Understanding the Time Value of Money

Learning Objectives

1. Explain the mechanics of compounding.
2. Use a financial calculator to determine the time value of money.
3. Understand the power of time in compounding.
4. Explain the importance of the interest rate in determining how an investment grows.
5. Calculate the present value of money to be received in the future.
6. Define an annuity and calculate its compound or future value.

Let’s just do what we always do—hijack some nuclear weapons and hold the world ransom.” These are the words of Dr. Evil, played by Mike Myers in the movie Austin Powers, International Man of Mystery. Frozen, along with his cat, Mr. Bigglesworth, in 1967 after escaping from the Electric Psychedelic Pussycat Swinger’s Club in London, Dr. Evil is thawed out in 1997 and immediately resumes his evil ways.

Dr. Evil continues with his plan: “Gentlemen, it has come to my attention that a breakaway Russian Republic Ripblackastan is about to transfer a nuclear warhead to the United Nations in a few days. Here’s the plan. We get the warhead, and we hold the world ransom for . . . one million dollars!”

Silence, followed by, “Umm, umm, umm” from Dr. Evil’s Number 2 man, played by Robert Wagner: “Don’t you think we should ask for more than a million dollars? A million dollars is not exactly a lot of money these days.”

“OK, then we hold the world ransom for $100 billion.”
A million dollars in 1967 certainly bought more than a million dollars in 1997 did. But consider this, if Dr. Evil had taken a million dollars in 1967 and put it in the stock market, it would have accumulated to over $30.7 million when he was thawed out in 1997. However, that $30+ million wouldn’t have the same purchasing power it did 30 years earlier. In fact, given the rate of inflation over that period, it would only purchase about one-fifth of what it would have in 1967.

Three decades is a long time. The year Dr. Evil was frozen one of the top-rated TV shows was *Bewitched*, and *The Monkees* took top honors at that year’s Emmy Awards for best comedy show. The Green Bay Packers won Super Bowl I. Thirty years later Dr. Evil woke up to those same sitcoms airing on “Nick at Night” and the Green Bay Packers winning Super Bowl XXXI. But times had changed—now the world was full of personal computers, compact disks, MP3s, and cable TV.

Now look to the future: For most of you, it will be well over 30 years before you retire. If you want to work out how much you will need for your golden years, how the heck do you look at today’s dollars and come up with a dollar figure? As we saw in **Principle 2: The Time Value of Money**, a dollar received today is worth more than a dollar received in the future. For one thing, a dollar received and invested today starts earning interest sooner than a dollar received and invested some time in the future. Remember, the **time value of money** means that we can’t compare amounts of money from two different periods without adjusting for this difference in value. Clearly, if you want a firm grasp on personal finance, it’s important to understand the time value of money.
Just how powerful is the time value of money? Think about this: if you were to invest $1,000 at 8 percent interest for 400 years, you would end up with $23 quadrillion—approximately $5 million per person on Earth. Of course, your investments won’t span 400 years—it’s doubtful that you’ll be cryogenically frozen like Dr. Evil and Austin Powers—but your investments will rely on the time value of money. If you manage properly, time can be the ace up your sleeve—the one that lets you pocket more than you would have imagined possible.

In personal finance, the time value of money is just as widespread as it is powerful. We’re always comparing money from different periods—for example, buying a bond today and receiving interest payments in the future, borrowing money to buy a house today and paying it back over the next 30 years, or determining exactly how much to save annually to achieve a certain goal. In fact, there’s very little in personal finance that doesn’t have some thread of the time value of money woven through it.

**Compound Interest and Future Values**

How does the time value of money turn small sums of money into extremely large sums of money? Through compound interest. **Compound interest** is basically interest paid on interest. If you take the interest you earn on an investment and reinvest it, you then start earning interest on the **principal** and the reinvested interest. In this way, the amount of interest you earn grows, or compounds.

**How Compound Interest Works**

Anyone who has ever had a savings account has received compound interest. For example, suppose you place $100 in a savings account that pays 6 percent interest annually. How will your savings grow? At the end of the first year you’ll have earned 6 percent or $6 on your initial deposit of $100, giving you a total of $106 in your savings account. That $106 is the **future value** (**FV**) of your investment, that is, the value of your investment at some future point in time. The mathematical formula illustrating the payment of interest is

\[
FV_1 = PV + PV(i) = PV(1 + i)
\]

(3.1)

where

\[FV_1 = \text{the future value of the investment at the end of 1 year}\]

\[i = \text{the annual interest rate}; \text{the interest earned is based on the balance at the beginning of the year and is paid at the end of the year. In this case } i = 6\% \text{ or, expressed in decimal form, } 0.06.\]

\[PV = \text{the present value, or the current value; that is, the value in today’s dollars of a sum of money}\]
In our example

\[ FV_1 = PV + PV(i) \]
\[ = PV(1 + i) \]
\[ = $100(1 + .06) \]
\[ = $100(1.06) \]
\[ = $106 \] \hspace{1cm} (3.1) \]

Assuming you leave the $6 interest payment in your savings account, known as reinvesting, what will your savings look like at the end of the second year? The future value of $106 at the end of the first year, \( FV_1 \), becomes the present value at the beginning of the second year. Inserting this number into equation (3.1), we get

\[ FV_2 = FV_1(1 + i) \] \hspace{1cm} (3.2) \]

which, for the example, gives

\[ FV_2 = $106(1.06) \]
\[ = $112.36 \]

What will your savings look like at the end of 3 years? 5 years? 10 years? Figure 3.1 illustrates how an investment of $100 would continue to grow for the first 10 years at a compound interest rate of 6 percent. Notice how the amount of interest earned annually increases each year because of compounding.

Why do you earn more interest during the second year than you did during the first? Simply because you now earn interest on the sum of the original principal, or present value, and the interest you earned in the first year. In effect, you are now earning interest on interest, which is the concept of compound interest.

How did we determine all the future values of your investment in Figure 3.1? We started with equations (3.1) and (3.2). Because those two equations share a common element, \( FV_1 \), we combine them to get

\[ FV_2 = FV_1(1 + i) \]
\[ = PV(1 + i)(1 + i) \]
\[ = PV(1 + i)^2 \] \hspace{1cm} (3.3) \]

To find what we had at the end of year 2, we just took the amount we had at the end of year 1 and multiplied it by \((1 + i)\).

We can generalize equation (3.3) to illustrate the value of your investment for any number of years by using the following equation:

\[ FV_n = PV(1 + i)^n \] \hspace{1cm} (3.4) \]

where

- \( FV_n \) = the future value of the investment at the end of \( n \) years
- \( n \) = the number of years during which the compounding occurs
- \( i \) = the annual interest rate
- \( PV \) = the present value, or the current value; that is, the value of a sum of money in today’s dollars
With annual compounding the interest is received at the end of each year and then reinvested back into the investment. Then, at the end of the second year, interest is earned on this new sum.

Equation (3.4) is the time value of money formula, and it will work for any investment that pays a fixed amount of interest, \( i \), for the life of the investment. As we work through this chapter, sometimes we will solve for \( i \) and other times we will solve for \( PV \) or \( n \). Regardless, equation (3.4) is the basis for almost all of our time value calculations.

**Example**

You receive a $1,000 academic award this year for being the best student in your personal finance course, and you place it in a savings account paying 5 percent annual interest compounded annually. How much will your account be worth in 10 years? Substituting \( PV = 1000 \), \( i = 5 \) percent, and \( n = 10 \) years into equation (3.4), you get

\[
FV_n = PV(1 + i)^n
\]

\[
= 1000(1 + 0.05)^{10}
\]

\[
= 1000(1.62889)
\]

\[
= 1628.89
\]

Thus, at the end of 10 years you will have $1,628.89 in your savings account. Unless, of course, you decide to add in or take out money along the way.
The Future-Value Interest Factor

Calculating future values by hand can be a serious chore. Luckily, you can use a calculator. Also, there are tables for the \((1 + i)^n\) part of the equation, which will now be called the future-value interest factor for \(i\) and \(n(FVIF_{i,n})\). These tables simplify your calculations by giving you the various values for combinations of \(i\) and \(n\).

