132. A person throws a rock upward from the edge of an 80-foot cliff. The height, \( h \), in feet, of the rock above the water at the bottom of the cliff after \( t \) seconds is described by the formula

\[
h = -16t^2 + 64t + 80.
\]

How long will it take for the rock to reach the water?

133. A rectangular swimming pool is 12 meters long and 8 meters wide. A tile border of uniform width is to be built around the pool using 120 square meters of tile. The tile is from a discontinued stock (so no additional materials are available), and all 120 square meters are to be used. How wide should the border be? Round to the nearest tenth of a meter. If zoning laws require at least a 2-meter-wide border around the pool, can this be done with the available tile?

**Group Exercise**

134. Each group member should find an “intriguing” algebraic formula that contains an expression in the form \( ax^2 + bx + c \) on one side. Consult college algebra books or liberal arts mathematics books to do so. Group members should select four of the formulas. For each formula selected, write and solve a problem similar to Exercises 99–102 in this exercise set.
to solve certain kinds of polynomial equations, equations involving rational exponents, and equations involving absolute value.

### Solve polynomial equations by factoring.

#### Polynomial Equations

The linear and quadratic equations that we studied in the first part of this chapter can be thought of as polynomial equations of degrees 1 and 2, respectively. By contrast, consider the following polynomial equations of degree greater than 2:

\[
3x^4 = 27x^2 \\
x^3 + x^2 = 4x + 4
\]

This equation is of degree 4 because 4 is the largest exponent.

This equation is of degree 3 because 3 is the largest exponent.

We can solve these equations by moving all terms to one side, thereby obtaining zero on the other side. We then use factoring and the zero-product principle.

#### EXAMPLE 1 Solving a Polynomial Equation by Factoring

Solve by factoring: \(3x^4 = 27x^2\).

**Solution**

**Step 1** Move all terms to one side and obtain zero on the other side. Subtract 27\(x^2\) from both sides.

\[
3x^4 = 27x^2 \\
3x^4 - 27x^2 = 27x^2 - 27x^2 \\
3x^4 - 27x^2 = 0
\]

This is the given equation.

Subtract 27\(x^2\) from both sides.

Simplify.

**Step 2** Factor. We can factor 3\(x^2\) from each term.

\[
3x^4 - 27x^2 = 0 \\
3x^2(x^2 - 9) = 0
\]

**Steps 3 and 4** Set each factor equal to zero and solve the resulting equations.

\[
3x^2 = 0 \quad \text{or} \quad x^2 - 9 = 0 \\
x^2 = 0 \quad x^2 = 9
\]

\[
x = ±\sqrt{0} \quad x = ±\sqrt{9} \\
x = 0 \quad x = ±3
\]

**Step 5** Check the solutions in the original equation. Check the three solutions, 0, -3, and 3, by substituting them into the original equation. Can you verify that the solution set is \{-3, 0, 3\}?
EXAMPLE 2  Solving a Polynomial Equation by Factoring

Solve by factoring: \( x^3 + x^2 = 4x + 4 \).

**Solution**

**Step 1** Move all terms to one side and obtain zero on the other side. Subtract 4x and subtract 4 from both sides.

\[
x^3 + x^2 - 4x - 4 = 4x + 4 - 4x - 4 = 0
\]

**Step 2** Factor. Because there are four terms, we use factoring by grouping. Group terms that have a common factor.

\[
x^3 + x^2 - 4x - 4 = 0
\]

Steps 3 and 4 Set each factor equal to zero and solve the resulting equations.

\[
x + 1 = 0 \quad \text{or} \quad x^2 - 4 = 0
\]

\[
x = -1 \quad \text{or} \quad x = \pm 2
\]

**Step 5** Check the solutions in the original equation. Check the three solutions, \(-1\), \(-2\), and 2, by substituting them into the original equation. Can you verify that the solution set is \(\{-2, -1, 2\}\)?

**Technology**

You can use a graphing utility to check the solutions of \( x^3 + x^2 - 4x - 4 = 0 \). Graph \( y = x^3 + x^2 - 4x - 4 \), as shown on the left. The x-intercepts are \(-2, -1, 2\), corresponding to the equation’s solutions.

2 Solve radical equations.

**Equations Involving Radicals**

A radical equation is an equation in which the variable occurs in a square root, cube root, or any higher root. An example of a radical equation is \( 28.5\sqrt{x} = 57 \). The variable occurs in a cube root.
The equation \( 28.5 \sqrt{x} = 57 \) can be used to find the area, \( x \), in square miles, of a Galápagos island with 57 species of plants. First, we isolate the radical by dividing both sides of the equation by 28.5.

\[
\frac{28.5 \sqrt{x}}{28.5} = \frac{57}{28.5}
\]

\[
\sqrt{x} = 2
\]

Next, we eliminate the radical by raising each side of the equation to a power equal to the index of the radical. Because the index is 3, we cube both sides of the equation.

\[
(\sqrt[3]{x})^3 = 2^3
\]

\[
x = 8
\]

Thus, a Galápagos island with 57 species of plants has an area of 8 square miles.

The Galápagos equation shows that solving equations involving radicals involves raising both sides of the equation to a power equal to the radicals index. All solutions of the original equation are also solutions of the resulting equation. However, the resulting equation may have some extra solutions that do not satisfy the original equation. Because the resulting equation may not be equivalent to the original equation, we must check each proposed solution by substituting it into the original equation. Let’s see exactly how this works.

**EXAMPLE 3 Solving an Equation Involving a Radical**

Solve: \( x + \sqrt{26 - 11x} = 4 \).

**Solution** To solve this equation, we isolate the radical expression \( \sqrt{26 - 11x} \) on one side of the equation. By squaring both sides of the equation, we can then eliminate the square root.

\[
x + \sqrt{26 - 11x} = 4
\]

\[
x + \sqrt{26 - 11x} - x = 4 - x
\]

\[
\sqrt{26 - 11x} = 4 - x
\]

\[
(\sqrt{26 - 11x})^2 = (4 - x)^2
\]

\[
26 - 11x = 16 - 8x + x^2
\]

Next, we need to write this quadratic equation in general form. We can obtain zero on the left side by subtracting 26 and adding \(11x\) on both sides.

\[
26 - 26 - 11x + 11x = 16 - 26 - 8x + 11x + x^2
\]

\[
0 = x^2 + 3x - 10
\]

\[
0 = (x + 5)(x - 2)
\]

\[
x + 5 = 0 \quad \text{or} \quad x - 2 = 0
\]

\[
x = -5 \quad \text{or} \quad x = 2
\]

We have not completed the solution process. Although -5 and 2 satisfy the squared equation, there is no guarantee that they satisfy the original equation. Thus, we must check the proposed solutions. We can do this using a graphing utility (see the technology box in the margin) or by substituting both proposed solutions into the given equation.
Solve and check: \( \sqrt{6x + 7} - x = 2 \).

When solving a radical equation, extra solutions may be introduced when you raise both sides of the equation to an even power. Such solutions, which are not solutions of the given equation, are called extraneous solutions.

The solution of radical equations with two or more square root expressions involves isolating a radical, squaring both sides, and then repeating this process.

Let’s consider an equation containing two square root expressions.

**EXAMPLE 4 Solving an Equation Involving Two Radicals**

Solve: \( \sqrt{3x + 1} - \sqrt{x + 4} = 1 \).

**Solution**

\[
\sqrt{3x + 1} - \sqrt{x + 4} = 1 \\
\sqrt{3x + 1} = \sqrt{x + 4} + 1 \\
(\sqrt{3x + 1})^2 = (\sqrt{x + 4} + 1)^2 \\
3x + 1 = x + 4 + 2\sqrt{x + 4} + 1 \\
2x - 3 = 2\sqrt{x + 4} \\
(2x - 3)^2 = (2\sqrt{x + 4})^2 \\
4x^2 - 12x + 9 = 4x + 4 \\
4x^2 - 16x + 5 = 0 \\
(x - 1)(4x - 5) = 0 \\
x = 1, \frac{5}{4} \\
\]

Check 1: \( x = 1 \)

\[
3(1) + 1 = 4 + 2\sqrt{1 + 4} + 1 \\
4 = 4 \checkmark \\
\]

Check 2: \( x = \frac{5}{4} \)

\[
3\left(\frac{5}{4}\right) + 1 = 4 + 2\sqrt{\frac{5}{4} + 4} + 1 \\
\frac{23}{4} = \frac{23}{4} \checkmark \\
\]

The solution set is \( \{1, \frac{5}{4}\} \).
The graph of has only one x-intercept at 5. This verifies that the solution set of is \(\{5\}\).

Technology
The graph of

\[
y = \sqrt{3x + 1} - \sqrt{x + 4} - 1
\]

has only one x-intercept at 5. This verifies that the solution set of \(\sqrt{3x + 1} - \sqrt{x + 4} = 1\) is \([5]\).

Check 0:

\[
\sqrt{3} \cdot 0 + 1 - \sqrt{0 + 4} = 1
\]

\[
\sqrt{1} - \sqrt{4} = 1
\]

\[
1 - 2 \frac{1}{4} = 1
\]

\[-1 = 1 \text{ False}
\]

Check 5:

\[
\sqrt{3} \cdot 5 + 1 - \sqrt{5 + 4} \frac{1}{2} = 1
\]

\[
\sqrt{16} - \sqrt{9} \frac{1}{2} = 1
\]

\[
4 - 3 \frac{1}{2} = 1
\]

\[1 = 1 \checkmark
\]

The false statement \(-1 = 1\) indicates that 0 is not a solution. It is an extraneous solution brought about by squaring each side of the equation. The only solution is 5, and the solution set is \([5]\).

Check Point
Solve and check: \(\sqrt{x + 5} - \sqrt{x - 3} = 2\).

Radicals and Windchill

The way that we perceive the temperature on a cold day depends on both air temperature and wind speed. The windchill temperature is what the air temperature would have to be with no wind to achieve the same chilling effect on the skin. The formula that describes windchill temperature, \(W\), in terms of the velocity of the wind, \(v\), in miles per hour, and the actual air temperature, \(t\), in degrees Fahrenheit, is

\[
W = 91.4 - \frac{(10.5 + 6.7\sqrt{v} - 0.45v)(457 - 5t)}{110}
\]

Use your calculator to describe how cold the air temperature feels (that is, the windchill temperature) when the temperature is 15° Fahrenheit and the wind is 5 miles per hour. Contrast this with a temperature of 40° Fahrenheit and a wind blowing at 50 miles per hour.
Because $\sqrt[b]{b}$ can be expressed as $b^{1/n}$, radical equations can be written using rational exponents. For example, the Galápagos equation

$$28.5\sqrt[3]{x} = 57$$

can be written

$$28.5x^{1/3} = 57.$$  

We solve this equation exactly as we did when it was expressed in radical form. First, isolate $x^{1/3}$.

$$\frac{28.5x^{1/3}}{28.5} = \frac{57}{28.5}$$

$$x^{1/3} = 2$$

Complete the solution process by raising both sides to the third power.

$$(x^{1/3})^3 = 2^3$$

$$x = 8$$

**Solving Radical Equations of the Form $x^{m/n} = k$**

Assume that $m$ and $n$ are positive integers, $n/m$ is in lowest terms, and $k$ is a real number.

1. Isolate the expression with the rational exponent.
2. Raise both sides of the equation to the $n/m$ power.

   **If $m$ is even:**
   
   $$x^{m/n} = k$$
   $$\left(x^{m/n}\right)^{n/m} = k^{n/m}$$
   $$x = \pm k^{n/m}$$

   **If $m$ is odd:**
   
   $$x^{m/n} = k$$
   $$\left(x^{m/n}\right)^{n/m} = k^{n/m}$$
   $$x = k^{n/m}$$

   It is incorrect to insert the $\pm$ symbol when the numerator of the exponent is odd. An odd index has only one root.

3. Check all proposed solutions in the original equation to find out if they are actual solutions or extraneous solutions.

**EXAMPLE 5 Solving Equations Involving Rational Exponents**

Solve:

a. $3x^{3/4} - 6 = 0$  

b. $x^{2/3} - \frac{3}{4} = -\frac{1}{4}$

**Solution**

a. Our goal is to isolate $x^{3/4}$. Then we can raise both sides of the equation to the $\frac{4}{3}$ power because $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$.

   $$3x^{3/4} - 6 = 0$$

   This is the given equation; we will isolate $x^{3/4}$.

   $$3x^{3/4} = 6$$

   Add 6 to both sides.

   $$\frac{3x^{3/4}}{3} = \frac{6}{3}$$

   Divide both sides by 3.

   $$x^{3/4} = 2$$

   Simplify.

   $$\left(x^{3/4}\right)^{4/3} = 2^{4/3}$$

   Raise both sides to the $\frac{4}{3}$ power. Because $\frac{4}{3} = 1 + \frac{1}{3}$ and $m$ is odd, we do not use the $\pm$ symbol.

   $$x = 2^{4/3}$$

   Simplify the left side: $(x^{3/4})^{4/3} = x^{\frac{3}{4} \cdot \frac{4}{3}} = x = x$.
The proposed solution is $2^{4/3}$. Complete the solution process by checking this value in the given equation.

$$3x^{3/4} - 6 = 0$$

This is the original equation.

$$3(2^{4/3})^{3/4} - 6 \neq 0$$

Substitute the proposed solution.

$$3 \cdot 2 - 6 \neq 0$$

$(2^{4/3})^{3/4} = 2^{12/12} = 2^1 = 2$.

$$0 = 0 \checkmark$$

The true statement shows that $2^{4/3}$ is a solution.

