deductive arguments about probabilities themselves, in which the probability of a certain combination of events is deduced from the probabilities of other events.*

In sum, the distinction between induction and deduction rests on the nature of the claims made by the two types of arguments about the relations between their premises and their conclusions. Thus we characterize the two types of arguments as follows: A deductive argument is one whose conclusion is claimed to follow from its premises with absolute necessity, this necessity not being a matter of degree and not depending in any way on whatever else may be the case. In sharp contrast, an inductive argument is one whose conclusion is claimed to follow from its premises only with probability, this probability being a matter of degree and dependent on what else may be the case.

1.6 Validity and Truth

A deductive argument is valid when it is successful. Its validity refers to the relation between its propositions—between the set of propositions that serve as the premises and the one proposition that serves as the conclusion of that argument. If the conclusion follows with logical necessity from the premises, we say that the argument is valid. Therefore validity can never apply to any single proposition by itself, because the needed relation cannot possibly be found within any one proposition.

Truth and falsity, on the other hand, are attributes of individual propositions. A single statement that serves as a premise in an argument may be true; the statement that serves as its conclusion may be false. This conclusion might have been validly inferred, but to say that any conclusion (or any single premise) is itself valid or invalid makes no sense.

Truth is the attribute of a proposition that asserts what really is the case. When I assert that Lake Superior is the largest of the five Great Lakes, I assert what really is the case, what is true. If I had claimed that Lake Michigan is the largest of the Great Lakes my assertion would not be in accord with the real world; therefore it would be false. This contrast between validity and truth is important: Truth and falsity are attributes of individual propositions or statements; validity and invalidity are attributes of arguments.

Just as the concept of validity cannot apply to single propositions, the concept of truth cannot apply to arguments. Of the several propositions in an

*If, for example, we learn that the probability of three successive heads in three tosses of a coin is 1/8, we may infer deductively that the probability of getting at least one tail in three tosses of a coin is 7/8. Other illustrations of such arguments are given in Chapter 14.
argument, some (or all) may be true and some (or all) may be false. However, the argument as a whole is neither true nor false. Propositions, which are statements about the world, may be true or false; deductive arguments, which consist of inferences from one set of propositions to other propositions, may be valid or invalid.

The relations between true (or false) propositions and valid (or invalid) arguments are critical and complicated. Those relations lie at the heart of deductive logic. Part II of this book is devoted largely to examination of those complex relations, but a preliminary discussion of the relation between validity and truth is in order here.

We begin by emphasizing that an argument may be valid even if one or more of its premises is not true. Every argument makes a claim about the relation between its premises and the conclusion drawn from them; that relation may hold even if the premises turn out to be false or the truth of the premises is in dispute. This point was made dramatically by Abraham Lincoln in 1858 in one of his debates with Stephen Douglas. Lincoln was attacking the *Dred Scott* decision of the Supreme Court, which had held that slaves who had escaped into Northern states must be returned to their owners in the South. Lincoln said:

I think it follows [from the *Dred Scott* decision] and submit to the consideration of men capable of arguing, whether as I state it in syllogistic form the argument has any fault in it:

Nothing in the Constitution or laws of any State can destroy a right distinctly and expressly affirmed in the Constitution of the United States.

The right of property in a slave is distinctly and expressly affirmed in the Constitution of the United States.

Therefore, nothing in the Constitution or laws of any State can destroy the right of property in a slave.

I believe that no fault can be pointed out in that argument; assuming the truth of the premises, the conclusion, so far as I have capacity at all to understand it, follows inevitably. There is a fault in it as I think, but the fault is not in the reasoning; but the falsehood in fact is a fault of the premises. I believe that the right of property in a slave is not distinctly and expressly affirmed in the Constitution, and Judge Douglas thinks it is. I believe that the Supreme Court and the advocates of that decision [the *Dred Scott* decision] may search in vain for the place in the Constitution where the right of property in a slave is distinctly and expressly affirmed. I say, therefore, that I think one of the premises is not true in fact.26

The reasoning in the argument that Lincoln recapitulates and attacks is not faulty—but its second premise (that “the right of property in a slave is . . . affirmed in the Constitution”) is plainly false. The conclusion has therefore not
been established. Lincoln’s logical point is correct and important: *An argument may be valid even when its conclusion and one or more of its premises are false.* The validity of an argument, we emphasize once again, depends only on the relation of the premises to the conclusion.

There are many possible combinations of true and false premises and conclusions in both valid and invalid arguments. Here follow seven illustrative arguments, each prefaced by the statement of the combination (or truth and validity) that it represents. With these illustrations (whose content is deliberately trivial) before us, we will be in a position to formulate some important principles concerning the relations between truth and validity.

I. Some *valid* arguments contain *only true* propositions—true premises and a true conclusion:

All mammals have lungs.
All whales are mammals.
Therefore all whales have lungs.

II. Some *valid* arguments contain *only false* propositions—false premises and a false conclusion:

All four-legged creatures have wings.
All spiders have four legs.
Therefore all spiders have wings.

This argument is valid because, if its premises were true, its conclusion would have to be true also—even though we know that in fact both the premises *and* the conclusion of this argument are false.

III. Some *invalid* arguments contain *only true* propositions—all their premises are true, and their conclusions are true as well:

If I owned all the gold in Fort Knox, then I would be wealthy.
I do not own all the gold in Fort Knox.
Therefore I am not wealthy.

The true conclusion of this argument does not follow from its true premises. This will be seen more clearly when the immediately following illustration is considered.

IV. Some *invalid* arguments contain *only true premises* and have a *false conclusion.* This is illustrated by an argument exactly like the previous one (III) in form, changed only enough to make the conclusion false.