Table 3.1 provides one such table (a more comprehensive version appears in Appendix A at the back of this book). Note that the amounts given in this table represent the value of \$1\) compounded at rate \(i\) at the end of the \(n\)th year. Thus, to calculate the future value of an initial investment, you need only determine the \(FVIF_{i,n}\) using a calculator or a table and multiply this amount by the initial investment. In effect, you can rewrite equation (3.4) as follows:

\[
\text{future value} = \text{present value} \times \text{future-value interest factor}
\]

or

\[
FV_n = PV(FVIF_{i,n}) \quad (3.4a)
\]

Let’s look back at the previous example of investing \$1,000\) at 5 percent compounded annually for 10 years. In Table 3.1 at the intersection of the \(n = 10\) row and the 5\% column, we find a value for \(FVIF_{5\%,10\text{yr}} = 1.629\). Thus,

\[
FV_{10} = \$1,000(1.629) = \$1,629
\]

This is the same answer we got before. You can also use this equation to solve for \(n\) and \(i\).

The average cost of a wedding in 2005 was approximately \$22,000. What will that wedding cost in 30 years, assuming a 4\% annual rate of inflation? We know that \(FV_n = PV(FVIF_{i\%,n\text{years}})\). To find the future value of a wedding in 30 years we need only multiply the present value (PV) of a wedding ($22,000) times the future-value interest factor for 4\% and 30 years (\(FVIF_{4\%,30\text{yr}}\)). To find the future-value interest factor for 4\% and 30 years, go to the \(FVIF\) table (see Appendix A) and simply move down the \(i = 4\%\) column until you reach its intersection with the \(n = 30\) years row: 3.243. Thus, the future value of a wedding in 30 years is:

\[
\text{future value} = \text{present value} \times \text{future-value interest factor}
\]

\[
FV = PV(FVIF_{4\%,30\text{yr}})
\]

\[
FV = \$22,000 (3.243)
\]

\[
= \$71,346
\]

Today’s average wedding will cost \$71,346 30 years from now!

The Rule of 72

Now you know how to determine the future value of any investment. What if all you want to know is how long it will take to double your money in that investment? One simple way to approximate how long it will take for a given sum to double in value is called the Rule of 72. This “rule” states that you can determine how many years it will take for a given sum to double by dividing the investment’s annual growth or interest rate into 72. For example, if an investment grows at an annual rate of 9 percent per year, according to the Rule of 72 it should take \(72/9 = 8\) years for that sum to double.

Example

**Rule of 72**

A helpful investment rule that states you can determine how many years it will take for a sum to double by dividing the annual growth rate into 72.
Keep in mind that this is not a hard and fast rule, just an approximation, but it’s a pretty good approximation at that. For example, the future-value interest factor from Table 3.1 for 8 years at 9 percent is 1.993, which is pretty close to the Rule of 72’s approximation of 2.0.

### Example

Using the “Rule of 72,” how long will it take to double your money if you invest it at 12 percent compounded annually?

\[
\text{numbers of years to double} = \frac{72}{\text{annual compound growth rate}}
\]

\[
= \frac{72}{12}
\]

\[
= 6 \text{ years}
\]

### Compound Interest with Nonannual Periods

Until now we’ve assumed that the compounding period is always annual. Sometimes, though, financial institutions compound interest on a quarterly, daily, or even continuous basis. What happens to your investment when your compounding period is nonannual? You earn more money faster. The sooner your interest is paid, the sooner you start earning interest on it, and the sooner you experience the benefits of compound interest.

The bottom line is that your money grows faster as the compounding period becomes shorter—for example, from annual compounding to monthly compounding. That’s because interest is earned on interest more frequently as the length of the compounding period declines.
Using a Financial Calculator

Time value of money calculations can be made simple with the aid of a financial calculator. If you don’t own a financial calculator, you can easily find one on the web—there’s an excellent one on the Website that accompanies this book (www.prenhall.com/keown). There’s even a little tutorial there. This calculator is illustrated in Figure 3.2. Before you try to whoop it up solving time value of money problems on your financial calculator, take note of a few keys that will prove necessary. There are five keys on a financial calculator that come into play:

\[ \text{N or Periods} \quad \text{I/Y or Rate} \quad \text{PV} \quad \text{PMT} \quad \text{FV} \]

Here's what they stand for:

- **N or Periods**: Stores (or calculates) the total number of payments or compounding periods.
- **I/Y or Rate**: Stores (or calculates) the interest or discount rate.
- **PV**: Stores (or calculates) the present value.
- **FV**: Stores (or calculates) the future value.
- **PMT**: Stores (or calculates) the dollar amount of each annuity payment. (We talk about these later in the chapter, but an annuity is a series of equal dollar payments for a specified number of time periods.)

And if you're using a Texas Instruments BA II Plus calculator, here's another one you’ll want to know about:

- **CPT**: This is the compute key on the Texas Instruments BA II Plus calculator, the calculator we use in examples in this text. If you want to compute the present value, you enter the known variables and press CPT PV.

Every calculator operates a bit differently with respect to entering variables. It is a good idea to become familiar with exactly how your calculator functions.

Figure 3.2 Time Value of Money Calculator on the Web site that Accompanies this Book
To solve a time value of money problem using a financial calculator, all you need to do is enter the appropriate numbers for three of the four variables and then press the key of the final variable to calculate its value.

Now let’s solve the previous example using a financial calculator. We were trying to find at what rate $11,167 must be compounded annually for it to grow to $20,000 in 10 years. All you have to do is input the known variables, then calculate the value of the one you’re looking for.

Enter 10

11,167 0 20,000

N I/Y PV PMT FV

Solve for 6.0

Why the negative sign before the $11,167? When using a financial calculator each problem will have at least one positive number and one negative number. In effect, a financial calculator sees money as “leaving your hands” and taking on a negative sign, or “returning to your hands” and taking on a positive sign. You’ll also notice that the answer appears as 6.0 rather than 0.06—when entering interest rates enter them as percentages rather than decimals, that is, 10 percent would be entered as 10, not .10.

**Calculator Tips: How to Get It Right**

Calculators are pretty easy to use. When people have problems with calculators, it is usually the result of a few common mistakes. Before you take a crack at solving a problem using a financial calculator:

1. Set your calculator to one payment per year. Some financial calculators use monthly payments as the default, so you will need to change it to annual payments.
2. Set your calculator to display at least four decimal places. Most calculators are preset to display only two decimal places. Because interest rates are so small, change your decimal setting to at least four.
3. Set your calculator to the “end” mode. Your calculator will assume cash flows occur at the end of each time period.

When you’re ready to work a problem, remember:

1. Every problem will have at least one positive and one negative number.
2. You must enter a zero for any variable that isn’t used in a problem, or you have to clear the calculator before beginning a new problem. If you don’t enter a value for one of the variables, your calculator won’t assume that the variable is zero. Instead, your calculator will assume it carries the same number as it did during the previous problem.
3. Enter the interest rate as a percent, not a decimal. That means 10% must be entered as 10 rather than .10.

**Compounding and the Power of Time**

Manhattan Island was purchased by Peter Minuit from Native Americans in 1626 for $24 in knickknacks and jewelry. If at the end of 1626 the Native Americans had invested their $24 at 8 percent compounded annually, it would be worth over $120.6 trillion today (by the end of 2006, 380 years later). That’s certainly enough to buy back all of Manhattan. In fact, with $120.6 trillion in the bank, the $90 billion to $100 billion you’d have to pay to buy back all of Manhattan would seem like pocket change. The story illustrates the incredible power of time in compounding. Let’s take a closer look.
The Power of Time

Why should you care about compounding? Well, the sooner you start saving for retirement and other long-term goals, the less painful the process of saving will be. Consider the tale of twin sisters who work at the Springfield DMV. Selma and Patty Bouvier decide to save for retirement, which is 35 years away. They’ll both receive an 8-percent annual return on their investment over the next 35 years.