The solution is $2^{4/3}$. The solution set is $\{2^{4/3}\}$.

b. To solve $x^{2/3} - \frac{1}{2} = -\frac{1}{2}$, our goal is to isolate $x^{2/3}$. Then we can raise both sides of the equation to the $2$ power because $\frac{1}{2}$ is the reciprocal of $\frac{2}{3}$.

$$x^{2/3} - \frac{1}{2} = -\frac{1}{2}$$

This is the given equation.

$$x^{2/3} = \frac{1}{2}$$

Add $\frac{1}{2}$ to both sides.

$$(x^{2/3})^{3/2} = (\frac{1}{2})^{3/2}$$

Raise both sides to the $\frac{3}{2}$ power. Because $\frac{3}{2} = \frac{2}{3}$ and $m$ is even, the $\pm$ symbol is necessary.

$$x = \pm \frac{1}{2}$$

Take a moment to verify that the solution set is $\{-\frac{1}{2}, \frac{1}{2}\}$.

Solve and check:

a. $5x^{3/2} - 25 = 0$  

b. $x^{2/3} - 8 = -4$.

### Equations That Are Quadratic in Form

Some equations that are not quadratic can be written as quadratic equations using an appropriate substitution. Here are some examples:

<table>
<thead>
<tr>
<th>Given Equation</th>
<th>Substitution</th>
<th>New Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^4 - 8x^2 - 9 = 0$ or $(x^2)^2 - 8x^2 - 9 = 0$</td>
<td>$t = x^2$</td>
<td>$t^2 - 8t - 9 = 0$</td>
</tr>
<tr>
<td>$5x^{2/3} + 11x^{1/3} + 2 = 0$ or $5(x^{1/3})^2 + 11x^{1/3} + 2 = 0$</td>
<td>$t = x^{1/3}$</td>
<td>$5t^2 + 11t + 2 = 0$</td>
</tr>
</tbody>
</table>

An equation that is **quadratic in form** is one that can be expressed as a quadratic equation using an appropriate substitution. Both of the preceding given equations are quadratic in form.

Equations that are quadratic in form contain an expression to a power, the same expression to that power squared, and a constant term. By letting $t$ equal the expression to the power, a quadratic equation in $t$ will result. Now it’s easy. Solve this quadratic equation for $t$. Finally, use your substitution to find the values for the variable in the given equation. Example 6 shows how this is done.
EXAMPLE 6  Solving an Equation Quadratic in Form

Solve: \( x^4 - 8x^2 - 9 = 0 \).

Solution  Notice that the equation contains an expression to a power, \( x^2 \), the same expression to that power squared, \( (x^2)^2 \), and a constant term, \(-9\). We let \( t \) equal the expression to the power. Thus,

\[
\begin{align*}
\text{let } t &= x^2. \\
\end{align*}
\]

Now we write the given equation as a quadratic equation in \( t \) and solve for \( t \).

\[
\begin{align*}
x^4 - 8x^2 - 9 &= 0 & \text{This is the given equation.} \\
(x^2)^2 - 8x^2 - 9 &= 0 & \text{The given equation contains } x^2 \text{ and } x^4 \text{ squared.} \\
t^2 - 8t - 9 &= 0 & \text{Replace } x^2 \text{ with } t. \\
(t - 9)(t + 1) &= 0 & \text{Factor.} \\
t - 9 &= 0 \text{ or } t + 1 &= 0 & \text{Apply the zero-product principle.} \\
t &= 9 \quad \text{ or } \quad t &= -1 & \text{Solve for } t.
\end{align*}
\]

We’re not done! Why not? We were asked to solve for \( x \) and we have values for \( t \). We use the original substitution, \( t = x^2 \), to solve for \( x \). Replace \( t \) with \( x^2 \) in each equation shown, namely \( t = 9 \) and \( t = -1 \).

\[
\begin{align*}
x^2 &= 9 & \Rightarrow & \quad x = \pm \sqrt{9} & \Rightarrow & \quad x = \pm 3 \\
x^2 &= -1 & \Rightarrow & \quad x = \pm \sqrt{-1} & \Rightarrow & \quad x = \pm i
\end{align*}
\]

The solution set is \( \{-3, 3, -i, i\} \).

EXAMPLE 7  Solving an Equation Quadratic in Form

Solve: \( 5x^{2/3} + 11x^{1/3} + 2 = 0 \).

Solution  Notice that the equation contains an expression to a power, \( x^{1/3} \), the same expression to that power squared, \( (x^{1/3})^2 \), and a constant term, \( 2 \). We let \( t \) equal the expression to the power. Thus,

\[
\begin{align*}
\text{let } t &= x^{1/3}. \\
\end{align*}
\]

Now we write the given equation as a quadratic equation in \( t \) and solve for \( t \).

\[
\begin{align*}
5x^{2/3} + 11x^{1/3} + 2 &= 0 & \text{This is the given equation.} \\
5(x^{1/3})^2 + 11x^{1/3} + 2 &= 0 & \text{The given equation contains } x^{1/3} \text{ and } x^{2/3} \text{ squared.} \\
5t^2 + 11t + 2 &= 0 & \text{Replace } x^{1/3} \text{ with } t. \\
(5t + 1)(t + 2) &= 0 & \text{Factor.} \\
5t + 1 &= 0 \text{ or } \quad t + 2 &= 0 & \text{Set each factor equal to } 0. \\
5t &= -1 \quad \text{ or } \quad t &= -2 & \text{Solve for } t. \\
t &= -\frac{1}{5} \quad \text{ or } \quad t &= -2
\end{align*}
\]
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Use the original substitution, \( t = x^{1/3} \), to solve for \( x \). Replace \( t \) with \( x^{1/3} \) in each of the preceding equations, namely \( t = -\frac{1}{5} \) and \( t = -2 \).

\[
\begin{align*}
x^{1/3} &= -\frac{1}{5} \quad & x^{1/3} &= -2 \\
(x^{1/3})^3 &= \left(-\frac{1}{5}\right)^3 \quad & (x^{1/3})^3 &= (-2)^3 \\
x &= -\frac{1}{125} \quad & x &= -8
\end{align*}
\]

Check these values to verify that the solution set is \( \{-\frac{1}{125}, -8\} \).

**5** Solve equations involving absolute value.

\[
\begin{align*}
|\frac{x}{2}| &= 2 \\
\frac{x}{2} &= \pm 2
\end{align*}
\]

*Figure 1.18 If \(|x| = 2\), then \( x = 2 \) or \( x = -2 \).*

**Technology**

You can use a graphing utility to verify the solution of an absolute value equation.

Consider, for example,

\[
|2x - 3| = 11
\]

Graph \( y_1 = |2x - 3| \) and \( y_2 = 11 \). The graphs are shown in a \([-10, 10, 1]\) by \([-1, 15, 1]\) viewing rectangle. The \( x \)-coordinates of the intersection points are \(-4\) and \(7\), verifying that \(\{-4, 7\}\) is the solution set.

**Equations Involving Absolute Value**

We have seen that the absolute value of \( x, \) \(|x|\), describes the distance of \( x \) from zero on a number line. Now consider absolute value equations, such as

\[
|x| = 2.
\]

This means that we must determine real numbers whose distance from the origin on the number line is 2. Figure 1.18 shows that there are two numbers such that \(|x| = 2\), namely, 2 or \(-2\). We write \( x = 2 \) or \( x = -2 \). This observation can be generalized as follows:

**Rewriting an Absolute Value Equation without Absolute Value Bars**

If \( c \) is a positive real number and \( X \) represents any algebraic expression, then \(|X| = c\) is equivalent to \( X = c \) or \( X = -c \).

**EXAMPLE 8** Solving an Equation Involving Absolute Value

Solve: \(|2x - 3| = 11\).

**Solution**

\[
\begin{align*}
|2x - 3| &= 11 \\
2x - 3 &= 11 \quad & 2x - 3 &= -11 \\
2x &= 14 \quad & 2x &= -8 \\
x &= 7 \quad & x &= -4
\end{align*}
\]

*Check 7:*

\[
\begin{align*}
|2x - 3| &= 11 \\
|2(7) - 3| &= 11 \\
|14 - 3| &= 11 \quad & |(-4) - 3| &= 11 \\
|11| &= 11 \quad & |-11| &= 11 \\
11 &= 11 \checkmark \quad & 11 &= 11 \checkmark
\end{align*}
\]

The solution set is \(\{-4, 7\}\).
Solve: \(|2x - 1| = 5\).

The absolute value of a number is never negative. Thus, if \(X\) is an algebraic expression and \(c\) is a negative number, then \(|X| = c\) has no solution. For example, the equation \(3x - 6 = -2\) has no solution because \(3x - 6\) cannot be negative. The solution set is \(\emptyset\), the empty set.

The absolute value of 0 is 0. Thus, if \(X\) is an algebraic expression and \(|X| = 0\), the solution is found by solving \(X = 0\). For example, the solution of \(|x - 2| = 0\) is obtained by solving \(x - 2 = 0\). The solution is 2 and the solution set is \(\{2\}\).

To solve some absolute value equations, it is necessary to first isolate the expression containing the absolute value symbols. For example, consider the equation

\[
3|2x - 3| - 8 = 25.
\]

We need to isolate \(|2x - 3|\). How can we isolate \(|2x - 3|\)? Add 8 to both sides of the equation and then divide both sides by 3.

\[
3|2x - 3| = 33 \quad \text{This is the given equation.}
\]

\[
|2x - 3| = 11 \quad \text{Divide both sides by 3.}
\]

This results in the equation we solved in Example 8.

**EXERCISE SET 1.6**

**Practice Exercises**

Solve each radical equation in Exercises 11–28. Check all proposed solutions.

11. \(\sqrt{3x + 18} = x\)  
12. \(\sqrt{20 - 8x} = x\)  
13. \(\sqrt{x + 3} = x - 3\)  
14. \(\sqrt{x + 10} = x - 2\)  
15. \(\sqrt{2x + 13} = x + 7\)  
16. \(\sqrt{6x + 1} = x - 1\)  
17. \(x - \sqrt{2x + 5} = 5\)  
18. \(x - \sqrt{x + 11} = 1\)  
19. \(\sqrt{3x + 10} = x + 4\)  
20. \(\sqrt{x} - 3 = x - 9\)  
21. \(\sqrt{x + 8} - \sqrt{x - 4} = 2\)

22. \(\sqrt{x + 5} - \sqrt{x - 3} = 2\)  
23. \(\sqrt{x - 5} - \sqrt{x - 8} = 3\)  
24. \(\sqrt{2x - 3} - \sqrt{x - 2} = 1\)  
25. \(\sqrt{2x + 3} + \sqrt{x - 2} = 2\)  
26. \(\sqrt{x + 2} + \sqrt{3x + 7} = 1\)  
27. \(\sqrt{3\sqrt{x} + 1} = \sqrt{3x - 5}\)  
28. \(\sqrt{1 + 4\sqrt{x}} = 1 + \sqrt{x}\)

Solve and check each equation with rational exponents in Exercises 29–38.

29. \(x^{3/2} = 8\)  
30. \(x^{3/2} = 27\)  
31. \((x - 4)^{3/2} = 27\)  
32. \((x + 5)^{3/2} = 8\)  
33. \(6x^{3/2} - 12 = 0\)  
34. \(8x^{5/3} - 24 = 0\)  
35. \((x - 4)^{2/3} = 16\)  
36. \((x + 5)^{2/3} = 4\)  
37. \((x^2 - x - 4)^{3/4} = 2\)  
38. \((x^2 - 3x + 3)^{3/2} - 1 = 0\)
Solve each equation in Exercises 39–58 by making an appropriate substitution.

39. \(x^4 - 5x^2 + 4 = 0\)
40. \(x^4 - 13x^2 + 36 = 0\)
41. \(9x^4 = 25x^2 - 16\)
42. \(4x^4 = 13x^2 - 9\)
43. \(x - 13 \sqrt{x} + 40 = 0\)
44. \(2x - 7 \sqrt{x} - 30 = 0\)
45. \(x^2 - x^3 - 20 = 0\)
46. \(x^2 - x^3 - 6 = 0\)
47. \(x^{3/2} - \frac{1}{x^{3/2}} - 6 = 0\)
48. \(2x^{2/3} + 7x^{1/3} - 15 = 0\)
49. \(x^{3/2} - 2x^{3/4} + 1 = 0\)
50. \(x^{2/5} + x^{1/5} - 6 = 0\)
51. \(2x - 3x^{1/2} + 1 = 0\)
52. \(x + 3x^{1/2} - 4 = 0\)
53. \((x - 5)^2 - 4(x - 5) - 21 = 0\)
54. \((x + 3)^2 + 7(x + 3) - 18 = 0\)
55. \((x^2 - x)^2 - 14(x^2 - x) + 24 = 0\)
56. \((x^2 - 2x)^2 - 11(x^2 - 2x) + 24 = 0\)
57. \(\left(y - \frac{8}{y}\right)^2 + 5\left(y - \frac{8}{y}\right) - 14 = 0\)
58. \(\left(y - \frac{10}{y}\right)^2 + 6\left(y - \frac{10}{y}\right) - 27 = 0\)

In Exercises 59–74, solve each absolute value equation or indicate the equation has no solution.

59. \(|x| = 8\)
60. \(|x| = 6\)
61. \(|x - 2| = 7\)
62. \(|x + 1| = 5\)
63. \(2|x - 1| = 5\)
64. \(2|x - 3| = 11\)
65. \(2|3x - 2| = 14\)
66. \(3|2x - 1| = 21\)
67. \(7|x| + 2 = 16\)
68. \(7|3x| + 2 = 16\)
69. \(|x + 1| + 5 = 3\)
70. \(|x + 1| + 6 = 2\)
71. \(|2x - 1| + 3 = 3\)
72. \(|3x - 2| + 4 = 4\)

Hint for Exercises 73–74: Absolute value expressions are equal when the expressions inside the absolute value bars are equal to or opposites of each other.

