploring the various immediate inferences that can be drawn from one or the other of them. For example, given that the proposition, “All surgeons are physicians,” is true, what can we know about the truth or falsehood of the proposition, “No nonsurgeons are nonphysicians?” Does this problematic proposition—or its contradictory or contrary—follow validly from the one given as true? To answer we proceed as follows: From what we are given, “All surgeons are physicians,” we can validly infer its contrapositive, “All nonphysicians are nonsurgeons.” From this, using conversion by limitation (valid according to the traditional view), we can derive “Some nonsurgeons are nonphysicians.” But this is the contradictory of the proposition in question.
(“No nonsurgeons are nonphysicians”), which is thus no longer problematic but known to be false.

In the very first chapter of this book we noted that a valid argument whose premises are true must have a true conclusion, but also that a valid argument whose premises are false can have a true conclusion. Thus, from the false premise, “All animals are cats,” the true proposition, “Some animals are cats,” follows by subalternation. And from the false proposition, “All parents are students,” conversion by limitation yields the true proposition, “Some students are parents.” Therefore, if a proposition is given to be false, and the question is raised about the truth or falsehood of some other, related proposition, the recommended procedure is to begin drawing immediate inferences from either (1) the contradictory of the proposition known to be false, or (2) the problematic proposition itself. The contradictory of a false proposition must be true, and all valid inferences from that will also be true propositions. And if we follow the other course and are able to show that the problematic proposition implies the proposition that is given is false, we know that it must itself be false. Here follows a table in which the forms of immediate inference—conversion, obversion, and contraposition—are fully displayed:

**Immediate Inferences: Conversion, Obversion, Contraposition**

<table>
<thead>
<tr>
<th>Convertend</th>
<th>Converse</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong>: All $S$ is $P$.</td>
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<tr>
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<td><strong>E</strong>: No $P$ is $S.$</td>
</tr>
<tr>
<td><strong>I</strong>: Some $S$ is $P.$</td>
<td><strong>I</strong>: Some $P$ is $S.$</td>
</tr>
<tr>
<td><strong>O</strong>: Some $S$ is not $P.$</td>
<td>(conversion not valid)</td>
</tr>
</tbody>
</table>

**Obversion**

<table>
<thead>
<tr>
<th>Obvertend</th>
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</tr>
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<tbody>
<tr>
<td><strong>A</strong>: All $S$ is $P.$</td>
<td><strong>E</strong>: No $S$ is non-$P$.</td>
</tr>
<tr>
<td><strong>E</strong>: No $S$ is $P.$</td>
<td><strong>A</strong>: All $S$ is non-$P$.</td>
</tr>
<tr>
<td><strong>I</strong>: Some $S$ is $P.$</td>
<td><strong>O</strong>: Some $S$ is not non-$P$.</td>
</tr>
<tr>
<td><strong>O</strong>: Some $S$ is not $P.$</td>
<td><strong>I</strong>: Some $S$ is non-$P$.</td>
</tr>
</tbody>
</table>

**Contraposition**

<table>
<thead>
<tr>
<th>Premise</th>
<th>Contrapositive</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong>: All $S$ is $P.$</td>
<td><strong>A</strong>: All non-$P$ is non-$S$.</td>
</tr>
</tbody>
</table>
EXERCISES

A. State the converses of the following propositions, and indicate which of them are equivalent to the given propositions.

*1. No people who are considerate of others are reckless drivers who pay no attention to traffic regulations.
2. All graduates of West Point are commissioned officers in the U.S. Army.
3. Some European cars are overpriced and underpowered automobiles.
4. No reptiles are warm-blooded animals.
*5. Some professional wrestlers are elderly persons who are incapable of doing an honest day’s work.

B. State the obverses of the following propositions.

*1. Some college athletes are professionals.
2. No organic compounds are metals.
3. Some clergy are not abstainers.
4. No geniuses are conformists.
*5. All objects suitable for boat anchors are objects that weigh at least fifteen pounds.

C. State the contrapositives of the following propositions and indicate which of them are equivalent to the given propositions.

*1. All journalists are pessimists.
2. Some soldiers are not officers.
3. All scholars are nondegenerates.
4. All things weighing less than fifty pounds are objects not more than four feet high.
*5. Some noncitizens are not nonresidents.

D. If “All socialists are pacifists” is true, what may be inferred about the truth or falsehood of the following propositions? That is, which can be
known to be true, which can be known to be false, and which are undetermined?

*1. Some nonpacifists are not nonsocialists.
2. No socialists are nonpacifists.
3. All nonsocialists are nonpacifists.
4. No nonpacifists are socialists.
*5. No nonsocialists are nonpacifists.
6. All nonpacifists are nonsocialists.
7. No pacifists are nonsocialists.
8. Some socialists are not pacifists.
9. All pacifists are socialists.
*10. Some nonpacifists are socialists.

E. If “No scientists are philosophers” is true, what may be inferred about the truth or falsehood of the following propositions? That is, which can be known to be true, which can be known to be false, and which are undetermined?

*1. No nonphilosophers are scientists.
2. Some nonphilosophers are not nonscientists.
3. All nonscientists are nonphilosophers.
4. No scientists are nonphilosophers.
*5. No nonscientists are nonphilosophers.
6. All philosophers are scientists.
7. Some nonphilosophers are scientists.
8. All nonphilosophers are nonscientists.
9. Some scientists are not philosophers.
*10. No philosophers are nonscientists.

F. If “Some saints were martyrs” is true, what may be inferred about the truth or falsehood of the following propositions? That is, which can be known to be true, which can be known to be false, and which are undetermined?

*1. All saints were martyrs.
2. All saints were nonmartyrs.
3. Some martyrs were saints.
4. No saints were martyrs.
*5. All martyrs were nonsaints.
6. Some nonmartyrs were saints.
7. Some saints were not nonmartyrs.
8. No martyrs were saints.
9. Some nonsaints were martyrs.
*10. Some martyrs were nonsaints.
11. Some saints were not martyrs.
12. Some martyrs were not saints.
13. No saints were nonmartyrs.
14. No nonsaints were martyrs.
*15. Some martyrs were not nonsaints.

G. If “Some merchants are not pirates” is true, what may be inferred about the truth or falsehood of the following propositions? That is, which can be known to be true, which can be known to be false, and which are undetermined?

*1. No pirates are merchants.
2. No merchants are nonpirates.
3. Some merchants are nonpirates.
4. All nonmerchants are pirates.
*5. Some nonmerchants are nonpirates.
6. All merchants are pirates.
7. No nonmerchants are pirates.
8. No pirates are nonmerchants.
9. All nonpirates are nonmerchants.
*10. Some nonpirates are not nonmerchants.
11. Some nonpirates are merchants.
12. No nonpirates are merchants.
13. Some pirates are merchants.
14. No merchants are nonpirates.
*15. No merchants are pirates.

5.7 Existential Import and the Interpretation of Categorical Propositions

Categorical propositions are the building blocks of arguments, and our aim throughout is to analyze and evaluate arguments. To do this we must be able to diagram and symbolize the A, E, I, and O propositions. But before we can do that we must confront and resolve a deep logical problem—one that has