Selma invests $2,000 per year at the end of each year only for the first 10 years of the 35-year period—for a total of $20,000 saved. Patty doesn’t start saving for 10 years and then saves $2,000 per year at the end of each year for the remaining 25 years—for a total of $50,000 saved. When they retire, Selma will have accumulated just under $200,000, while Patty will have accumulated just under $150,000, despite the fact that Selma saved for only 10 years while Patty saved for 25 years. Figure 3.3 presents their results and illustrates the power of time in compounding.

Let’s look at another example to see what this really means to you. The compound growth rate in the stock market over the period 1926–2004 was approximately 10.4 percent. Although the rate of return on stocks has been far from constant over this period, assume for the moment that you could earn a constant annual return of 10.4 percent compounded annually on an investment in stocks. If you invested $500 in stocks at the beginning of 1926 and earned 10.4 percent compounded annually, your investment would have grown to $1,511,698 by the end of 2006 (81 years). That would make you one wealthy senior citizen.

Figure 3.3 The Power of Time in Compounding

<table>
<thead>
<tr>
<th>Initial Investment</th>
<th>Contribution Years</th>
<th>Years Investing</th>
<th>Total Investment</th>
<th>Return at End of 35 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20,000</td>
<td>1–10</td>
<td>10</td>
<td>$20,000</td>
<td>$198,422</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$146,212</td>
</tr>
<tr>
<td>$50,000</td>
<td>11–35</td>
<td>25</td>
<td>$50,000</td>
<td></td>
</tr>
</tbody>
</table>

1See Ibbotson Associates, Bonds, Bills, & Inflation 2005 Yearbook™ (Chicago, 2005). The power of compounding is truly amazing. Say you’re 22 and intend to retire at age 65—43 years from now. If you place $14,201 in stocks and they earn 10.4 percent compounded annually over those 43 years, you’d have accumulated $1 million by retirement—all in just 15,695 days!
Let’s look at one example that illustrates the danger in just looking at the bottom-line numbers without considering the time value of money. One of today’s “hot” collectibles is the Schwinn Deluxe Tornado boys’ bicycle, which sold for $49.95 in 1959. In 2006, 47 years later, a Schwinn Tornado in mint condition is selling on eBay for $772.50, which is 15.47 times its original cost. At first glance you might view this as a 1,547-percent return—but you’d be ignoring the time value of money. At what rate did this investment really compound? The answer is 6.00 percent per year, which ignores any storage costs that might have been incurred. The Schwinn may provide a great ride, but given what you just saw common stocks doing over the same period, it doesn’t provide a very good return.

The Importance of the Interest Rate

It’s not just time that makes money grow in value, it’s also the interest rate. Most people understand that a higher interest rate earns you more money—that’s why some people are willing to buy a risky bond issued by MGM Grand that pays 11 percent rather than a very safe bond issued by the government that pays only 5.0 percent—but most people don’t understand just how dramatic a difference the interest rate can make. This brings us back to Principle 9: The Best Protection Is Knowledge.

Without an understanding of investment concepts such as the time value of money, you’re a prime target for bad advice. You’re also at a real disadvantage because you might not be able to take advantage of good deals and even understand basic financial principles, such as those that apply to interest rates. The bottom line is it’s much easier to do things correctly if you understand what you’re doing. Let’s take a closer look at interest rates.

Obviously, the choice of interest rate plays a critical role in how much an investment grows. But do small changes in the interest rate have much of an impact on future values? To answer this question, let’s look back to Peter Minuit’s purchase of Manhattan. If the Native Americans had invested their $24 at 10 percent rather than 8 percent compounded annually at the end of 1626, they would have $129 quadrillion by the end of 2006. That’s 129 moved over 15 decimal places, or $129,000,000,000,000,000. Actually, that’s enough to buy back not only Manhattan Island, but the entire world and still have plenty left over!

Now let’s assume a lower interest rate, say 6 percent. In that case the $24 would have grown to a mere $99.2 billion—one thousandth of what it grew to at 8 percent, and only one millionth of what it would have grown to at 10 percent. With today’s real estate prices, you might be able to buy Manhattan, but you probably couldn’t pay your taxes!

To illustrate the power of a high interest rate in compounding, let’s look at a “daily double.” A “daily double” simply means that your money doubles each day. In effect, it assumes an interest rate of 100 percent compounded on a daily basis. Let’s see what can happen to a penny over a month’s worth of daily doubles, assuming that the month has 31 days in it. The first day begins with 1¢, the second day it compounds to 2¢, the third day it becomes 4¢, the fourth day 8¢, the fifth day 16¢, and so forth. As shown in Table 3.2, by day 20 it would have grown to $5,242.88, and by day 31 it would have grown to over $10 million. This explains why Albert Einstein once marveled that “Compound interest is the eighth wonder of the world.”
Discount Rate
The interest rate used to bring future dollars back to the present.

Present Value
Up until this point we’ve been moving money forward in time; that is, we know how much we have to begin with and are trying to determine how much that sum will grow in a certain number of years when compounded at a specific rate. We’re now going to look at the reverse question: What’s the value in today’s dollars of a sum of money to be received in the future? That is, what’s the present value?

Why is present value important to us? It lets us strip away the effects of inflation and see what future cash flows are worth in today’s dollars. It also lets us compare dollar values from different periods. In later chapters we’ll use the present value to determine how much to pay for stocks and bonds.

In finding the present value of a future sum, we’re moving future money back to the present. What we’re doing is, in fact, nothing other than inverse compounding. In compounding we talked about the compound interest rate and the initial investment; in determining the present value we will talk about the discount rate and present value.

When we use the term “discount rate,” we mean the interest rate used to bring future money back to present, that is, the interest rate used to “discount” that future money back to present. For example, if we expected to receive a sum of money in 10 years and wanted to know what it would buy in today’s dollars, we would discount that future sum of money back to present at the anticipated inflation rate. Other than that, the technique and the terminology remain the same, and the mathematics are simply reversed.

Let’s return to equation (3.4), the time value of money equation. We now want to solve for present value instead of future value. To do this we can simply rearrange the terms in equation (3.4) and we get

\[ PV = FV_n \left[ \frac{1}{(1 + i)^n} \right] \]  

(3.5)

where

\( FV_n \) = the future value of the investment at the end of \( n \) years

\( n \) = the number of years until the payment will be received

\( i \) = the annual discount (or interest) rate

\( PV \) = the present value of the future sum of money

Table 3.2  The Daily Double

<table>
<thead>
<tr>
<th>Day</th>
<th>“Daily double”: 1¢ at 100% Compounded Daily Would Become</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1</td>
<td>$.01</td>
</tr>
<tr>
<td>Day 2</td>
<td>.02</td>
</tr>
<tr>
<td>Day 3</td>
<td>.04</td>
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<td>.08</td>
</tr>
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<td>Day 5</td>
<td>.16</td>
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<td>Day 7</td>
<td>.64</td>
</tr>
<tr>
<td>Day 8</td>
<td>1.28</td>
</tr>
<tr>
<td>Day 15</td>
<td>163.84</td>
</tr>
<tr>
<td>Day 20</td>
<td>5,242.88</td>
</tr>
<tr>
<td>Day 25</td>
<td>167,772.16</td>
</tr>
<tr>
<td>Day 30</td>
<td>5,368,709.12</td>
</tr>
<tr>
<td>Day 31</td>
<td>10,737,418.24</td>
</tr>
</tbody>
</table>
Present-Value Interest Factor \( (PVIF_{i,n}) \)
The value \( [1/(1 + i)^n] \) used as a multiplier to calculate an amount's present value.