73. \(|3x - 1| = |x + 5|\)
74. \(|2x - 7| = |x + 3|\)

Solve each equation in Exercises 75–84 by the method of your choice.

75. \(x + 2 \sqrt{x} - 3 = 0\)
76. \(x^3 + 3x^2 - 4x - 12 = 0\)
77. \((x + 4)^{1/2} = 8\)
78. \((x^2 - 1)^2 - 2(x^2 - 1) = 3\)
79. \(\sqrt{4x + 15} - 2x = 0\)
80. \(x^{2/5} - 1 = 0\)
81. \(|x^2 + 2x - 36| = 12\)
82. \(\sqrt{3x + 1} - \sqrt{x - 1} = 2\)
83. \(x^3 - 2x^2 = x - 2\)
84. \(|x^2 + 6x + 1| = 8\)

Application Exercises

First the good news: The graph shows that U.S. seniors’ scores in standard testing in science have improved since 1982. Now the bad news: The highest possible score is 500, and in 1970, the average test score was 304.

**U.S. Seniors’ Test Scores in Science**

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1982</td>
<td>280</td>
</tr>
<tr>
<td>1986</td>
<td>287</td>
</tr>
<tr>
<td>1990</td>
<td>288</td>
</tr>
<tr>
<td>1992</td>
<td>293</td>
</tr>
<tr>
<td>1994</td>
<td>293</td>
</tr>
<tr>
<td>1996</td>
<td>295</td>
</tr>
<tr>
<td>1998</td>
<td>296</td>
</tr>
<tr>
<td>1999</td>
<td>297</td>
</tr>
</tbody>
</table>

**Source:** National Assessment of Educational Progress

The formula

\[ S = 4\sqrt{x} + 280 \]

models the average science test score, \(S\), \(x\) years after 1982. Use the formula to solve Exercises 85–86.

85. When will the average science score return to the 1970 average of 304?
86. When will the average science test score be 300?

Out of a group of 50,000 births, the number of people, \(y\), surviving to age \(x\) is modeled by the formula

\[ y = 5000\sqrt{100 - x} \]

The graph of the formula is shown. Use the formula to solve Exercises 87–88.

87. To what age will 40,000 people in the group survive? Identify the solution as a point on the graph of the formula.
88. To what age will 35,000 people in the group survive? Identify the solution as a point on the graph of the formula.
For each planet in our solar system, its year is the time it takes the planet to revolve once around the sun. The formula
\[ E = 0.2x^{3/2} \]
models the number of Earth days in a planet’s year, \( E \), where \( x \) is the average distance of the planet from the sun, in millions of kilometers. Use the formula to solve Exercises 89–90.

89. We, of course, have 365 Earth days in our year. What is the average distance of Earth from the sun? Use a calculator and round to the nearest million kilometers.

90. There are approximately 88 Earth days in the year of the planet Mercury. What is the average distance of Mercury from the sun? Use a calculator and round to the nearest million kilometers.

Use the Pythagorean Theorem to solve Exercises 91–92.

91. Two vertical poles of lengths 6 feet and 8 feet stand 10 feet apart (see the figure). A cable reaches from the top of one pole to some point on the ground between the poles and then to the top of the other pole. Where should this point be located to use 18 feet of cable?

92. Towns \( A \) and \( B \) are located 6 miles and 3 miles, respectively, from a major expressway. The point on the expressway closest to town \( A \) is 12 miles from the point on the expressway closest to town \( B \). Two new roads are to be built from \( A \) to the expressway and then to \( B \). (See the figure at the top of the next column.)

Exercise Set 1.6 • 143

93. Without actually solving the equation, give a general description of how to solve \( x^3 - 5x^2 - x + 5 = 0 \).

94. In solving \( \sqrt{3x + 4} = \sqrt{2x + 4} = 2 \), why is it a good idea to isolate a radical term? What if we don’t do this and simply square each side? Describe what happens.

95. What is an extraneous solution to a radical equation?

96. Explain how to recognize an equation that is quadratic in form. Provide two original examples with your explanation.

97. Describe two methods for solving this equation: \( x - 5\sqrt{x} + 4 = 0 \).

98. Explain how to solve an equation involving absolute value.

99. Explain why the procedure that you explained in Exercise 98 does not apply to the equation \( |x - 2| = -3 \). What is the solution set for this equation?

100. Describe the trend shown by the graph in Exercises 87–88. When is the rate of decrease most rapid? What does this mean about survival rate by age?

Technology Exercises

In Exercises 101–103, use a graphing utility and the graph’s x-intercepts to solve each equation. Check by direct substitution. A viewing rectangle is given.

101. \( x^3 + 3x^2 - x - 3 = 0 \)
\([-6, 6, 1] \text{ by } [-6, 6, 1]\)

102. \( -x^4 + 4x^3 - 4x^2 = 0 \)
\([-6, 6, 1] \text{ by } [-9, 2, 1]\)

103. \( \sqrt{2x + 13} - x - 5 = 0 \)
\([-5, 5, 1] \text{ by } [-5, 5, 1]\)

104. Use a graphing utility to obtain the graph of the formula in Exercises 87–88. Then use the TRACE feature to trace along the curve until you reach the point that visually shows the solution to Exercise 87 or 88.
106. Solve: \( \sqrt{6x - 2} = \sqrt{2x + 3} - \sqrt{4x - 1} \).

107. Solve without squaring both sides:

\[
5 - \frac{2}{x} = \sqrt{5 - \frac{2}{x}}.
\]

108. Solve for \( x \):

\[
\sqrt[3]{x} \sqrt{x} = 9.
\]

109. Solve for \( x \):

\[
x^{\frac{5}{6}} + x^{\frac{2}{3}} - 2x^{\frac{1}{2}} = 0.
\]

### SECTION 1.7 Linear Inequalities

**Objectives**

1. Graph an inequality’s solution set.
2. Use set-builder and interval notations.
3. Use properties of inequalities to solve inequalities.
4. Solve compound inequalities.
5. Solve inequalities involving absolute value.

Rent-a-Heap, a car rental company, charges $125 per week plus $0.20 per mile to rent one of their cars. Suppose you are limited by how much money you can spend for the week: You can spend at most $335. If we let \( x \) represent the number of miles you drive the heap in a week, we can write an inequality that models the given conditions.

Using the commutative property of addition, we can express this inequality as

\[ 0.20x + 125 \leq 335. \]

The form of this inequality is \( ax + b \leq c \), with \( a = 0.20 \), \( b = 125 \), and \( c = 335 \). Any inequality in this form is called a linear inequality in one variable. The greatest exponent on the variable in such an inequality is 1. The symbol between \( ax + b \) and \( c \) can be \( \leq \) (is less than or equal to), \( < \) (is less than), \( \geq \) (is greater than or equal to), or \( > \) (is greater than).
In this section, we will study how to solve linear inequalities such as $0.20x + 125 \leq 335$. **Solving an inequality** is the process of finding the set of numbers that make the inequality a true statement. These numbers are called the **solutions** of the inequality, and we say that they **satisfy** the inequality. The set of all solutions is called the **solution set** of the inequality. We begin by discussing how to graph and how to represent these solution sets.

### Graphs of Inequalities; Interval Notation

There are infinitely many solutions to the inequality $x > -4$, namely all real numbers that are greater than $-4$. Although we cannot list all the solutions, we can make a drawing on a number line that represents these solutions. Such a drawing is called the **graph of the inequality**.

Graphs of solutions to linear inequalities are shown on a number line by shading all points representing numbers that are solutions. Parentheses indicate endpoints that are not solutions. Square brackets indicate endpoints that are solutions.

**EXAMPLE 1** **Graphing Inequalities**

Graph the solutions of:

a. $x < 3$   b. $x \geq -1$   c. $-1 < x \leq 3$.

**Solution**

a. The solutions of $x < 3$ are all real numbers that are less than 3. They are graphed on a number line by shading all points to the left of 3. The parenthesis at 3 indicates that 3 is not a solution, but numbers such as 2.9999 and 2.6 are. The arrow shows that the graph extends indefinitely to the left.

b. The solutions of $x \geq -1$ are all real numbers that are greater than or equal to $-1$. We shade all points to the right of $-1$ and the point for $-1$ itself. The bracket at $-1$ shows that $-1$ is a solution of the given inequality. The arrow shows that the graph extends indefinitely to the right.

c. The inequality $-1 < x \leq 3$ is read “$-1$ is less than $x$ and $x$ is less than or equal to 3,” or “$x$ is greater than $-1$ and less than or equal to 3.” The solutions of $-1 < x \leq 3$ are all real numbers between $-1$ and 3, not including $-1$ but including 3. The parenthesis at $-1$ indicates that $-1$ is not a solution. By contrast, the bracket at 3 shows that 3 is a solution. Shading indicates the other solutions.
Use set-builder and interval notations.

Let \( a \) and \( b \) be real numbers such that

<table>
<thead>
<tr>
<th>Interval Notation</th>
<th>Set-Builder Notation</th>
<th>Graph</th>
</tr>
</thead>
</table>
| \((a, b)\)        | \(\{x \mid a < x < b\}\) | ![Graph](a,b) |}
| \([a, b]\)        | \(\{x \mid a \leq x \leq b\}\) | ![Graph](a, b) |}
| \([a, b)\)        | \(\{x \mid a \leq x < b\}\) | ![Graph](a, b) |}
| \((a, b]\)        | \(\{x \mid a < x \leq b\}\) | ![Graph](a, b] |}
| \((a, \infty)\)   | \(\{x \mid x > a\}\) | ![Graph](a, \infty) |}
| \([a, \infty)\)   | \(\{x \mid x \geq a\}\) | ![Graph](a, \infty) |}
| \((-\infty, b)\)  | \(\{x \mid x < b\}\) | ![Graph](-\infty, b) |}
| \((-\infty, b]\)  | \(\{x \mid x \leq b\}\) | ![Graph](-\infty, b] |}
| \((-\infty, \infty)\) | \(\mathbb{R}\) (set of all real numbers) | ![Graph](-\infty, \infty) |}

Table 1.5 Intervals on the Real Number Line

Graph the solutions of:
- a. \(x \leq 2\)
- b. \(x > -4\)
- c. \(2 \leq x < 6\).

Now that we know how to graph the solution set of an inequality such as \(x > -4\), let’s see how to represent the solution set. One method is with set-builder notation. Using this method, the solution set of \(x > -4\) can be expressed as \(\{x \mid x > -4\}\).

We read this as “the set of all real numbers \(x\) such that \(x\) is greater than \(-4\).”

Another method used to represent solution sets of inequalities is interval notation. Using this notation, the solution set of \(x > -4\) is expressed as \((-4, \infty)\). The parenthesis at \(-4\) indicates that \(-4\) is not included in the interval. The infinity symbol, \(\infty\), does not represent a real number. It indicates that the interval extends indefinitely to the right.

Table 1.5 lists nine possible types of intervals used to describe subsets of real numbers.

**EXAMPLE 2** Intervals and Inequalities

Express the intervals in terms of inequalities and graph:
- a. \((-1, 4]\)
- b. \([2.5, 4]\)
- c. \((-4, \infty)\).

**Solution**

- a. \((-1, 4]\) = \(\{x \mid -1 < x \leq 4\}\)
- b. \([2.5, 4]\) = \(\{x \mid 2.5 \leq x \leq 4\}\)
- c. \((-4, \infty)\) = \(\{x \mid x > -4\}\)
Express the intervals in terms of inequalities and graph:

a. \([-2, 5]\)  
b. \([1, 3.5]\)  
c. \((-\infty, -1)\).

**Solving Linear Inequalities**

Back to our question: How many miles can you drive your Rent-a-Heap car if you can spend at most $335 per week? We answer the question by solving

\[0.20x + 125 \leq 335\]

for \(x\). The solution procedure is nearly identical to that for solving

\[0.20x + 125 = 335.\]

Our goal is to get \(x\) by itself on the left side. We do this by subtracting 125 from both sides to isolate \(0.20x\):

\[0.20x + 125 \leq 335\]

\[0.20x + 125 - 125 \leq 335 - 125\]

\[0.20x \leq 210.\]

Finally, we isolate \(x\) from \(0.20x\) by dividing both sides of the inequality by 0.20:

\[\frac{0.20x}{0.20} \leq \frac{210}{0.20}\]

\[x \leq 1050.\]

With at most $335 per week to spend, you can travel at most 1050 miles.

We started with the inequality \(0.20x + 125 \leq 335\) and obtained the inequality \(x \leq 1050\) in the final step. Both of these inequalities have the same solution set, namely \(\{x \mid x \leq 1050\}\). Inequalities such as these, with the same solution set, are said to be equivalent.

We isolated \(x\) from \(0.20x\) by dividing both sides of \(0.20x \leq 210\) by 0.20, a positive number. Let’s see what happens if we divide both sides of an inequality by a negative number. Consider the inequality \(10 < 14\). Divide 10 and 14 by \(-2\):

\[\frac{10}{-2} = -5\]  and  \[\frac{14}{-2} = -7.\]

Because \(-5\) lies to the right of \(-7\) on the number line, \(-5\) is greater than \(-7\):

\[-5 > -7.\]

Notice that the direction of the inequality symbol is reversed:

\[10 < 14\]

\[-5 > -7.\]

In general, when we multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol is reversed. When we reverse the direction of the inequality symbol, we say that we change the sense of the inequality.