If Bill Gates owned all the gold in Fort Knox, then Bill Gates would be wealthy.
Bill Gates does not own all the gold in Fort Knox.
Therefore Bill Gates is not wealthy.
The premises of this argument are true, but its conclusion is false. Such an argument cannot be valid because it is impossible for the premises of a valid argument to be true and its conclusion to be false.

V. Some valid arguments have false premises and a true conclusion:

All fishes are mammals.
All whales are fishes.
Therefore all whales are mammals.

The conclusion of this argument is true, as we know; moreover, it may be validly inferred from these two premises, both of which are wildly false.

VI. Some invalid arguments also have false premises and a true conclusion:

All mammals have wings.
All whales have wings.
Therefore all whales are mammals.

From Examples V and VI taken together, it is clear that we cannot tell from the fact that an argument has false premises and a true conclusion whether it is valid or invalid.

VII. Some invalid arguments, of course, contain all false propositions—false premises and a false conclusion:

All mammals have wings.
All whales have wings.
Therefore all mammals are whales.

These seven examples make it clear that there are valid arguments with false conclusions (Example II), as well as invalid arguments with true conclusions (Examples III and VI). Hence it is clear that the truth or falsity of an argument’s conclusion does not by itself determine the validity or invalidity of that argument. Moreover, the fact that an argument is valid does not guarantee the truth of its conclusion (Example II).

Two tables (referring to the seven preceding examples) will make very clear the variety of possible combinations. The first table shows that invalid arguments can have every possible combination of true and false premises and conclusions:

<table>
<thead>
<tr>
<th>Invalid Arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>True Premises</strong></td>
</tr>
<tr>
<td>True Conclusion</td>
</tr>
<tr>
<td>Example III</td>
</tr>
<tr>
<td>Example VI</td>
</tr>
</tbody>
</table>
The second table shows that valid arguments can have only three of those combinations of true and false premises and conclusions:

<table>
<thead>
<tr>
<th>Valid Arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>True Premises</td>
</tr>
<tr>
<td>Example I</td>
</tr>
<tr>
<td>False Premises</td>
</tr>
</tbody>
</table>

The one blank position in the second table exhibits a fundamental point: *If an argument is valid and its premises are true, we may be certain that its conclusion is true also.* To put it another way: *If an argument is valid and its conclusion is false, not all of its premises can be true.* Some perfectly valid arguments do have false conclusions, but any such argument must have at least one false premise.

When an argument is valid and all of its premises are true, we call it *sound.* The conclusion of a sound argument obviously must be true—and only a sound argument can establish the truth of its conclusion. If a deductive argument is not sound—that is, if the argument is not valid or if not all of its premises are true—it fails to establish the truth of its conclusion even if in fact the conclusion is true.

To test the truth or falsehood of premises is the task of science in general, because premises may deal with any subject matter at all. The logician is not (professionally) interested in the truth or falsehood of propositions so much as in the logical relations between them. By *logical relations between propositions* we mean those relations that determine the correctness or incorrectness of the arguments in which they occur. The task of determining the correctness or incorrectness of arguments falls squarely within the province of logic. The logician is interested in the correctness even of arguments whose premises may be false.

Why do we not confine ourselves to arguments with true premises, ignoring all others? Because the correctness of arguments whose premises are not known to be true may be of great importance. In science, for example, we verify theories by *deducing* testable consequences from uncertain theoretical premises—but we cannot know beforehand which theories are true. In everyday life also, we must often choose between alternative courses of action, first seeking to deduce the consequences of each. To avoid deceiving ourselves, we must reason correctly about the consequences of the alternatives, taking each as a premise. If we were interested only in arguments with true premises, we would not know which set of consequences to trace out until we knew which of the alternative premises was true. But if we knew which of the alternative premises was true, we would not need to reason about it at all, because our purpose was to help us decide which alternative premise to *make* true. To confine our attention to arguments with premises known to be true would therefore be self-defeating.
Effective methods for establishing the validity or invalidity of deductive arguments are presented and explained at length in Part II of this book.

**EXERCISES**

Construct a series of deductive arguments, on any subject of your choosing, each with only two premises, having the following characteristics:

1. A valid argument with one true premise, one false premise, and a false conclusion
2. A valid argument with one true premise, one false premise, and a true conclusion
3. An invalid argument with two true premises and a false conclusion
4. An invalid argument with two true premises and a true conclusion
5. A valid argument with two false premises and a true conclusion
6. An invalid argument with two false premises and a true conclusion
7. An invalid argument with one true premise, one false premise, and a true conclusion
8. A valid argument with two true premises and a true conclusion

**SUMMARY**

The most fundamental concepts of logic are introduced in this chapter.

In Section 1.1 we explained why logic is defined as the study of the methods and principles used to distinguish correct from incorrect reasoning.

In Section 1.2 we gave an account of propositions, which may be asserted or denied, and which are either true or false, and of arguments—the central concern of logicians—which are clusters of propositions of which one is the conclusion and the others are the premises offered in its support.

In Section 1.3 we discussed difficulties in the recognition of arguments, arising from the variety of ways in which the propositions they contain may be expressed, and sometimes even the absence of their express statement in arguments called enthymemes.

In Section 1.4 we discussed the differences between arguments and explanations, showing why this distinction often depends on the context, and the intent of the passage in that context.

In Section 1.5 we explained the fundamental difference between deductive arguments, whose conclusions may be certain (if the premises are true and the reasoning valid), and inductive arguments, aiming to establish matters of fact, whose conclusions may be very probable but are never certain.