Because the mathematical procedure for determining the present value is exactly the inverse of determining the future value, the relationships among \( n \), \( i \), and \( PV \) are just the opposite of those we observed in future value. The present value of a future sum of money is inversely related to both the number of years until the payment will be received and the discount rate. Figure 3.4 shows this relationship graphically.

To help us compute present values, we once again have some handy tables. This time they calculate the \( [1/(1 + i)^n] \) part of the equation, which we call the present-value interest factor for \( i \) and \( n \), or \( PVIF_{i,n} \). These tables simplify the math by giving us the various values for combinations of \( i \) and \( n \) defined as \( [1/(1 + i)^n] \). Appendix B at the back of this book presents fairly complete versions of these tables, and an abbreviated version appears in Table 3.3.

**Table 3.3** Present Value of $1 (single amount), \( PVIF_{i,n} \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
<th>8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.962</td>
<td>.952</td>
<td>.943</td>
<td>.935</td>
<td>.926</td>
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<tr>
<td>2</td>
<td>.925</td>
<td>.907</td>
<td>.890</td>
<td>.873</td>
<td>.857</td>
</tr>
<tr>
<td>3</td>
<td>.889</td>
<td>.864</td>
<td>.840</td>
<td>.816</td>
<td>.794</td>
</tr>
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<td>.855</td>
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</tr>
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<tr>
<td>8</td>
<td>.731</td>
<td>.677</td>
<td>.627</td>
<td>.582</td>
<td>.540</td>
</tr>
<tr>
<td>9</td>
<td>.703</td>
<td>.645</td>
<td>.592</td>
<td>.544</td>
<td>.500</td>
</tr>
<tr>
<td>10</td>
<td>.676</td>
<td>.614</td>
<td>.558</td>
<td>.508</td>
<td>.463</td>
</tr>
<tr>
<td>11</td>
<td>.650</td>
<td>.585</td>
<td>.527</td>
<td>.475</td>
<td>.429</td>
</tr>
</tbody>
</table>

**Instructions:** Each present-value interest factor (PVIF\(_{6\%,10\,\text{years}}\)) corresponds to a specific time period and interest rate. To find the present-value interest factor for 6 percent and 10 years (PVIF\(_{6\%,10\,\text{years}}\)), simply move down the \( i = 6\% \) column until you reach its intersection with the \( n = 10\,\text{years} \) row: 0.558. The present value is then calculated as follows:

\[
pv = future\,value \times present-value\,interest\,factor\n\]

\[
PV = FV \times PVIF_{6\%,10\,\text{years}}\n\]
A close examination of Table 3.3 shows that the values in these tables are the inverse of the tables found in Appendix A and Table 3.1. Of course, this inversion makes sense because the values in Appendix A are \((1 + i)^n\) and those in Appendix B are \([1/(1 + i)^n]\). To determine the present value of a sum of money to be received at some future date, you need only determine the value of the appropriate \(PVIF_{i,n}\) by using a calculator or consulting the tables, and multiply it by the future value. In effect, you can use the new notation and rewrite equation (3.5) as follows:

\[
\text{present value} = \text{future value} \times \text{present-value interest factor}
\]

or

\[
PV = FV_n (PVIF_{i,n})
\]  

(3.5a)

Example

You’re on vacation in Florida and you see an advertisement stating that you’ll receive $100 simply for taking a tour of a model condominium. However, when you investigate, you discover that the $100 is in the form of a savings bond that will not pay you the $100 for 10 years. What is the present value of $100 to be received 10 years from today if your discount rate is 6 percent? By looking at the \(n = 10\) row and \(i = 6\%\) column of Table 3.3, you find the \(PVIF_{6\%, 10 \text{ yr}}\) is 0.558. Substituting \(FV_{10} = 100\) and \(PVIF_{6\%, 10 \text{ yr}} = 0.558\) into equation (3.5a), you find

\[
PV = 100(PVIF_{6\%, 10 \text{ yr}})
\]

\[
= 100(0.558)
\]

\[
= 55.80
\]

Thus, the value in today’s dollars of that $100 savings bond is only $55.80. Not a bad take for touring a condo, but it’s not a hundred bucks.

Calculator Clues

Calculating a Present Value

Note for all calculations in this chapter: If you don’t have a financial calculator handy, you can use the one that is located on Web site that accompanies this book: www.prenhall.com/keown.

In the example above, we’re calculating the present value of $100 to be received in 10 years given a 6 percent interest or discount rate. For any value that does not appear in the calculations we’ll enter a value of 0.

Enter

\[
\begin{array}{ccc}
\text{N} & \text{I/Y} & \text{PV} & \text{PMT} & \text{FV} \\
10 & 6 & 0 & 100 & \\
\end{array}
\]

Solve for

\(-55.84\)

You’ll notice that this calculator solution is slightly different from the answer we just got using the tables; that’s just a matter of rounding error. You’ll also notice we get a negative value for the answer. Remember, that’s because financial calculators view money like a bank does. You deposit money in the bank (the sign is negative because the money “leaves your hands”), and later you take money out of the bank (the sign is positive because the money “returns to your hands”). Every problem with two cash flows will have one with a positive sign and one with a negative sign.
Example

Let’s consider the impatient son of wealthy parents who wants his inheritance NOW! He’s been promised $500,000 in 40 years. Assuming the appropriate discount rate (i.e., the interest rate used to bring future money back to the present) is 6 percent, what is the present value of the $500,000? We know that 

$$ PV = FV \times (PVIF_{i\%, n \text{ years}}) $$

To find the present value of the estate we need only multiply the future value ($FV$), which is $500,000 times the present-value interest factor for 6% and 40 years ($PVIF_{6\%, 40 \text{ yr}}$). To find the present-value interest factor for 6% and 40 years, go to the $PVIF$ table (see Appendix B) and simply move down the $i = 6\%$ column until you reach its intersection with the $n = 40 \text{ year}$ row: 0.097. Thus, the present value of the estate is:

$$ PV = FV \times (PVIF_{i\%, n \text{ years}}) $$

$$ PV = 500,000 \times (0.097) $$

$$ PV = 48,500 $$

That $500,000 the son is to receive in 40 years is worth only $48,500 in today’s dollars. Another way of looking at this problem is that if you deposit $48,500 in the bank today earning 6 percent annually, in 40 years you’d have $500,000.

Calculator Clues

Calculating a Present Value

In this example, you’re solving for the present value of $500,000 to be received in 40 years given a 6 percent interest or discount rate.

Enter 40 6 0 500,000

Solve for $N \ I/Y \ PV \ PMT \ FV$

As expected, you get a negative sign on the PV. Try entering a higher value for I/Y and see what happens to the PV. Again, you’ll notice a slight difference in the calculator solution due to rounding error and the negative sign that the solution takes on.

STOP & THINK

Why should you be interested in stripping away the effects of inflation from money you receive in the future? Because the dollar value of future money is not as important as that money’s purchasing power. For example, you might be excited if you were told you would receive $1 million in 20 years. However, if you then found out that in 20 years a new car will cost $800,000, your average monthly food bill will be $15,000, and a typical month’s rent on your apartment will be $30,000, you would have a different view of the $1 million. Using the time value of money to strip away the effects of inflation allows you to calculate the value of a future amount in terms of the purchasing power of today’s dollars.

Example

What is the present value of an investment that yields both $500 to be received in 5 years and $1,000 to be received in 10 years if the discount rate is 4 percent? Substituting the values of $n = 5$, $i = 4\%$, and $FV_5 = 500$; and $n = 10$, $i = 4\%$, and $FV_{10} = 1,000$ into equation (3.5a) and adding these values together, we find
Present values are comparable and can be added together because they are measured in the same time period’s dollars. You’ll notice in the calculator solution that this is a three-step process—the first two steps calculate the present value of the $500 and $1,000 future amounts, and the third step adds them together.