We can isolate a variable in a linear inequality the same way we can isolate a variable in a linear equation. The following properties are used to create equivalent inequalities:
EXAMPLE 3 Solving a Linear Inequality

Solve and graph the solution set on a number line:

\[3 - 2x < 11.\]

Solution

\[
\begin{align*}
3 - 2x &< 11 & \text{This is the given inequality.} \\
3 - 2x - 3 &< 11 - 3 & \text{Subtract 3 from both sides.} \\
-2x &< 8 & \text{Simplify.} \\
\frac{-2x}{-2} &> \frac{8}{-2} & \text{Divide both sides by -2 and reverse the sense of the inequality.} \\
x &> -4 & \text{Simplify.}
\end{align*}
\]

The solution set consists of all real numbers that are greater than \(-4\), expressed as \(\{x \mid x > -4\}\) in set-builder notation. The interval notation for this solution set is \((-4, \infty)\). The graph of the solution set is shown as follows:

-4 -3 2 0 1 2 3 4 + x

Check Point

Solve and graph the solution set on a number line:

\[2 - 3x \leq 5.\]
EXAMPLE 4 Solving a Linear Inequality

Solve and graph the solution set: \(7x + 15 \geq 13x + 51\).

Solution

We will collect variable terms on the left and constant terms on the right.

\[
7x + 15 \geq 13x + 51
\]

This is the given inequality.

\[
7x + 15 - 13x \geq 13x + 51 - 13x
\]

Subtract \(13x\) from both sides.

\[
-6x + 15 \geq 51
\]

Simplify.

\[
-6x + 15 - 15 \geq 51 - 15
\]

Subtract 15 from both sides.

\[
-6x \geq 36
\]

Simplify.

\[
\frac{-6x}{-6} \leq \frac{36}{-6}
\]

Divide both sides by \(-6\) and reverse the sense of the inequality.

\[
x \leq -6
\]

Simplify.

The solution set consists of all real numbers that are less than or equal to \(-6\), expressed as \(\{x \mid x \leq -6\}\). The interval notation for this solution set is \((-\infty, -6]\). The graph of the solution set is shown as follows:

```
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2
```

Check Point

Solve and graph the solution set: \(6 - 3x \leq 5x - 2\).

Technology

You can use a graphing utility to verify that \((-\infty, -6]\) is the solution set of

\[
7x + 15 \geq 13x + 51
\]

For what values of \(x\) does the graph of \(y = 7x + 15\) lie above or on the graph of \(y = 13x + 51\)?

The graphs are shown on the left in a \([-10, 2, 1]\) by \([-40, 5, 5]\) viewing rectangle. Look closely at the graphs. Can you see that the graph of \(y = 7x + 15\) lies above or on the graph of \(y = 13x + 51\) when \(x \leq -6\), or on the interval \((-\infty, -6]\)?

Solving Compound Inequalities

We now consider two inequalities such as

\[-3 < 2x + 1 \text{ and } 2x + 1 \leq 3\]

expressed as a **compound inequality**

\[-3 < 2x + 1 \leq 3\]

The word “and” does not appear when the inequality is written in the shorter form, although it is implied. The shorter form enables us to solve both inequalities at once. By performing the same operation on all three parts of the inequality, our goal is to **isolate \(x\) in the middle**.
EXAMPLE 5  Solving a Compound Inequality

Solve and graph the solution set:

\[-3 < 2x + 1 \leq 3.\]

**Solution**  We would like to isolate \(x\) in the middle. We can do this by first subtracting 1 from all three parts of the compound inequality. Then we isolate \(x\) from \(2x\) by dividing all three parts of the inequality by 2.

\[
\begin{align*}
-3 < 2x + 1 & \leq 3 \\
-3 - 1 < 2x + 1 - 1 & \leq 3 - 1 \\
-4 < 2x & \leq 2 \\
-2 < x & \leq 1
\end{align*}
\]

The solution set consists of all real numbers greater than \(-2\) and less than or equal to \(1\), represented by \(\{x \mid -2 < x \leq 1\}\) in set-builder notation and \((-2, 1]\) in interval notation. The graph is shown as follows:

Graph showing \([-3, 1]\) by \([-5, 5, 1]\)

Check Point  Solve and graph the solution set:  \(1 \leq 2x + 3 < 11.\)

5 Solve inequalities involving absolute value.

Solving Inequalities with Absolute Value

We know that \(|x|\) describes the distance of \(x\) from zero on a real number line. We can use this geometric interpretation to solve an inequality such as

\[|x| < 2.\]

This means that the distance of \(x\) from 0 is less than 2, as shown in Figure 1.19. The interval shows values of \(x\) that lie less than 2 units from 0. Thus, \(x\) can lie between \(-2\) and \(2\). That is, \(x\) is greater than \(-2\) and less than \(2\). We write \((-2, 2)\) or \(\{x \mid -2 < x < 2\}\).

Some absolute value inequalities use the “greater than” symbol. For example, \(|x| > 2\) means that the distance of \(x\) from 0 is greater than 2, as shown in Figure 1.20. Thus, \(x\) can be less than \(-2\) or greater than \(2\). We write \(x < -2\) or \(x > 2\).

These observations suggest the following principles for solving inequalities with absolute value:

**Solving an Absolute Value Inequality**

If \(X\) is an algebraic expression and \(c\) is a positive number,

1. The solutions of \(|X| < c\) are the numbers that satisfy \(-c < X < c\).
2. The solutions of \(|X| > c\) are the numbers that satisfy \(X < -c\) or \(X > c\).

These rules are valid if \(<\) is replaced by \(\leq\) and \(>\) is replaced by \(\geq\).
EXAMPLE 6  Solving an Absolute Value Inequality with $<$

Solve and graph the solution set: $|x - 4| < 3$.

Solution

$$|x| < c \text{ means } -c < x < c.$$  

$|x - 4| < 3$ means $-3 < x - 4 < 3$.

We solve the compound inequality by adding 4 to all three parts.

$$-3 < x - 4 < 3$$

$$-3 + 4 < x - 4 + 4 < 3 + 4$$

$$1 < x < 7$$

The solution set is all real numbers greater than 1 and less than 7, denoted by $\{x | 1 < x < 7\}$ or $(1, 7)$. The graph of the solution set is shown as follows:

Check Point 6 Solve and graph the solution set: $|x - 2| < 5$.

EXAMPLE 7  Solving an Absolute Value Inequality with $\geq$

Solve and graph the solution set: $|2x + 3| \geq 5$.

Solution

$$|x| \geq c \text{ means } x \leq -c \text{ or } x \geq c.$$  

$|2x + 3| \geq 5$ means $2x + 3 \leq -5$ or $2x + 3 \geq 5$.

We solve each of these inequalities separately.

$$2x + 3 \leq -5 \text{ or } 2x + 3 \geq 5$$

$$2x + 3 - 3 \leq -5 - 3 \text{ or } 2x + 3 - 3 \geq 5 - 3$$

$$2x \leq -8 \text{ or } 2x \geq 2$$

$$\frac{2x}{2} \leq \frac{-8}{2} \text{ or } \frac{2x}{2} \geq \frac{2}{2}$$

$$x \leq -4 \text{ or } x \geq 1$$

The solution set is $\{x | x \leq -4 \text{ or } x \geq 1\}$, that is, all $x$ in $(-\infty, -4]$ or $[1, \infty)$. The graph of the solution set is shown as follows:

Study Tip

The graph of the solution set for $|X| > c$ will be divided into two intervals. The graph of the solution set for $|X| < c$ will be a single interval.
Solve and graph the solution set: \(|2x - 5| \geq 3\).

**Applications**

Our next example shows how to use an inequality to select the better deal between two pricing options. We will use our five-step strategy for solving problems using mathematical models.

**EXAMPLE 8 Creating and Comparing Mathematical Models**

Acme Car rental agency charges $4 a day plus $0.15 a mile, whereas Interstate rental agency charges $20 a day and $0.05 a mile. Under what conditions is the daily cost of an Acme rental a better deal than an Interstate rental?

**Solution**

**Step 1** Let \(x\) represent one of the quantities. We are looking for the number of miles driven in a day to make Acme the better deal. Thus, let \(x = \) the number of miles driven in a day.

**Step 2** Represent other quantities in terms of \(x\). We are not asked to find another quantity, so we can skip this step.

**Step 3** Write an inequality in \(x\) that describes the conditions.

This is the inequality that models the verbal conditions.

\[ 4 + 0.15x < 20 + 0.05x \]

Subtract \(0.05x\) from both sides.

\[ 4 + 0.15x - 0.05x < 20 + 0.05x - 0.05x \]

Simplify.

\[ 4 + 0.1x < 20 \]

Subtract 4 from both sides.

\[ 4 + 0.1x - 4 < 20 - 4 \]

Simplify.

\[ 0.1x < 16 \]

Divide both sides by 0.1.

\[ \frac{0.1x}{0.1} < \frac{16}{0.1} \]

Simplify.

\[ x < 160 \]

Thus, driving fewer than 160 miles per day makes Acme the better deal.
Step 5  Check the proposed solution in the original wording of the problem.
One way to do this is to take a mileage less than 160 miles per day to see if Acme is the better deal. Suppose that 150 miles are driven in a day.

\[
\text{Cost for Acme} = 4 + 0.15(150) = 26.50
\]

\[
\text{Cost for Interstate} = 20 + 0.05(150) = 27.50
\]

Acme has a lower daily cost, making it the better deal.

A car can be rented from Basic Rental for $260 per week with no extra charge for mileage. Continental charges $80 per week plus 25 cents for each mile driven to rent the same car. Under what conditions is the rental cost for Basic Rental a better deal than Continental’s?

EXERCISE SET 1.7

Practice Exercises
In Exercises 1–12, graph the solutions of each inequality on a number line.

1. \( x > 6 \)
2. \( x > -2 \)
3. \( x < -4 \)
4. \( x < 0 \)
5. \( x \geq -3 \)
6. \( x \geq -5 \)
7. \( x \leq 4 \)
8. \( x \leq 7 \)
9. \(-2 < x \leq 5 \)
10. \(-3 \leq x < 7 \)
11. \(-1 < x \leq 4 \)
12. \(-7 \leq x < 0 \)

In Exercises 13–26, express each interval in terms of an inequality on a number line.

13. \((1, 6]\)
14. \((-2, 4]\)
15. \([-5, 2)\)
16. \([-4, 3)\)
17. \([-3, 1)\)
18. \([-2, 5)\)
19. \((2, \infty)\)
20. \((3, \infty)\)
21. \([-3, \infty)\)
22. \([-5, \infty)\)
23. \((-\infty, 3)\)
24. \((-\infty, 2)\)
25. \((-\infty, 5, 5]\)

In Exercises 27–48, express each inequality in terms of an inequality and graph the interval on a number line. Express the solution set using interval notation.

27. \(5x + 11 < 26\)
28. \(2x + 5 < 17\)
29. \(3x - 7 \geq 13\)
30. \(8x - 2 \geq 14\)
31. \(-9x \geq 36\)
32. \(-5x \leq 30\)
33. \(8x - 11 \leq 3x - 13\)
34. \(18x + 45 \leq 12x - 8\)
35. \(4(x + 1) + 2 \geq 3x + 6\)
36. \(8x + 3 > 3(2x + 1) + x + 5\)
37. \(2x - 11 < -3(x + 2)\)
38. \(-4(x + 2) > 3x + 20\)
39. \(1 - (x + 3) \geq 4 - 2x\)
40. \(5(3 - x) \leq 3x - 1\)
41. \(\frac{x}{3} - \frac{3}{5} \leq \frac{x}{2} + 1\)
42. \(\frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}\)
43. \(1 - \frac{x}{2} > 4\)
44. \(7 - \frac{4x}{5} < \frac{3}{5}\)
45. \(\frac{x}{6} - \frac{4}{9} \geq \frac{x - 2}{18} + \frac{5}{18}\)
46. \(\frac{4x - 3}{6} + 2 \leq \frac{2x - 1}{12}\)
47. \(4(3x - 2) - 3x < 3(1 + 3x) - 7\)
48. \(3(x - 8) - 2(10 - x) > 5(x - 1)\)

Solve each inequality in Exercises 49–56 and graph the solution set on a number line. Express the solution set using interval notation.

49. \(6 < x + 3 < 8\)
50. \(7 < x + 5 < 11\)
51. \(-3 \leq x - 2 < 1\)
52. \(-6 < x - 4 \leq 1\)
53. \(-11 < 2x - 1 < -5\)
54. \(3 \leq 4x - 3 < 19\)
55. \(-3 \leq \frac{2x}{3} - 5 < -1\)
56. \(-6 \leq \frac{1}{2}x - 4 < -3\)

Solve each inequality in Exercises 57–84 by first rewriting each one as an equivalent inequality without absolute value bars. Graph the solution set on a number line. Express the solution set using interval notation.

57. \(|x| < 3\)
58. \(|x| < 5\)
59. \(|x - 1| \leq 2\)
60. \(|x + 3| \leq 4\)
61. \(|2x - 6| < 8\)
62. \(|3x + 5| < 17\)
63. \(|2(x - 1) + 4| \leq 8\)
64. \(|3(x - 1) + 2| \leq 20\)
65. \(\frac{2y + 6}{3} < 2\)
66. \(\frac{3(x - 1)}{4} < 6\)
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67. \(|x| > 3\)
68. \(|x| > 5\)
69. \(|x - 1| \geq 2\)
70. \(|x + 3| \geq 4\)
71. \(3x - 8 > 7\)
72. \(|5x - 2| > 13\)
73. \(\frac{2x + 2}{4} \geq 2\)
74. \(\left|\frac{3x - 3}{9}\right| \geq 1\)
75. \(3 - \frac{2}{3}x > 5\)
76. \(3 - \frac{3}{4}x > 9\)
77. \(3|x - 1| + 2 \geq 8\)
78. \(-2|4 - x| \geq -4\)
79. \(3 < |2x - 1|\)
80. \(5 \geq |4 - x|\)
81. \(12 < \left|-2x + \frac{6}{7}\right| + \frac{3}{7}\)
82. \(1 < \left|x - \frac{11}{3}\right| + \frac{7}{3}\)
83. \(4 + \left|3 - \frac{x}{3}\right| \geq 9\)
84. \(2 - \frac{x}{2} - 1 \leq 1\)

Application Exercises

The bar graph shows how we spend our leisure time. Let \(x\) represent the percentage of the population regularly participating in an activity. In Exercises 85–92, write the name or names of the activity described by the given inequality or interval.