**Calculator Clues**

**Calculating Present Value When There Is More Than One Future Cash Flow**

In the example above there are two cash flows, the first is $500 at the end of the fifth year, and the second is $1,000 at the end of the tenth year. You want to calculate their present value given a 4 percent interest or discount rate. All you have to do is bring each flow back to present, and then add them together. You can add these present values together because they both are measured in the same period’s dollars.

**STEP 1:** Calculate the present value of the $500 at the end of year 5:

Enter 

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>0</td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

Solve for $PV = 500(0.822) = 411$

**STEP 2:** Calculate the present value of the $1,000 at the end of year 10:

Enter 

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>0</td>
<td>1,000</td>
<td></td>
</tr>
</tbody>
</table>

Solve for $PV = 1,000(0.676) = 676$

**STEP 3:** Add up the two present values:

$411 + 676 = 1,087$

**Annuities**

To this point, we’ve been examining single deposits—moving them back and forth in time. Now we’re going to examine annuities. Most people deal with a great number of annuities. Pension funds, insurance obligations, and interest received from bonds all involve annuities. An annuity is a series of equal dollar payments coming at the end of each time period for a specified number of time periods (years, months, etc.). Because annuities occur frequently in finance—for example, as bond interest payments and mortgage payments—they are treated specially. Although compounding and determining the present value of an annuity can be done using equations (3.4) and (3.5), these calculations can be time-consuming, especially for larger annuities. Thus, we have modified the formulas to deal directly with annuities.
## Compound Annuities

A **compound annuity** involves depositing or investing an equal sum of money at the end of each year (or time period) for a certain number of years (or time periods, e.g., months) and allowing it to grow. Perhaps you are saving money for education, a new car, or a vacation home. In each case you’ll want to know how much your savings will have grown by some point in the future.

Actually, you can find the answer by using equation (3.4) and compounding each of the individual deposits to its future value. For example, if to provide for a college education you are going to deposit $500 at the end of each year for the next 5 years in a bank where it will earn 6 percent interest, how much will you have at the end of 5 years? Compounding each of these values using equation (3.4), you find that you will have $2,818.50 at the end of 5 years.

As Table 3.4 shows, all we’re really doing in the preceding calculation is summing up a number of future values. To simplify this process once again, there are tables providing the future-value interest factor for an annuity for $i$ and $n$ ($FVIFA_{i,n}$). Appendix C provides a fairly complete version of these tables, and Table 3.5 presents an abbreviated version. Using this new factor, we can calculate the future value of an annuity as follows:

$$\text{future value of an annuity} = \text{annual payment} \times \text{future-value interest factor of an annuity}$$

or

$$FV_n = PMT(FVIFA_{i,n}) \quad (3.6)$$

Using the future-value interest factor for an annuity ($FVIFA$) to solve our previous example involving 5 years of deposits of $500, invested at 6 percent interest, we would look in the $i = 6\%$ column and $n = 5$ row and find the value of the $FVIFA_{6\%, 5\text{ yr}}$ to be 5.637. Substituting this value into equation (3.6), we get

$$FV_5 = $500(FVIFA_{6\%, 5\text{ yr}})$$
$$FV_5 = $500(5.637)$$
$$= $2,818.50$$

This is the same answer we obtained earlier. (If it weren’t, I’d need to get a new job!)

Rather than ask how much you’ll accumulate if you deposit an equal sum in a savings account each year, a more common question is, how much must you deposit each year to accumulate a certain amount of savings? This question often

### Table 3.4 Illustration of a 5-Year $500 Annuity Compounded at 6%**

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar deposits at end of year</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>$500.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>530.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>562.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>595.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>631.00</td>
</tr>
<tr>
<td>Future value of the annuity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$2,818.50</td>
</tr>
</tbody>
</table>
arises when saving for large expenditures, such as retirement or a down payment on a home.

For example, you may know that you’ll need $10,000 for education in 8 years. How much must you put away at the end of each year at 6 percent interest to have the college money ready? In this case, you know the values of \( n \), \( i \), and \( FV \) in equation (3.6), but you don’t know the value of \( PMT \). Substituting these example values in equation (3.6), you find

\[
\frac{10,000}{9.897} = PMT
\]

Thus, you must invest $1,010.41 at the end of each year at 6 percent interest to accumulate $10,000 at the end of 8 years.

For a moment let’s use the future value of an annuity and think back to the discussion of the power of time. There’s no question of the power of time. One way to illustrate this power is to look at how much you’d have to save each month to reach some far-off goal. For example, you’d like to save up $50,000 by the time you turn 60 to use to see a Rolling Stones concert. (There’s a good chance they’ll still be on tour and that concert tickets will cost that much.)

If you can invest your money at 12 percent and start saving when you turn 21, making your last payment on your sixtieth birthday, you’ll need to put aside only $4.25 per month. If you started at age 31, that figure would be $14.31 per month.

Table 3.5 **Future Value of a Series of Equal Annual Deposits (annuity), \( FVIFA_{i, n} \)**

<table>
<thead>
<tr>
<th>( n )</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
<th>8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>2.040</td>
<td>2.050</td>
<td>2.060</td>
<td>2.070</td>
<td>2.080</td>
</tr>
<tr>
<td>3</td>
<td>3.122</td>
<td>3.152</td>
<td>3.184</td>
<td>3.215</td>
<td>3.246</td>
</tr>
<tr>
<td>5</td>
<td>5.416</td>
<td>5.526</td>
<td>5.637</td>
<td>5.751</td>
<td>5.867</td>
</tr>
<tr>
<td>7</td>
<td>7.898</td>
<td>8.142</td>
<td>8.394</td>
<td>8.654</td>
<td>8.923</td>
</tr>
<tr>
<td>9</td>
<td>10.583</td>
<td>11.027</td>
<td>11.491</td>
<td>11.978</td>
<td>12.488</td>
</tr>
</tbody>
</table>

Thus, you must invest $1,010.41 at the end of each year at 6 percent interest to accumulate $10,000 at the end of 8 years.

For a moment let’s use the future value of an annuity and think back to the discussion of the power of time. There’s no question of the power of time. One way to illustrate this power is to look at how much you’d have to save each month to reach some far-off goal. For example, you’d like to save up $50,000 by the time you turn 60 to use to see a Rolling Stones concert. (There’s a good chance they’ll still be on tour and that concert tickets will cost that much.)

If a couple goes out to dinner and a movie four times a month at $75 an outing and cuts this down to two times per month, they will save $1,800 per year. If they take this saved money and invest it at the end of each year, earning 10 percent compounded annually (ignoring taxes), in 30 years they would accumulate $296,089!
However, if you waited until age 51, it would rise to $217.35 per month. When it comes to compounding, time is on your side.

**Example**

If you deposit $2,000 in an individual retirement account (IRA) at the end of each year and it grows at a rate of 10 percent per year, how much will you have at the end of 40 years? We know that \( FV_n = PMT(FVIFA_{10\%, \ n \ years}) \). To find what your $2,000 annual deposit will have grown to after 40 years (\( FV_{40} \)), we need only multiply the annual payment (\( PMT \)), which is $2,000, times the future-value interest factor for an annuity at 10% and 40 years (\( FVIFA_{10\%, \ 40 \ yr} \)).

To find the future-value interest factor for an annuity at 10% and 40 years, go to the \( FVIFA \) table (see Appendix C) and simply move down the \( i = 10\% \) column until you reach its intersection with the \( n = 40 \) years row: 442.58. Thus, the future value after 40 years of an annual deposit of $2,000 per year is:

\[
FV_n = PMT(FVIFA_{10\%, \ 40 \ yr}) = 2,000(442.58) = 885,160
\]

You’ll have $885,160 in 40 years!

**Calculator Clues**

**Future Value of an Annuity**

In this example you’re going to deposit $2,000 at the end of each year for 40 years in an account that earns 10 percent per year, and you want to know its value at the end of the fortieth year.

Enter

\[
N \quad I/Y \quad PV \quad PMT \quad FV
\]

Solve for

\[
-885,185.11
\]

As expected, you get a negative sign on the PV. What happens if you enter a higher value for I/Y?

**Example**

Let’s take one more look at the power of compounding. Assume you empty the change out of your pocket each day—averaging a dollar a day—and set it aside. Then, at the end of each year, you invest it at 12 percent. If you began doing this at age 18, 50 years later you would have accumulated $876,007. If you waited until you were 33 to begin your pocket-emptying ritual, you’d accumulate only $157,557. Keep in mind that between the time you were 18 and when you turned 33 you invested only a total of $5,475. Remember Principle 15: Just Do It! In the world of investing, time is your best friend.

**Calculator Clues**

**Future Value of an Annuity**

At the end of each year for 50 years you deposit $365 in an account that earns 12 percent.

Enter

\[
N \quad I/Y \quad PV \quad PMT \quad FV
\]

Solve for

\[
-876,006.66
\]
Present-Value Interest Factor for an Annuity

A multiplier used to determine the present value of an annuity. The present-value interest factors are found in Appendix D.

Table 3.6 Illustration of a 5-Year $500 Annuity Discounted Back to the Present at 6%

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollars received at end of year</td>
<td></td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Present value of the annuity</td>
<td>$2,106.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As expected, you get a negative sign on the PV. What happens if you waited until you were 33 instead of 18 to begin? Then you’d only be investing for 35 years, so you change N to 35 and solve it again:

Enter

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>12</td>
<td>0</td>
<td>365</td>
<td></td>
</tr>
</tbody>
</table>

Solve for PV

-157,557.18

Present Value of an Annuity

In planning your finances, you need to examine the relative value of all your annuities. To compare them, you need to know the present value of each. Although you can find the present value of an annuity by using the present value table in Appendix B, this process can be tedious, particularly when the annuity lasts for several years. If you wish to know what $500 received at the end of the next 5 years is worth to you given the appropriate discount rate of 6 percent, you can separately bring each of the $500 flows back to the present at 6 percent using equation (3.6) and then add them together. Thus, the present value of this annuity is $2,106.00. As Table 3.6 shows, all we’re really doing in this calculation is adding up present values. Because annuities occur so frequently in personal finance, the process of determining the present value of an annuity has been simplified by defining the present-value interest factor for an annuity for i and n (PVIFA\(_{i, n}\)). The PVIFA\(_{i, n}\) is simply the sum of the PVIFs for years 1 to n. Tables for values of PVIFA\(_{i, n}\) have once again been compiled for various combinations of i and n. Appendix D provides a fairly complete version of these tables, and Table 3.7 provides an abbreviated version.

Using this new factor, we can determine the present value of an annuity as follows:

\[
\text{present value of an annuity} = \text{annual payment} \times \text{present-value interest factor of an annuity}
\]

or

\[
PV = PMT(PVIFA_{i, n})
\] (3.7)

Using the PVIFA to solve our previous example involving $500 received annually and discounted back to the present at 6 percent, we would look in the i = 6% column
Table 3.7  
**Present Value of a Series of Annual Deposits (annuity), \( PVIFA_{i,n} \)**

Instructions: Each present-value interest factor for an annuity \( (PVIFA_{i%, n \text{ years}}) \) corresponds to a specific time period (number of years) and interest rate. For example, to find the present-value interest factor of an annuity for 6 percent and 5 years \( (PVIFA_{6\%, 5 \text{ yr}}) \), simply move down the \( i = 6\% \) column until you reach its intersection with the \( n = 5 \) years row: 4.212. The future value is then calculated as follows:

\[
pv = \text{annual payment} \times \text{present-value interest factor for an annuity}
\]

\[
\text{or } PV = PMT(PVIFA_{i\%, n \text{ years}})
\]

<table>
<thead>
<tr>
<th>( n )</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
<th>8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.962</td>
<td>0.952</td>
<td>0.943</td>
<td>0.935</td>
<td>0.926</td>
</tr>
<tr>
<td>2</td>
<td>1.886</td>
<td>1.859</td>
<td>1.833</td>
<td>1.808</td>
<td>1.783</td>
</tr>
<tr>
<td>3</td>
<td>2.775</td>
<td>2.723</td>
<td>2.673</td>
<td>2.624</td>
<td>2.577</td>
</tr>
<tr>
<td>4</td>
<td>3.630</td>
<td>3.546</td>
<td>3.465</td>
<td>3.387</td>
<td>3.312</td>
</tr>
<tr>
<td>5</td>
<td>4.452</td>
<td>4.329</td>
<td>4.212</td>
<td>4.100</td>
<td>3.993</td>
</tr>
<tr>
<td>6</td>
<td>5.242</td>
<td>5.076</td>
<td>4.917</td>
<td>4.767</td>
<td>4.623</td>
</tr>
<tr>
<td>7</td>
<td>6.002</td>
<td>5.786</td>
<td>5.582</td>
<td>5.389</td>
<td>5.206</td>
</tr>
<tr>
<td>8</td>
<td>6.733</td>
<td>6.463</td>
<td>6.210</td>
<td>5.971</td>
<td>5.747</td>
</tr>
<tr>
<td>10</td>
<td>8.111</td>
<td>7.722</td>
<td>7.360</td>
<td>7.024</td>
<td>6.710</td>
</tr>
<tr>
<td>11</td>
<td>8.760</td>
<td>8.306</td>
<td>7.887</td>
<td>7.499</td>
<td>7.139</td>
</tr>
</tbody>
</table>

and the \( n = 5 \) row and find the \( PVIFA_{6\%, 5 \text{ yr}} \) to be 4.212. Substituting the appropriate values into equation (3.7), we find

\[
PV = PMT(PVIFA_{6\%, 5 \text{ yr}})
\]

\[
PV = $500(4.212)
\]

\[
PV = $2,106
\]

Again, we get the same answer we previously did. (We’re on a roll now!) We didn’t get the same answer just because we’re smart. Actually, we got the same answer both times because the \( PVIFA \) tables are calculated by adding up the values in the \( PVIF \) table.

**Example**

As part of a class action law suit settlement against Lee’s “Press On Abs” (they caused a nasty rash), you are slated to receive $1,000 at the end of each year for the next 10 years. What is the present value of this 10-year, $1,000 annuity discounted back to the present at 5%? Substituting \( n = 10 \) years, \( i = 5\% \), and \( PMT = $1,000 \) into equation (3.7), you find

\[
PV = 1,000(PVIFA_{5\%, 10 \text{ yr}})
\]

Determining the value for the \( PVIFA_{5\%, 10 \text{ yr}} \) from Table 3.7, row \( n = 10 \), column \( i = 5\% \), and substituting it into our equation, we get

\[
PV = 1,000(7.722)
\]

\[
PV = 7,722
\]

Thus, the present value of this annuity is $7,722.
Chapter 3 • Understanding the Time Value of Money

Calculator Clues

Present Value of an Annuity

In this example, you’re solving for the present value of a 10-year, $1,000 annuity discounted back to the present at 5%.

Enter

\[
\begin{array}{cccc}
N & 10 \\
I/Y & 5 \\
PV & \text{1,000} \\
PMT & 0 \\
FV & \text{7,721.73}
\end{array}
\]

Solve for

As expected, you get a negative sign on the PV. What happens if you enter a higher value for I/Y?