Percentage of U.S. Population Participating in Each Activity on a Regular Basis

<table>
<thead>
<tr>
<th>Activity</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercise</td>
<td>61%</td>
</tr>
<tr>
<td>Movies</td>
<td>60%</td>
</tr>
<tr>
<td>Gardening</td>
<td>55%</td>
</tr>
<tr>
<td>Amusement Parks</td>
<td>51%</td>
</tr>
<tr>
<td>Home Improvement</td>
<td>47%</td>
</tr>
<tr>
<td>Playing Sports</td>
<td>39%</td>
</tr>
<tr>
<td>Sports Events</td>
<td>37%</td>
</tr>
</tbody>
</table>

Source: U.S. Census Bureau

85. \(x < 40\%\)
86. \(x < 50\%\)
87. \([51\%, 61\%]\)
88. \([47\%, 60\%]\)
89. \([51\%, 61\%]\)
90. \([47\%, 60\%]\)
91. \([39\%, 55\%]\)
92. \([37\%, 47\%]\)

The line graph at the top of the next column shows the declining consumption of cigarettes in the United States. The data shown by the graph can be modeled by

\[ N = 550 - 9x \]

where \(N\) is the number of cigarettes consumed, in billions, \(x\) years after 1988. Use this formula to solve Exercises 93–94.

93. How many years after 1988 will cigarette consumption be less than 370 billion cigarettes each year? Which years does this describe?
94. Describe how many years after 1988 cigarette consumption will be less than 325 billion cigarettes each year. Which years are included in your description?
95. The formula for converting Fahrenheit temperature, \(F\), to Celsius temperature, \(C\), is

\[ C = \frac{5}{9}(F - 32) \]

If Celsius temperature ranges from 15\(^\circ\) to 35\(^\circ\), inclusive, what is the range for the Fahrenheit temperature? Use interval notation to express this range.
96. The formula

\[ T = 0.01x + 56.7 \]

models the global mean temperature, \(T\), in degrees Fahrenheit, of Earth \(x\) years after 1905. For which range of years was the global mean temperature at least 56.7\(^\circ\)F and at most 57.2\(^\circ\)F?

The three television programs viewed by the greatest percentage of U.S. households in the twentieth century are shown in the table. The data are from a random survey of 4000 TV households by Nielsen Media Research. In Exercises 97–98, let \(x\) represent the actual viewing percentage in the U.S. population.

TV Programs with the Greatest U.S. Audience Viewing Percentage of the Twentieth Century

<table>
<thead>
<tr>
<th>Program</th>
<th>Viewing Percentage in Survey</th>
</tr>
</thead>
<tbody>
<tr>
<td>“M<em>A</em>S*H”</td>
<td>60.2%</td>
</tr>
<tr>
<td>Feb. 28, 1983</td>
<td></td>
</tr>
<tr>
<td>“Dallas”</td>
<td>53.3%</td>
</tr>
<tr>
<td>Nov. 21, 1980</td>
<td></td>
</tr>
<tr>
<td>“Roots” Part 8</td>
<td>51.1%</td>
</tr>
<tr>
<td>Jan. 30, 1977</td>
<td></td>
</tr>
</tbody>
</table>

Source: Nielsen Media Research
97. The inequality \(|x - 60.2| \leq 1.6\) describes the actual viewing percentage for “M*A*S*H” in the U.S. population. Solve the inequality and interpret the solution. Explain why the surveys margin of error is \(\pm 1.6\%\).

98. The inequality \(|x - 51.1| \leq 1.6\) describes the actual viewing percentage for “Roots” Part 8 in the U.S. population. Solve the inequality and interpret the solution. Explain why the surveys margin of error is \(\pm 1.6\%\).

99. If a coin is tossed 100 times, we would expect approximately 50 of the outcomes to be heads. It can be demonstrated that a coin is unfair if \(h\), the number of outcomes that result in heads, satisfies \(\frac{h - 50}{\sqrt{50}} \geq 1.645\). Describe the number of outcomes that determine an unfair coin that is tossed 100 times.

100. The inequality \(|T - 57| \leq 7\) describes the range of monthly average temperature, \(T\), in degrees Fahrenheit, for San Francisco, California. The inequality \(|T - 50| \leq 22\) describes the range of monthly average temperature, \(T\), in degrees Fahrenheit, for Albany, New York. Solve each inequality and interpret the solution. Then describe at least three differences between the monthly average temperatures for the two cities.

In Exercises 101–110, use the five-step strategy for solving word problems. Give a linear inequality that models the verbal conditions and then solve the problem.

101. A truck can be rented from Basic Rental for \$50 a day plus \$0.20 per mile. Continental charges \$20 per day plus \$0.50 per mile to rent the same truck. How many miles must be driven in a day to make the rental cost for Basic Rental a better deal than Continental’s?

102. You are choosing between two long-distance telephone plans. Plan A has a monthly fee of \$15 with a charge of \$0.08 per minute for all long-distance calls. Plan B has a monthly fee of \$3 with a charge of \$0.12 per minute for all long-distance calls. How many minutes of long-distance calls in a month make plan A the better deal?

103. A city commission has proposed two tax bills. The first bill requires that a homeowner pay \$1800 plus 3\% of the assessed home value in taxes. The second bill requires taxes of \$200 plus 8\% of the assessed home value. What price range of home assessment would make the first bill a better deal?

104. A local bank charges \$8 per month plus \$5 per check. The credit union charges \$2 per month plus \$6 per check. How many check should be written each month to make the credit union a better deal?

105. A company manufactures and sells blank audiocassette tapes. The weekly fixed cost is \$10,000 and it cost \$0.40 to produce each tape. The selling price is \$2.00 per tape. How many tapes must be produced and sold each week for the company to have a profit gain?

106. A company manufactures and sells personalized stationery. The weekly fixed cost is \$3000 and it cost \$3.00 to produce each package of stationery. The selling price is \$5.50 per package. How many packages of stationery must be produced and sold each week for the company to have a profit gain?

107. An elevator at a construction site has a maximum capacity of 2800 pounds. If the elevator operator weighs 265 pounds and each cement bag weighs 65 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

108. An elevator at a construction site has a maximum capacity of 3000 pounds. If the elevator operator weighs 245 pounds and each cement bag weighs 95 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

109. On two examinations, you have grades of 86 and 88. There is an optional final examination, which counts as one grade. You decide to take the final in order to get a course grade of A, meaning a final average of at least 90.

a. What must you get on the final to earn an A in the course?

b. By taking the final, if you do poorly, you might risk the B that you have in the course based on the first two exam grades. If your final average is less than 80, you will lose your B in the course. Describe the grades on the final that will cause this happen.

110. Parts for an automobile repair cost \$175. The mechanic charges \$34 per hour. If you receive an estimate for at least \$226 and at most \$294 for fixing the car, what is the time interval that the mechanic will be working on the job?

Writing in Mathematics

111. When graphing the solutions of an inequality, what does a parenthesis signify? What does a bracket signify?

112. When solving an inequality, when is it necessary to change the sense of the inequality? Give an example.

113. Describe ways in which solving a linear inequality is similar to solving a linear equation.

114. Describe ways in which solving a linear inequality is different than solving a linear equation.

115. What is a compound inequality and how is it solved?

116. Describe how to solve an absolute value inequality involving the symbol \(<\). Give an example.

117. Describe how to solve an absolute value inequality involving the symbol \(>\). Give an example.

118. Explain why \(|x| < -4\) has no solution.

119. Describe the solution set of \(|x| > -4\).

120. The formula \(V = 3.5x + 120\) models Super Bowl viewers, \(V\), in millions, \(x\) years after 1990. Use the formula to write a word problem that can be solved using a linear inequality. Then solve the problem.
127. What’s wrong with this argument? Suppose $x$ and $y$ represent two real numbers, where $x > y$.

$$2 > 1 \quad \text{This is a true statement.}$$
$$2(y - x) > 1(y - x) \quad \text{Multiply both sides by } y - x.$$
$$2y - 2x > y - x \quad \text{Use the distributive property.}$$
$$y - 2x > -x \quad \text{Subtract } y \text{ from both sides.}$$
$$y > x \quad \text{Add } 2x \text{ to both sides.}$$

The final inequality, $y > x$, is impossible because we were initially given $x > y$. 

128. The graphs of $y = 6$, $y = 3(-x - 5) - 9$, and $y = 0$ are shown in the figure. The graphs were obtained using a graphing utility and a viewing rectangle. Use the graphs to write the solution set for the compound inequality. Express the solution set using interval notation.

$$0 < 3(-x - 5) - 9 < 6.$$
Not afraid of heights and cutting-edge excitement? How about sky diving? Behind your exhilarating experience is the world of algebra. After you jump from the airplane, your height above the ground at every instant of your fall can be described by a formula involving a variable that is squared. At some point, you’ll need to open your parachute. How can you determine when you must do so? Let \( x \) represent the number of seconds you are falling. You can compute when to open the parachute by solving an inequality that takes on the form \( ax^2 + bx + c < 0 \). Such an inequality is called a quadratic inequality.

### Definition of a Quadratic Inequality

A quadratic inequality is any inequality that can be put in one of the forms

- \( ax^2 + bx + c < 0 \)
- \( ax^2 + bx + c > 0 \)
- \( ax^2 + bx + c \leq 0 \)
- \( ax^2 + bx + c \geq 0 \)

where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \).

In this section we establish the basic techniques for solving quadratic inequalities. We will use these techniques to solve inequalities containing quotients, called rational inequalities. Finally, we will consider a formula that models the position of any free-falling object. As a sky diver, you could be that free-falling object!

### Solving Quadratic Inequalities

Graphs can help us to visualize the solutions of quadratic inequalities. The cuplike graph of \( y = x^2 - 7x + 10 \) is shown in Figure 1.21. The \( x \)-intercepts, 2 and 5, are boundary points between where the graph lies above the \( x \)-axis, shown in blue, and where the graph lies below the \( x \)-axis, shown in red. These boundary points play a critical role in solving quadratic inequalities.

**Figure 1.21**
Procedure for Solving Quadratic Inequalities

1. Express the inequality in the general form
   \[ ax^2 + bx + c > 0 \quad \text{or} \quad ax^2 + bx + c < 0. \]

2. Solve the equation \( ax^2 + bx + c = 0 \). The real solutions are the boundary points.

3. Locate these boundary points on a number line, thereby dividing the number line into test intervals.

4. Choose one representative number within each test interval. If substituting that value into the original inequality produces a true statement, then all real numbers in the test interval belong to the solution set. If substituting that value into the original inequality produces a false statement, then no real numbers in the test interval belong to the solution set.

5. Write the solution set, selecting the interval(s) that produced a true statement. The graph of the solution set on a number line usually appears as

This procedure is valid if \(<\) is replaced by \(\leq\) and \(>\) is replaced by \(\geq\).

EXAMPLE 1 Solving a Quadratic Inequality

Solve and graph the solution set on a real number line: \( x^2 - 7x + 10 < 0 \).

Solution

Step 1 Write the inequality in general form. The inequality is given in this form, so this step has been done for us.

Step 2 Solve the related quadratic equation. This equation is obtained by replacing the inequality sign by an equal sign. Thus, we will solve \( x^2 - 7x + 10 = 0 \).

\[
\begin{align*}
  x^2 - 7x + 10 &= 0 & \text{This is the related quadratic equation.} \\
  (x - 2)(x - 5) &= 0 & \text{Factor.} \\
  x - 2 &= 0 & \quad \text{or} \quad x - 5 = 0 \quad \text{Set each factor equal to 0.} \\
  x &= 2 & \quad \text{or} \quad x &= 5 \quad \text{Solve for } x.
\end{align*}
\]

The boundary points are 2 and 5.

Step 3 Locate the boundary points on a number line. The number line with the boundary points is shown as follows:

The boundary points divide the number line into three test intervals, namely \((-\infty, 2), (2, 5), \text{ and } (5, \infty)\).
Step 4 Take one representative number within each test interval and substitute that number into the original inequality.

<table>
<thead>
<tr>
<th>Test Interval</th>
<th>Representative Number</th>
<th>Substitute into ( x^2 - 7x + 10 &lt; 0 )</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, 2))</td>
<td>0</td>
<td>(0^2 - 7 \cdot 0 + 10 &lt; 0)</td>
<td>False</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(10 &lt; 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>((-\infty, 2)) does not belong to the solution set.</td>
<td></td>
</tr>
<tr>
<td>((2, 5))</td>
<td>3</td>
<td>(3^2 - 7 \cdot 3 + 10 &lt; 0)</td>
<td>True</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9 - 21 + 10 &lt; 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2 &lt; 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>((2, 5)) belongs to the solution set.</td>
<td></td>
</tr>
<tr>
<td>((5, \infty))</td>
<td>6</td>
<td>(6^2 - 7 \cdot 6 + 10 &lt; 0)</td>
<td>False</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(36 - 42 + 10 &lt; 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4 &lt; 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>((5, \infty)) does not belong to the solution set.</td>
<td></td>
</tr>
</tbody>
</table>

Step 5 The solution set consists of the intervals that produce a true statement. Our analysis shows that the solution set is the interval \((2, 5)\). The graph in Figure 1.22 confirms that \(x^2 - 7x + 10 < 0\) (lies below the \(x\)-axis) in this interval. The graph of the solution set on a number line is shown as follows:

Figure 1.22 The graph lies below the \(x\)-axis between the boundary points 2 and 5, in the interval \((2, 5)\).

Check Point

Solve and graph the solution set on a real number line:

\[x^2 + 2x - 3 < 0.\]

**EXAMPLE 2** Solving a Quadratic Inequality

Solve and graph the solution set: \(2x^2 + x \geq 15\).

**Solution**

Step 1 Write the inequality in general form. We can write \(2x^2 + x \geq 15\) in standard form by subtracting 15 from both sides. This will give us zero on the right.

\[2x^2 + x - 15 \geq 15 - 15\]
\[2x^2 + x - 15 \geq 0\]

Step 2 Solve the related quadratic equation. This equation is obtained by replacing the inequality sign by an equal sign. Thus, we will solve \(2x^2 + x - 15 = 0\).

\[2x^2 + x - 15 = 0\] This is the related quadratic equation.
\[(2x - 5)(x + 3) = 0\] Factor.
\[2x - 5 = 0\] or \[x + 3 = 0\] Set each factor equal to 0.
\[x = \frac{5}{2}\] or \[x = -3\] Solve for \(x\).

The boundary points are \(-3\) and \(\frac{5}{2}\).
Solve and graph the solution set: \( x^3 - x \geq 20 \).
Many students want to solve
by first multiplying both sides
by to clear fractions.
This is incorrect. The problem
is that contains a
variable and can be positive or
negative, depending on the
value of $x$. Thus, we do not
know whether or not to
reverse the sense of the
inequality.

\[
\begin{align*}
x + 3 &> 0 \quad \text{and} \quad x - 7 > 0 \\
x + 3 &< 0 \quad \text{and} \quad x - 7 < 0.
\end{align*}
\]

Consequently, we solve $\frac{x + 3}{x - 7} > 0$ using boundary points to divide the
number line into test intervals. Then we select one representative number in each
interval to determine whether that interval belongs to the solution set. Example 3
illustrates how this is done.

**EXAMPLE 3 Using Test Numbers to Solve a Rational Inequality**

Solve and graph the solution set: $\frac{x + 3}{x - 7} > 0$.

**Solution** We begin by finding values of $x$ that make the numerator and
denominator 0.

\[
\begin{align*}
x + 3 & = 0 \\
x - 7 & = 0
\end{align*}
\]

Set the numerator and denominator equal to 0.

\[
\begin{align*}
x & = -3 \\
x & = 7
\end{align*}
\]

Solve.

The boundary points are $-3$ and 7. We locate these numbers on a number line
as follows:

These boundary points divide the number line into three test intervals, namely $(-\infty, -3)$, $(-3, 7)$, and $(7, \infty)$. Now, we take one representative number from each test interval and substitute that number into the original inequality.

<table>
<thead>
<tr>
<th>Test Interval</th>
<th>Representative Number</th>
<th>Substitute into $\frac{x + 3}{x - 7} &gt; 0$</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-\infty, -3)$</td>
<td>$-4$</td>
<td>$\frac{-4 + 3}{-4 - 7} &gt; 0$</td>
<td>$(\infty, -3)$ belongs to the solution set.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{-1}{11} &gt; 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{1}{11} &gt; 0$, True</td>
<td></td>
</tr>
<tr>
<td>$(-3, 7)$</td>
<td>$0$</td>
<td>$\frac{0 + 3}{0 - 7} &gt; 0$</td>
<td>$(-3, 7)$ does not belong to the solution set.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{-3}{7} &gt; 0$, False</td>
<td></td>
</tr>
<tr>
<td>$(7, \infty)$</td>
<td>$8$</td>
<td>$\frac{8 + 3}{8 - 7} &gt; 0$</td>
<td>$(7, \infty)$ belongs to the solution set.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{11}{1} &gt; 0$, True</td>
<td></td>
</tr>
</tbody>
</table>
Our analysis shows that the solution set is
\[ (-\infty, -3) \quad \text{or} \quad (7, \infty). \]
The graph of the solution set on a number line is shown as follows:

![Graph of solution set]

**Check Point** 3  Solve and graph the solution set: \( \frac{x - 5}{x + 2} > 0. \)

The first step in solving a rational inequality is to bring all terms to one side, obtaining zero on the other side. Then express the nonzero side as a single quotient. At this point, we follow the same procedure as in Example 3, finding values of the variable that make the numerator and denominator 0. These values serve as boundary points that separate the number line into intervals.

**EXAMPLE 4  Solving a Rational Inequality**

Solve and graph the solution set: \( \frac{x + 1}{x + 3} \leq 2. \)

**Solution**

**Step 1  Express the inequality so that one side is zero and the other side is a single quotient.** We subtract 2 from both sides to obtain zero on the right.

\[
\frac{x + 1}{x + 3} \leq 2 
\]

This is the given inequality.

\[
\frac{x + 1}{x + 3} - 2 \leq 0
\]

Subtract 2 from both sides, obtaining 0 on the right.

\[
\frac{x + 1}{x + 3} - \frac{2(x + 3)}{x + 3} \leq 0
\]

The least common denominator is \( x + 3. \) Express 2 in terms of this denominator.

\[
\frac{x + 1 - 2(x + 3)}{x + 3} \leq 0
\]

Subtract rational expressions.

\[
\frac{x + 1 - 2x - 6}{x + 3} \leq 0
\]

Apply the distributive property.

\[
\frac{-x - 5}{x + 3} \leq 0
\]

Simplify.

**Step 2  Find boundary points by setting the numerator and the denominator equal to zero.**

\[
-x - 5 = 0 \quad \text{and} \quad x + 3 = 0
\]

Set the numerator and denominator equal to 0. These are the values that make the previous quotient zero or undefined.

\[
x = -5 \quad \text{and} \quad x = -3
\]

Solve for \( x. \)

The boundary points are \(-5\) and \(-3.\) Because equality is included in the given less-than-or-equal-to symbol, we include the value of \( x \) that causes the quotient \( \frac{-x - 5}{x + 3} \) to be zero. Thus, \(-5\) is included in the solution set. By contrast, we do not include \(-3\) in the solution set because \(-3\) makes the denominator zero.
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Step 3 Locate boundary points on a number line. The number line, with the boundary points, is shown as follows:

The boundary points divide the number line into three test intervals, namely $(-\infty, -5]$, $[-5, -3)$, and $(-3, \infty)$.

Step 4 Take one representative number within each test interval and substitute that number into the original inequality.

<table>
<thead>
<tr>
<th>Test Interval</th>
<th>Representative Number</th>
<th>Substitute into $\frac{x + 1}{x + 3} \leq 2$</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-\infty, -5]$</td>
<td>-6</td>
<td>$\frac{-6 + 1}{-6 + 3} \leq 2$</td>
<td>$(-\infty, -5]$ belongs to the solution set.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{5}{3} \leq 2$</td>
<td></td>
</tr>
<tr>
<td>$[-5, -3)$</td>
<td>-4</td>
<td>$\frac{-4 + 1}{-4 + 3} \leq 2$</td>
<td>$[-5, -3)$ does not belong to the solution set.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{3}{3} \leq 2$</td>
<td></td>
</tr>
<tr>
<td>$(-3, \infty)$</td>
<td>0</td>
<td>$\frac{0 + 1}{0 + 3} \leq 2$</td>
<td>$(-3, \infty)$ belongs to the solution set.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{1}{3} \leq 2$</td>
<td></td>
</tr>
</tbody>
</table>

Step 5 The solution set consists of the intervals that produce a true statement. Our analysis shows that the solution set is

$(-\infty, -5]$ or $(-3, \infty)$.

The graph of the solution set on a number line is shown as follows:

Check Point 4 Solve and graph the solution set: $\frac{2x}{x + 1} \leq 1$.

Applications

We are surrounded by evidence that the world is profoundly mathematical. For example, did you know that every time you throw an object vertically upward, its changing height above the ground can be described by a mathematical formula? The same formula can be used to describe objects that are falling, such as the sky divers shown in the opening to this section.
The Position Formula for a Free-Falling Object Near Earth’s Surface

An object that is falling or vertically projected into the air has its height above the ground, \( s \), in feet, given by

\[
s = -16t^2 + v_0t + s_0
\]

where \( v_0 \) is the original velocity (initial velocity) of the object, in feet per second, \( t \) is the time that the object is in motion, in seconds, and \( s_0 \) is the original height (initial height) of the object, in feet.

In Example 5, we solve a quadratic inequality in a problem about the position of a free-falling object.

**EXAMPLE 5 Using the Position Model**

A ball is thrown vertically upward from the top of the Leaning Tower of Pisa (176 feet high) with an initial velocity of 96 feet per second (Figure 1.23). During which time period will the ball’s height exceed that of the tower?

**Solution**

\[
s = -16t^2 + v_0t + s_0
\]

This is the position formula for a free-falling object.

\[
s = -16t^2 + 96t + 176
\]

Because \( v_0 \) (initial velocity) = 96 and \( s_0 \) (initial position) = 176, substitute these values into the formula.

\[
-16t^2 + 96t + 176 > 176
\]

This is the inequality implied by the problem’s question. We must find \( t \).

\[
-16t^2 + 96t > 0
\]

Subtract 176 from both sides.

\[
-16t^2 + 96t = 0
\]

Solve the related quadratic equation.

\[-16t(t - 6) = 0\]

Factor.

\[-16t = 0 \text{ or } t - 6 = 0
\]

Set each factor equal to 0.

\[
t = 0 \quad t = 6
\]

Solve for \( t \). The boundary points are 0 and 6.

\[
0 < t < 6
\]

Locate these values on a number line, with \( t \geq 0 \).

The intervals are \((-\infty, 0), (0, 6)\) and \((6, \infty)\). For our purposes, the mathematical model is useful only from \( t = 0 \) until the ball hits the ground. (By setting \(-16t^2 + 96t + 176\) equal to zero, we find \( t \approx 7.47 \); the ball hits the ground after approximately 7.47 seconds.) Thus, we use \((0, 6)\) and \((6, 7.47)\) for our test intervals.
Technology

The graphs of
\[ y_1 = -16x^2 + 96x + 176 \]
and
\[ y_2 = 176 \]
are shown in a viewing rectangle. The graphs show that the ball’s height exceeds that of the tower between 0 and 6 seconds, excluding \( t = 0 \) and \( t = 6 \).

Check Point

An object is propelled straight up from ground level with an initial velocity of 80 feet per second. Its height at time \( t \) is described by
\[ s = -16t^2 + 80t \]
where the height, \( s \), is measured in feet and the time, \( t \), is measured in seconds. In which time interval will the object be more than 64 feet above the ground?

EXERCISE SET 1.8

Practice Exercises

Solve each quadratic inequality in Exercises 1–28, and graph the solution set on a real number line. Express each solution set in interval notation.

1. \( (x - 4)(x + 2) > 0 \)
2. \( (x + 3)(x - 5) > 0 \)
3. \( (x - 7)(x + 3) \leq 0 \)
4. \( (x + 1)(x - 7) \leq 0 \)
5. \( x^2 - 5x + 4 > 0 \)
6. \( x^2 - 4x + 3 < 0 \)
7. \( x^2 + 5x + 4 > 0 \)
8. \( x^2 + x - 6 > 0 \)
9. \( x^2 - 6x + 9 < 0 \)
10. \( x^2 - 2x + 1 > 0 \)
11. \( x^2 - 6x + 8 \leq 0 \)
12. \( x^2 - 2x - 3 \geq 0 \)
13. \( 3x^2 + 10x - 8 \leq 0 \)
14. \( 9x^2 + 3x - 2 \geq 0 \)
15. \( 2x^2 + x < 15 \)
16. \( 6x^2 + x > 1 \)
17. \( 4x^2 + 7x < -3 \)
18. \( 3x^2 + 16x < -5 \)
19. \( 5x \leq 2 - 3x^2 \)
20. \( 4x^2 + 1 \geq 4x \)
21. \( x^2 - 4x \geq 0 \)
22. \( x^2 + 2x < 0 \)
23. \( 2x^2 + 3x > 0 \)
24. \( 3x^2 - 5x \leq 0 \)
25. \( -x^2 + x \geq 0 \)
26. \( -x^2 + 2x \geq 0 \)
27. \( |x^2 + 2x - 36| > 12 \)
28. \( |x^2 + 6x + 1| > 8 \)

Solve each rational inequality in Exercises 29–48, and graph the solution set on a real number line. Express each solution set in interval notation.

29. \( \frac{x - 4}{x + 3} > 0 \)
30. \( \frac{x + 5}{x - 2} > 0 \)
31. \( \frac{x + 3}{x + 4} < 0 \)
32. \( \frac{x + 5}{x + 2} < 0 \)
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33. \[ \frac{-x + 2}{x - 4} \geq 0 \]
34. \[ \frac{-x - 3}{x + 2} \leq 0 \]
35. \[ \frac{4 - 2x}{3x + 4} \leq 0 \]
36. \[ \frac{3x + 5}{5 - 2x} \geq 0 \]
37. \[ \frac{x}{x - 3} > 0 \]
38. \[ \frac{x + 4}{x} > 0 \]
39. \[ \frac{x + 1}{x + 3} < 2 \]
40. \[ \frac{x}{x - 1} > 2 \]
41. \[ \frac{x + 4}{2x - 1} \leq 3 \]
42. \[ \frac{1}{x - 3} < 1 \]
43. \[ \frac{x - 2}{x + 2} \leq 2 \]
44. \[ \frac{x}{x + 2} \geq 2 \]
45. \[ \frac{3}{x + 3} > \frac{3}{x - 2} \]
46. \[ \frac{1}{x + 1} > \frac{2}{x - 1} \]
47. \[ \frac{x^2 - x - 2}{x^2 - 4x + 3} > 0 \]
48. \[ \frac{x^2 - 3x + 2}{x^2 - 2x - 3} > 0 \]

**Application Exercises**

**Use the position formula**

\[ s = -16t^2 + v_0t + s_0 \]

\((v_0 = \text{initial velocity}, s_0 = \text{initial position}, t = \text{time})\)

*to answer Exercises 49–52. If necessary, round answers to the nearest hundredth of a second.*

49. A projectile is fired straight upward from ground level with an initial velocity of 80 feet per second. During which interval of time will the projectile’s height exceed 96 feet?