As with the other problems involving compounding and present-value tables, given any three of the four unknowns in equation (3.7), we can solve for the fourth. In the case of the \(PVIFA\) table, we may be interested in solving for \(PMT\), if we know \(i\), \(n\), and \(PV\). The financial interpretation of this action would be: How much can be
withdrawn, perhaps as a pension or to make loan payments, from an account that earns $i$ percent compounded annually for each of the next $n$ years if you wish to have nothing left at the end of $n$ years?

**Amortized Loans**

You’re not always on the receiving end of an annuity. More often, your annuity will involve paying off a loan in equal installments over time. Loans that are paid off this way, in equal periodic payments, are called amortized loans. Examples of amortized loans include car loans and mortgages.

Suppose you borrowed $6,000 at 15-percent interest to buy a car and wish to repay it in four equal payments at the end of each of the next 4 years. We can use equation (3.7) to determine what the annual payments will be and solve for the value of $PMT$, the annual annuity. Again, you know three of the four values in that equation, $PV$, $i$, and $n$. $PV$, the present value of the future annuity, is $6,000; i$, the annual interest rate, is 15 percent; and $n$, the number of years for which the annuity will last, is 4 years. $PMT$, the annuity payment received (by the lender and paid by you) at the end of each year, is unknown. Substituting these values into equation (3.7) you find

$$PV = PMT(PVIFA_{i\%, \, n \, yr})$$

$$6,000 = PMT(PVIFA_{15\%, \, 4 \, yr})$$

$$6,000 = PMT(2.855)$$

$$6,000/2.855 = PMT(2.855)/2.855$$

$$2,101.58 = PMT$$

To repay the principal and interest on the outstanding loan in 4 years, the annual payments would be $2,101.58. The breakdown of interest and principal payments is given in the loan amortization schedule in Figure 3.5. As you can see, the interest payment declines each year as the loan outstanding declines.

**Figure 3.5** Loan Amortization Schedule Involving a $6,000 Loan at 15% to Be Repaid in 4 years

```
Year | Principal | Interest
---------|-----------|--------
1     |           |        |
2     |           |        |
3     |           |        |
4     |           |        |

Total Principal Paid = $6,000.00
Total Interest Paid = $2,436.32
```
**Calculator Clues**  
**Calculating a Loan Payment**

Calculating loan payments is easy with a financial calculator. In the example above you want to determine the loan payments on a $6,000 loan at 15 percent that you want to pay off with four equal payments at the end of each of the next 4 years. All you need to do is plug your numbers into the calculator and solve for PMT. PV is $6,000, because that’s how much you’ve borrowed today, and FV is 0 because you will have the loan paid off after 4 years.

Enter  
\[
\begin{array}{cccc}
N & I/Y & PV & PMT & FV \\
4 & 15 & 6,000 & 0 & 0 \\
\end{array}
\]

Solve for PMT

\[
\text{PMT} = -2,101.59
\]

As expected, PMT takes on a negative sign.

**Solving for I/Y and N Using the Tables or a Financial Calculator**

As you might expect, you can solve for I/Y and N using either the tables or a financial calculator. With the tables, you first find what table value you’re looking for, then you see what column (if you’re solving for I/Y) or row (if you’re solving for N) it is in. With a financial calculator, all you do is enter the known variables and solve it. Take a look at the examples below.

Let’s assume that the DaimlerChrysler Corporation has guaranteed that the price of a new Jeep will always be $20,000. You’d like to buy one, but currently you have only $7,752. How many years will it take for your initial investment of $7,752 to grow to $20,000 if it is invested at 9 percent compounded annually? We can use equation (3.4a) to solve for this problem as well. Substituting the known values in equation (3.4a), you find

\[
FV_n = PV(FVIF_{i, n}) \\
$20,000 = \frac{7,752(FVIF_{9\%, n \text{ yr}})}{7,752} \\
2.58 = FVIF_{9\%, n \text{ yr}}
\]

Thus, you’re looking for a value of 2.58 in the \( FVIF_{i, n} \) tables, and you know it must be in the 9% column. To finish solving the problem, look down the 9-percent column for the value closest to 2.58. You’ll find that it occurs in the \( n = 11 \) row. Thus, it will take 11 years for an initial investment of $7,752 to grow to $20,000 if it is invested at 9 percent compounded annually.

**Calculator Clues**  
**Solving for N—the Number of Payments**

Solving for the number of payments using a financial calculator is simple. To solve for N, enter the known variables and solve. In this example, how many years would it take for $7,752 to grow to $20,000 at 9%?

Enter  
\[
\begin{array}{cccc}
N & I/Y & PV & PMT & FV \\
9 & -7,752 & 0 & 20,000 & 0 \\
\end{array}
\]

Solve for N

\[
N = 10.998
\]
The answer is 10.998 or about 11 years. You’ll notice we gave the present value, $7,752, a negative sign and the future value, $20,000, a positive sign. Why? Because a calculator looks at cash flows like it’s a bank. You deposit your money in the bank (and the sign is negative because the money “leaves your hands”), and later you take your money out of the bank (the sign is positive because the money “returns to your hands”). As a result, every problem will have a positive and negative sign on the cash flows.

Now let’s solve for the compound annual growth rate. In 10 years you’d really like to have $20,000 to buy a new Jeep, but you have only $11,167. At what rate must your $11,167 be compounded annually for it to grow to $20,000 in 10 years? Substituting the known variables into equation (3.4a), you get

\[ \frac{FV_n}{PV} = \frac{PVIF_{i,n}}{PVIF_{i,10\text{yr}}} \]

You know to look in the \( n = 10 \) row of the \( PVIF_{i,n} \) tables for a value of 1.791, and you find this in the \( i = 6\% \) column. Thus, if you want your initial investment of $11,167 to grow to $20,000 in 10 years, you must invest it at 6 percent.

**Calculator Clues**

**Solving for I/Y— the Rate of Return**

Finding a rate of return using a financial calculator is simple. To solve for I/Y, enter the known variables and solve. For example, what rate did an initial investment of $5,000 that grew to $40,000 in 20 year grow at?

Enter 10 11,167 0 20,000

Solve for

\[ 6.0009 \]

The answer is 6.0009—about 6 percent. Just as when you solved for N, you gave the present value, $11,167, a negative sign and the annual payment, $20,000, a positive sign.

**Perpetuities**

A **perpetuity** is an annuity that continues forever. That is, every year from its establishment, this investment pays the same dollar amount and never stops paying. Determining the present value of a perpetuity is delightfully simple: You divide the payment amount by the discount rate. For example, the present value of a perpetuity that pays a constant dividend of $10 per share forever if the appropriate discount rate is 5 percent is $10/0.05 = $200. Thus, the equation representing the present value of a perpetuity is

\[ PV = \frac{PP}{i} \]

where

\[ PV = \text{the present value of the perpetuity} \]

\[ PP = \text{the annual dollar amount provided by the perpetuity} \]

\[ i = \text{the annual interest (or discount) rate} \]
SUMMARY

Almost every decision in personal finance involves the techniques of compounding and time value of money—putting aside money now to achieve some future goal. The cornerstone of the time value of money is the concept of compound interest, which is interest paid on interest.