50. A projectile is fired straight upward from ground level with an initial velocity of 128 feet per second. During which interval of time will the projectile’s height exceed 128 feet?

51. A ball is thrown vertically upward with a velocity of 64 feet per second from the top edge of a building 80 feet high. For how long is the ball higher than 96 feet?

52. A diver leaps into the air at 20 feet per second from a diving board that is 10 feet above the water. For how many seconds is the diver at least 12 feet above the water?

53. The formula

\[ H = \frac{15}{8}x^2 - 30x + 200 \]

models heart rate, \(H\), in beats per minute, \(x\) minutes after a strenuous workout.

a. What is the heart rate immediately following the workout?

b. According to the model, during which intervals of time after the strenuous workout does the heart rate exceed 110 beats per minute? For which of these intervals has model breakdown occurred? Which interval provides a more realistic answer? How did you determine this?

*The bar graph at the top of the next column shows the cost of Medicare, in billions of dollars, projected through 2005. The data can be modeled by a linear model, \(y = 27x + 163\); a quadratic model, \(y = 1.2x^2 + 15.2x + 181.4\).*

54. The graph indicates that Medicare spending will reach $458 billion in 2005. Find the amount predicted by each of the formulas for that year. How well do the formulas model the value in the graph? Which formula serves as a better model for that year?

55. For which years does the quadratic model indicate that Medicare spending will exceed $356.6 billion?

56. For which years does the quadratic model indicate that Medicare spending will exceed $629.4 billion?

*Source: Congressional Budget Office*

57. Describe the company’s production level so that the average cost of producing each wheelchair does not exceed $425. Use a rational inequality to solve the problem. Then explain how your solution is shown on the graph.
58. Describe the company’s production level so that the average cost of producing each wheelchair does not exceed $410. Use a rational inequality to solve the problem. Then explain how your solution is shown on the graph.

Writing in Mathematics

59. What is a quadratic inequality?
60. What is a rational inequality?
61. Describe similarities and differences between the solutions of \((x - 2)(x + 5) \geq 0\) and \(\frac{x - 2}{x + 5} \geq 0\).

Technology Exercises

Solve each inequality in Exercises 62–65 using a graphing utility.

62. \(x^2 + 3x - 10 > 0\)  
63. \(2x^2 + 5x - 3 \leq 0\)  
64. \(x^3 + x^2 - 4x - 4 > 0\)  
65. \(\frac{x - 4}{x - 1} \leq 0\)

Critical Thinking Exercises

66. Which one of the following is true?
   a. The solution set of \(x^2 > 25\) is \((5, \infty)\).
   b. The inequality \(\frac{x - 2}{x + 3} < 2\) can be solved by multiplying both sides by \(x + 3\), resulting in the equivalent inequality \(x - 2 < 2(x + 3)\).
   c. \((x + 3)(x - 1) \equiv 0\) and \(\frac{x + 3}{x - 1} \equiv 0\) have the same solution set.
   d. None of these statements is true.
67. Write a quadratic inequality whose solution set is \([-3, 5]\).
68. Write a rational inequality whose solution set is \((-\infty, -4)\) or \([3, \infty)\).

In Exercises 69–72, use inspection to describe each inequality’s solution set. Do not solve any of the inequalities.

69. \((x - 2)^2 > 0\)  
70. \((x - 2)^2 \leq 0\)  
71. \((x - 2)^2 < -1\)  
72. \(\frac{1}{(x - 2)^2} > 0\)

In Exercises 73–74, use the method for solving quadratic inequalities to solve each higher-order polynomial inequality.

73. \(x^3 + x^2 - 4x - 4 > 0\)  
74. \(x^3 + 2x^2 - x - 2 \geq 0\)

75. The graphing utility screen shows the graph of \(y = 4x^2 - 8x + 7\).

a. Use the graph to describe the solution set of \(4x^2 - 8x + 7 > 0\).
   b. Use the graph to describe the solution set of \(4x^2 - 8x + 7 < 0\).
   c. Use an algebraic approach to verify each of your descriptions in parts (a) and (b).

76. The graphing utility screen shows the graph of \(y = \sqrt{27 - 3x^2}\). Write and solve a quadratic inequality that explains why the graph only appears for \(-3 \leq x \leq 3\).

Group Exercise

77. This exercise is intended as a group learning experience and is appropriate for groups of three to five people. Before working on the various parts of the problem, reread the description of the position formula on page 164.
   a. Drop a ball from a height of 3 feet, 6 feet, and 12 feet. Record the number of seconds it takes for the ball to hit the ground.
   b. For each of the three initial positions, use the position formula to determine the time required for the ball to hit the ground.
   c. What factors might result in differences between the times that you recorded and the times indicated by the formula?
   d. What appears to be happening to the time required for a free-falling object to hit the ground as its initial height is doubled? Verify this observation algebraically and with a graphing utility.
   e. Repeat part (a) using a sheet of paper rather than a ball. What differences do you observe? What factor seems to be ignored in the position formula?
   f. What is meant by the acceleration of gravity and how does this number appear in the position formula for a free-falling object?
Summary

DEFINITIONS AND CONCEPTS

1.1 Graphs and Graphing Utilities

a. The rectangular coordinate system consists of a horizontal number line, the \( x \)-axis, and a vertical number line, the \( y \)-axis, intersecting at their zero points, the origin. Each point in the system corresponds to an ordered pair of real numbers \((x, y)\). The first number in the pair is the \( x \)-coordinate; the second number is the \( y \)-coordinate. See Figure 1.1 on page 76.

b. An ordered pair is a solution of an equation in two variables if replacing the variables by the corresponding coordinates results in a true statement. The ordered pair is said to satisfy the equation. The graph of the equation is the set of all points whose coordinates satisfy the equation. One method for graphing an equation is to plot ordered-pair solutions and connect them with a smooth curve or line.

c. An \( x \)-intercept of a graph is the \( x \)-coordinate of a point where the graph intersects the \( x \)-axis. The \( y \)-coordinate corresponding to a graph's \( x \)-intercept is always zero.

d. A \( y \)-intercept of a graph is the \( y \)-coordinate of a point where the graph intersects the \( y \)-axis. The \( x \)-coordinate corresponding to a graph's \( y \)-intercept is always zero.

1.2 Linear Equations

a. A linear equation in one variable \( x \) can be written in the form \( ax + b = 0, a \neq 0 \).

b. The procedure for solving a linear equation is given in the box on page 86.

c. If an equation contains fractions, begin by multiplying both sides by the least common denominator, thereby clearing fractions.

d. If an equation contains rational expressions with variable denominators, avoid in the solution set any values of the variable that make a denominator zero.

e. An identity is an equation that is true for all real numbers for which both sides are defined. A conditional equation is not an identity and is true for at least one real number. An inconsistent equation is an equation that is not true for even one real number.

1.3 Formulas and Applications

a. A formula is an equation that uses letters to express a relationship between two or more variables.

b. Mathematical modeling is the process of finding equations and formulas to describe real-world phenomena. Such equations and formulas, together with the meaning assigned to the variables, are called mathematical models. Mathematical models can be formed from verbal models or from actual data.

c. A five-step procedure for solving problems using mathematical models is given in the box on page 97.

1.4 Complex Numbers

a. The imaginary unit \( i \) is defined as

\[ i = \sqrt{-1}, \text{ where } i^2 = -1. \]

The set of numbers in the form \( a + bi \) is called the set of complex numbers; \( a \) is the real part and \( b \) is the imaginary part. If \( b = 0 \), the complex number is a real number. If \( b \neq 0 \), the complex number is an imaginary number. Complex numbers in the form \( bi \) are called pure imaginary numbers.

b. Rules for adding and subtracting complex numbers are given in the box on page 109.

c. To multiply complex numbers, multiply as if they are polynomials. After completing the multiplication, replace \( i^2 \) with \(-1\).
DEFINITIONS AND CONCEPTS

d. The complex conjugate of \( a + bi \) is \( a - bi \) and vice versa. The multiplication of complex conjugates gives a real number:
\[
(a + bi)(a - bi) = a^2 + b^2.
\]
e. To divide complex numbers, multiply the numerator and the denominator by the complex conjugate of the denominator.

f. When performing operations with square roots of negative numbers, begin by expressing all square roots in terms of \( i \). The principal square root of \( -b \) is defined by
\[
\sqrt{-b} = i\sqrt{b}.
\]

1.5 Quadratic Equations

a. A quadratic equation in \( x \) can be written in the general form \( ax^2 + bx + c = 0, a \neq 0 \).

b. The procedure for solving a quadratic equation by factoring and the zero-product principle is given in the box on pages 115–116.

c. The procedure for solving a quadratic equation by the square root method is given in the box on page 118.

d. All quadratic equations can be solved by completing the square. Isolate the binomial with the two variable terms on one side of the equation. If the coefficient of the \( x^2 \)-term is not one, divide each side of the equation by this coefficient. Then add the square of half the coefficient of \( x \) to both sides.

e. All quadratic equations can be solved by the quadratic formula
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]
The formula is derived by completing the square of the equation \( ax^2 + bx + c = 0 \).

f. The discriminant, \( b^2 - 4ac \), indicates the number and type of solutions to the quadratic equation \( ax^2 + bx + c = 0 \), shown in Table 1.3 on page 124.

g. Table 1.4 on page 125 shows the most efficient technique to use when solving a quadratic equation.

1.6 Other Types of Equations

a. Some polynomial equations of degree 3 or greater can be solved by moving all terms to one side, obtaining zero on the other side, factoring, and using the zero-product principle. Factoring by grouping is often used.

b. A radical equation is an equation in which the variable occurs in a square root, cube root, and so on. A radical equation can be solved by isolating the radical and raising both sides of the equation to a power equal to the radicals index. When raising both sides to an even power, check all proposed solutions in the original equation. Eliminate extraneous solutions from the solution set.

c. A radical equation with rational exponents can be solved by isolating the expression with the rational exponent and raising both sides of the equation to a power that is the reciprocal of the rational exponent. See the details in the box on page 137.

d. An equation is quadratic in form if it can be written in the form \( at^2 + bt + c = 0 \), where \( t \) is an algebraic expression and \( a \neq 0 \). Solve for \( t \) and use the substitution that resulted in this equation to find the values for the variable in the given equation.

e. Absolute value equations in the form \( |X| = c, c > 0 \), can be solved by rewriting the equation without absolute value bars: \( X = c \) or \( X = -c \).
1.1 Graph each equation in Exercises 1–4.

Let

1. \( y = 2x - 2 \)
2. \( y = x^2 - 3 \)
3. \( y = x \)
4. \( y = |x| - 2 \)

5. What does a \([-20, 40, 10]\) by \([-5, 5, 1]\) viewing rectangle mean? Draw axes with tick marks and label the tick marks to illustrate this viewing rectangle.

In Exercises 6–8, use the graph and determine the \( x \)-intercepts, if any, and the \( y \)-intercepts, if any. For each graph, tick marks along the axes represent one unit each.

6. 

7. 

8. 

9. What percentage of Americans who are 75 have Alzheimer’s disease?
10. What age represents 50% prevalence of Alzheimer’s disease?
11. Describe the trend shown by the graph.

1.2 In Exercises 12–17, solve and check each linear equation.

12. \( 2x - 5 = 7 \)
13. \( 5x + 20 = 3x \)
14. \( 7(x - 4) = x + 2 \)
15. \( 1 - 2(6 - x) = 3x + 2 \)
Exercises 18–22 contain equations with constants in denominators. Solve each equation and check by the method of your choice.

18. \( \frac{2x}{3} = \frac{x}{6} + 1 \)
19. \( \frac{x}{2} - \frac{1}{10} = \frac{x}{5} + \frac{1}{2} \)
20. \( \frac{2x}{3} = 6 - \frac{x}{4} \)
21. \( \frac{x}{4} = 2 + \frac{x - 3}{3} \)
22. \( \frac{3x + 1}{3} - \frac{13}{2} = \frac{1 - x}{4} \)

Exercises 23–26 contain equations with variables in denominators. 

a. List the value or values representing restriction(s) on the variable.  
b. Solve the equation.

23. \( \frac{9}{4} - \frac{1}{2x} = \frac{4}{x} \)
24. \( \frac{7}{x - 5} + 2 = \frac{x + 2}{x - 5} \)
25. \( \frac{1}{x - 1} - \frac{1}{x + 1} = \frac{2}{x^2 - 1} \)
26. \( \frac{4}{x + 2} + \frac{2}{x - 4} = \frac{30}{x^2 - 2x - 8} \)

In Exercises 27–29, determine whether each equation is an identity, a conditional equation, or an inconsistent equation.

27. \( \frac{1}{x + 5} = 0 \)
28. \( 7x + 13 = 4x - 10 + 3x + 23 \)
29. \( 7x + 13 = 3x - 10 + 2x + 23 \)

1.3

30. The percentage, \( P \), of U.S. adults who read the daily newspaper can be modeled by the formula

\[ P = -0.7x + 80 \]

where \( x \) is the number of years after 1965. In which year will 52% of U.S. adults read the daily newspaper?

31. Suppose you were to list in order, from least to most, the family income for every U.S. family. The median income is the income in the middle of this list of ranked data. This income can be modeled by the formula

\[ I = 1321.7(x - 1980) + 21,153. \]

In this formula, \( I \) represents median family income in the United States and \( x \) is the actual year, beginning in 1980. When was the median income $45,875?

In Exercises 32–39, use the five-step strategy given in the box on page 97 to solve each problem.

32. The cost of raising a child through the age of 17 varies by income group. The cost in middle-income families exceeds that of low-income families by $63 thousand, and the cost of high-income families is $3 thousand less than twice that of low-income families. Three children, one in a low-income family, one in a middle-income family, and one in a high-income family, will cost a total of $756 thousand to raise through the age of 17. Find the cost of raising a child in each of the three income groups. (Source: The World Almanac; low annual income is less than $36,800, middle is $36,800–$61,900, and high exceeds $61,900.)

33. In 2000, the average weekly salary for workers in the United States was $567. If this amount is increasing by $15 yearly, in how many years after 2000 will the average salary reach $702. In which year will that be?

34. You are choosing between two long-distance telephone plans. One plan has a monthly fee of $15 with a charge of $0.05 per minute. The other plan has a monthly fee of $5 with a charge of $0.07 per minute. For how many minutes of long-distance calls will the costs for the two plans be the same?

35. You inherit $10,000 with the stipulation that for the first year the money must be placed in two investments paying 8% and 12% annual interest, respectively. How much should be invested at each rate if the total interest earned for the year is to be $950?

36. The length of a rectangular football field is 14 meters more than twice the width. If the perimeter is 346 meters, find the field’s dimensions.

37. The bus fare in a city is $1.50. People who use the bus have the option of purchasing a monthly coupon book for $25.00. With the coupon book, the fare is reduced to $0.25. Determine the number of times in a month the bus must be used so that the total monthly cost without the coupon book is the same as the total monthly cost with the coupon book.

38. A salesperson earns $300 per week plus 5% commission of sales. How much must be sold to earn $800 in a week?

39. A study entitled Performing Arts—The Economic Dilemma documents the relationship between the number of concerts given by a major orchestra and the attendance per concert. For each additional concert given per year, attendance per concert drops by approximately eight people. If 50 concerts are given, attendance per concert is 2987 people. How many concerts should be given to ensure an audience of 2627 people at each concert?

In Exercises 40–42, solve each formula for the specified variable.

40. \( V = \frac{1}{3} Bh \) for \( h \)

41. \( F = f(1 - M) \) for \( M \)

42. \( T = gr + gvt \) for \( g \)

1.4

In Exercises 43–52, perform the indicated operations and write the result in standard form.

43. \( (8 - 3i) - (17 - 7i) \)

44. \( 4i(3i - 2) \)

45. \( (7 - 5i)(2 + 3i) \)

46. \( (3 - 4i)^2 \)

47. \( (7 + 8i)(7 - 8i) \)

48. \( \frac{6}{5 + i} \)

49. \( \frac{3 + 4i}{4 - 2i} \)

50. \( \sqrt{-32} - \sqrt{-18} \)

51. \( (-2 + \sqrt{-100})^2 \)

52. \( \frac{4 + \sqrt{-8}}{2} \)
1.5

Solve each equation in Exercises 53–54 by factoring.
53. \(2x^2 + 15x = 8\)  
54. \(5x^2 + 20x = 0\)

Solve each equation in Exercises 55–56 by the square root method.
55. \(2x^2 - 3 = 125\)  
56. \((3x - 4)^2 = 18\)

In Exercises 57–58, determine the constant that should be added to the binomial so that it becomes a perfect square trinomial. Then write and factor the trinomial.
57. \(x^2 + 20x\)  
58. \(x^2 - 3x\)

Solve each equation in Exercises 59–60 by completing the square.
59. \(x^2 - 12x + 27 = 0\)  
60. \(3x^2 - 12x + 11 = 0\)

Solve each equation in Exercises 61–63 using the quadratic formula.
61. \(x^2 = 2x + 4\)  
62. \(x^2 - 2x + 19 = 0\)  
63. \(2x^2 = 3 - 4x\)

Compute the discriminant of each equation in Exercises 64–65. What does the discriminant indicate about the number and type of solutions?
64. \(x^2 - 4x + 13 = 0\)  
65. \(9x^2 = 2 - 3x\)

Solve each equation in Exercises 66–71 by the method of your choice.
66. \(2x^2 - 11x + 5 = 0\)  
67. \((3x + 5)(x - 3) = 5\)  
68. \(3x^2 - 7x + 1 = 0\)  
69. \(x^2 - 9 = 0\)  
70. \((x - 3)^2 = 25 = 0\)  
71. \(3x^2 - x + 2 = 0\)

72. The weight of a human fetus is modeled by the formula \(W = 3t^2\), where \(W\) is the weight, in grams, and \(t\) is the time, in weeks, \(0 \leq t \leq 39\). After how many weeks does the fetus weigh 1200 grams?

73. The alligator, an endangered species, is the subject of a protection program. The formula \(P = -10x^2 + 475x + 3500\) models the alligator population, \(P\), after \(x\) years of the protection program, where \(0 \leq x \leq 12\). After how many years is the population up to 7250?

74. The graph of the alligator population described in Exercise 73 is shown over time. Identify your solution in Exercise 73 as a point on the graph.

75. An architect is allowed 15 square yards of floor space to add a small bedroom to a house. Because of the room’s design in relationship to the existing structure, the width of the rectangular floor must be 7 yards less than two times the length. Find the length and width of the rectangular floor that the architect is permitted.

76. A building casts a shadow that is double the length of its height. If the distance from the end of the shadow to the top of the building is 300 meters, how high is the building? Round to the nearest meter.

1.6

Solve each polynomial equation in Exercises 77–78.
77. \(2x^4 = 50x^2\)  
78. \(2x^3 - x^2 - 18x + 9 = 0\)

Solve each radical equation in Exercises 79–80.
79. \(\sqrt{2x - 3} + x = 3\)  
80. \(\sqrt{x - 4} + \sqrt{x + 1} = 5\)

Solve the equations containing absolute value in Exercises 83–84.
83. \(x^4 - 5x^2 + 4 = 0\)  
84. \(x^{1/2} + 3x^{1/4} - 10 = 0\)

Solve each equation in Exercises 87–90 by the method of your choice.
87. \(3x^{4/3} - 5x^{2/3} + 2 = 0\)  
88. \(2\sqrt{x - 1} = x\)  
89. \(|x - 5| - 3 = 0\)  
90. \(x^3 + 2x^2 = 9x + 18\)

91. The distance to the horizon that you can see, \(D\), in miles, from the top of a mountain \(H\) feet high is modeled by the formula \(D = \sqrt{2H}\). You’ve hiked to the top of a mountain with views extending 50 miles to the horizon. How high is the mountain?

1.7

In Exercises 92–94, graph the solutions of each inequality on a number line.
92. \(x > 5\)  
93. \(x \leq 1\)  
94. \(-3 \leq x < 0\)

In Exercises 95–97, express each interval in terms of an inequality, and graph the interval on a number line.
95. \((-2, 3]\)  
96. \([-1.5, 2]\)  
97. \((-\infty, 1)\)

Solve each linear inequality in Exercises 98–103 and graph the solution set on a number line. Express each solution set in interval notation.
98. \(-6x + 3 \leq 15\)  
99. \(6x - 9 \geq -4x - 3\)
100. \(\frac{x}{3} - \frac{3}{4} > \frac{x}{2}\)  
101. \(6x + 5 > -2(x - 3) - 25\)
102. \(3(2x - 1) - 2(x - 4) \geq 7 + 2(3 + 4x)\)  
103. \(7 < 2x + 3 \leq 9\)
Chapter 1 Test

1. Graph \( y = x^2 - 4 \) by letting \( x \) equal integers from \(-3\) through \(3\).

2. The graph of \( y = -\frac{1}{3}x + 3 \) is shown in a \([-6, 6, 1]\) by \([-6, 6, 1]\) viewing rectangle. Determine the \( x \)-intercepts, if any, and the \( y \)-intercepts, if any.

3. The graph shows the unemployment rate in the United States from 1990 through 2000. For the period shown, during which year did the unemployment rate reach a maximum? Estimate the percentage of the work force unemployed, to the nearest tenth of a percent, at that time.

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1.8

Solve each quadratic inequality in Exercises 111–112, and graph the solution set on a real number line. Express each solution set in interval notation.

111. \( 2x^2 + 7x \leq 4 \)  
112. \( 2x^2 > 6x - 3 \)

Solve each rational inequality in Exercises 113–114, and graph the solution set on a real number line. Express each solution set in interval notation.

113. \( \frac{x - 6}{x + 2} > 0 \)  
114. \( \frac{x + 3}{x - 4} \leq 5 \)

115. Use the position formula

\[ s = -16t^2 + v_0t + s_0 \]

...to solve this problem. A projectile is fired vertically upward from ground level with an initial velocity of 48 feet per second. During which time period will the projectile's height exceed 32 feet?

Find the solution set for each equation in Exercises 4–16.

4. \( 7(x - 2) = 4(x + 1) - 21 \)
5. \( \frac{2x - 3}{4} = \frac{x - 4}{2} - \frac{x + 1}{4} \)
6. \( \frac{2}{x - 3} - \frac{4}{x + 3} = \frac{8}{x^2 - 9} \)
7. \( 2x^2 - 3x - 2 = 0 \)  
8. \( (3x - 1)^2 = 75 \)
9. \( x(x - 2) = 4 \)  
10. \( 4x^2 = 8x - 5 \)
11. \( x^3 - 4x^2 - x + 4 = 0 \)  
12. \( \sqrt{x - 3} + 5 = x \)
13. \( \sqrt{x + 4} + \sqrt{x - 1} = 5 \)  
14. \( 5x^{\frac{3}{2}} - 10 = 0 \)
15. \( x^{\frac{2}{3}} - 9x^{\frac{1}{3}} + 8 = 0 \)  
16. \( \frac{2}{3}x - 6 = 2 \)

Solve each inequality in Exercises 17–22. Express the answer in interval notation and graph the solution set on a number line.

17. \( 3(x + 4) \geq 5x - 12 \)  
18. \( \frac{x}{6} + \frac{1}{8} \leq \frac{x - 3}{4} \)
19. \( -3 \leq \frac{2x + 5}{3} < 6 \)  
20. \( |3x + 2| \geq 3 \)
21. \( x^2 < x + 12 \)  
22. \( \frac{2x + 1}{x - 3} > 3 \)

In Exercises 23–25, perform the indicated operations and write the result in standard form.

23. \( (6 - 7i)(2 + 5i) \)  
24. \( \frac{5}{2 - i} \)
25. $2\sqrt{-49} + 3\sqrt{-64}$

In Exercises 26–27, solve each formula for the specified variable.

26. $V = \frac{1}{3} lwh$ for $h$

27. $y - y_i = m(x - x_i)$ for $x$

The male minority? The graphs show enrollment in U.S. colleges, with projections from 2000 to 2009. The trend indicated by the graphs is among the hottest topics of debate among college-admission officers. Some private liberal arts colleges have quietly begun special efforts to recruit men—including admissions preferences for them.

Exercises 28–29 are based on the data shown by the graphs.

28. The data for the men can be modeled by the formula

$$N = 0.01x + 3.9$$

where $N$ represents enrollment, in millions, $x$ years after 1984. According to the formula, when will the projected enrollment for men be 4.1 million? How well does the formula describe enrollment for that year shown by the line graph?

The graphs show enrollment in U.S. Colleges, with projections from 2000 to 2009. The trend indicated by the graphs is among the hottest topics of debate among college-admission officers. Some private liberal arts colleges have quietly begun special efforts to recruit men—including admissions preferences for them.

29. The data for the women can be modeled from the following verbal description:

In 1984, 4.1 million women were enrolled. Female enrollment has increased by 0.07 million per year since then. According to the verbal model, when will the projected enrollment for women be 5.71 million? How well does the verbal model describe enrollment for that year shown by the line graph?

30. On average, the number of unhealthy air days per year in Los Angeles exceeds three times that of New York City by 48 days. If Los Angeles and New York City combined have 268 unhealthy air days per year, determine the number of unhealthy days for the two cities. (Source: U.S. Environmental Protection Agency)

31. The costs for two different kinds of heating systems for a three-bedroom home are given in the following table. After how many years will total costs for solar heating and electric heating be the same? What will be the cost at that time?

<table>
<thead>
<tr>
<th>System</th>
<th>Cost to Install</th>
<th>Operating Cost/Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>$29,700</td>
<td>$150</td>
</tr>
<tr>
<td>Electric</td>
<td>$5000</td>
<td>$1100</td>
</tr>
</tbody>
</table>

32. You placed $10,000 in two investments paying 8% and 10% annual interest, respectively. At the end of the year, the total interest from these investments was $940. How much was invested at each rate?

33. The length of a rectangular carpet is 4 feet greater than twice its width. If the area is 48 square feet, find the carpet's length and width.

34. A vertical pole is to be supported by a wire that is 26 feet long and anchored 24 feet from the base of the pole. How far up the pole should the wire be attached?

35. You take a summer job selling medical supplies. You are paid $600 per month plus 4% of the sales price of all the supplies you sell. If you want to earn more than $2500 per month, what value of medical supplies must you sell?