With the time value of money, you can determine how much an investment will grow over time using the following formula:

\[ FV_n = PV(1 + i)^n \]  \hspace{1cm} (3.4)

To simplify these calculations, there are tables for the \((1 + i)^n\) part of the equation (the future-value interest factor for \(i\) and \(n\), or \(FVIF_{i,n}\)). In effect, you can rewrite equation (3.4) as follows:

\[ \text{future value} = \text{present value} \times \text{future-value interest factor} \]

\[ FV_n = PV(FVIF_{i,n}) \]  \hspace{1cm} (3.4a)

It is also important to understand the role of the interest rate in determining how large an investment grows. Together, time and the interest rate determine how much you will need to save in order to achieve your goals. You can also use the Rule of 72 to determine how long it will take to double your invested money. This “rule” is only an approximation:

\[ \text{number of years to double} = \frac{72}{\text{annual compound growth rate}} \]

Many times we also want to solve for present value instead of future value. We use the following formula to do this:

\[ PV = FV_n \left[ \frac{1}{1 + i} \right] \]  \hspace{1cm} (3.5)

\[ PV = FV_n(PVIF_{i,n}) \]  \hspace{1cm} (3.5a)

An annuity is a series of equal annual dollar payments coming at the end of each year for a specified number of years. Because annuities occur frequently in finance—for example, as bond interest payments and mortgage payments—they are treated specially. A compound annuity involves depositing or investing an equal sum of money at the end of each year for a certain number of years and allowing it to grow.

\[ \text{future value of an annuity} = \text{annual payment} \times \text{future-value interest factor of an annuity} \]

or

\[ FV_n = PMT(FVIFA_{i,n}) \]  \hspace{1cm} (3.6)

To find the present value of an annuity, we use the following formula:

\[ \text{present value} = \text{annual payment} \times \text{present-value interest factor of an annuity} \]

or

\[ PV = PMT(PVIFA_{i,n}) \]  \hspace{1cm} (3.7)
Many times annuities involve paying off a loan in equal installments over time. Loans that are paid off this way, in equal periodic payments, are called amortized loans. Examples of amortized loans include car loans and mortgages.

**REVIEW QUESTIONS**

1. What is compound interest? How is compound interest related to the time value of money?
2. What is “future value” and why is it important to calculate?
3. Describe how the Rule of 72 can be used to make financial planning decisions.
4. What four variables are necessary to solve a time value of money problem with a compound interest table or financial calculator? Which of these variables equals zero when solving a simple annuity problem? Why?
5. Explain the concept of the time value of money. Explain two ways this concept is relevant in financial planning.
6. What two factors most affect how much people need to save to achieve their financial goals?
7. Why do you think that Albert Einstein once called compound interest the “eighth wonder of the world”?
8. Why do investors require a greater expected return for an investment of longer maturity although the required return to reach a goal, with an equal initial investment, decreases with the longer time horizon?
9. What is the most commonly used discount rate when calculating present value?
10. Why is the interest rate in a time value of money calculation sometimes referred to as the discount rate? Why is it also called “inverse compounding”?
11. Explain in terms of the future-value interest factor why, given a certain goal, that as the period of time to invest increases the required periodic investment decreases.
12. What is the primary difference between an annuity and a compound annuity?
13. What is the relationship between present-value and future-value interest factors and present and future interest factors for annuities?
14. Define an amortized loan and give two common examples.
15. Why is it necessary to use a negative present value when solving for N (the number of payments) or I/Y (the rate of return)? Similarly, why does the answer have a negative sign if positive payments were used when solving for a future value on a calculator?
16. What is a perpetuity? Name an example of a perpetuity (payments or receipt of income) in personal finance.

**PROBLEMS AND ACTIVITIES**

1. Your mother just won $250,000 for splitting a Nobel Prize with three coworkers. If she invests her prize money in a diversified equity portfolio returning 8 percent per year, approximately how long will it take her to become a millionaire, before accounting for taxes?
2. Linda Baer has saved $5,000 for a previously owned vehicle. Ignoring taxes and assuming her money is invested in a flexible withdrawal CD earning 5% compounded annually, how long will it take to buy a car that costs $7,500? (Hint: The answer is between 6 and 10 years.)
3. Paul Ramos just graduated from college and landed his first “real” job, which pays $23,000 a year. In 10 years, what will he need to earn to maintain the same purchasing power if inflation averages 3.5 percent?
4. Anthony and Michelle Mitchell just got married and received $30,000 in cash gifts for their wedding. If they place half of this money in a fixed rate investment earning 12 percent compounded annually, how much will they have on their twenty-fifth anniversary? Would the future value be larger or smaller if the compounding period was 6 months? How much more or less would they have earned with this shorter compounding period?
5. Calculate the future value of $5,000 earning 10 percent assuming an annual compounding period.
Calculate the future value of $5,000 earning 10 percent assuming simple interest (the interest earned does not earn future interest). How much interest did the interest earn?

6. Ahmed Mustafa just turned 22 and wants to have $10,000 saved by his thirtieth birthday. Assuming no additional deposits, if he currently has $6,000 in an intermediate-term bond fund earning a 5 percent yield, will he reach his goal? If not, what rate of return is required to meet his goal?

7. If another Austin Powers movie is released in 2007, and Dr. Evil, now armed with a financial calculator, wanted to hold the Earth ransom for $7,039,988.71; what inflation rate would Dr. Evil use to make his ransom equivalent to $1 million in 1967? (Hint: Inflation is compounded on an annual basis.)

8. When a small child, Derek’s grandfather established a trust fund for him to receive $20,000 on his thirty-fifth birthday. (Any earnings beyond $20,000 reverted to his grandfather.) Derek just turned 23. What is the value of his trust today if the trust fund earns 7 percent interest? What is the present value if he had to wait until age 40 to receive the money?

9. You and 11 coworkers just won $12 million from the state lottery. Assuming you each receive your share over 20 years and that the state lottery earns a 5.5 percent return on its funds, what is the present value of your prize before taxes if you request the “up-front cash” option?

10. Richard Gorman is 65 years old and about to retire. He has $500,000 saved to supplement his pension and Social Security and would like to withdraw it in equal annual dollar amounts so that nothing is left after 15 years. How much does he have to withdraw each year if he earns 7 percent on his money?

11. Assume you are 25 and earn $35,000 per year, never expect to receive a raise, and plan to retire at age 55. If you invest 5 percent of your salary in a 401(k) plan returning 10 percent annually and the company provides a $0.50 per $1.00 match on your contributions up to 3 percent of salary, what is your estimated future value? How much can you withdraw monthly if you want to deplete your account over 30 years?

12. Joe Eiss, 22, just started working full-time and plans to deposit $3,000 annually into an IRA earning 12 percent interest compounded annually. How much would he have in 20 years? 30 years? 40 years? If he changed his investment period and instead invested $250 monthly and the investment also changed to monthly compounding, how much would he have after the same three time periods? Comment on the differences over time.

SUGGESTED PROJECTS

1. Ask older friends or relatives about the cost of specific items (e.g., a gallon of gas, cup of coffee, etc.) during their youth. Also inquire about average wages in the past. Compare the amounts given to current expenses and income levels. Explain your findings using time value of money concepts.

2. Calculate what an individual retirement arrangement (IRA) would be worth if you begin contributing $3,000 annually following graduation. Define your own retirement age and make any other assumptions needed. Explain the key factors that will influence the amount of money saved at retirement.

3. Develop and solve a future value, a present value, a future value of an annuity, and a present value of an annuity problem. Establish the three known variables in each problem and solve for the fourth. Do not always solve for the same variable. Explain the results.

4. Using a financial calculator, determine the future value of $5,000 invested at 12 percent for 40 years with annual compounding. What rate of interest would result in the same future value if the compounding period were changed to monthly? (Hint: Change the compounding period and solve the equation in reverse for the interest rate.)

5. Research the cost of five products or services that were advertised in newspapers or magazines published more than 20 years ago. Then find the current cost of these items. Calculate the rate of inflation for each item between the two points in time.

6. Visit a local financial institution and record the interest rates and minimum balance requirements for the various financial products offered. Using the information gathered, determine how much interest would be earned if you deposited $2,500 in each account. For each, calculate your total return subtracting any applicable service charges.
Part 1 • Financial Planning

Jenny Smith, 27, just received a promotion at work that increased her annual salary to $37,000. She is eligible to participate in her employer’s 401(k) plan to which the employer matches dollar-for-dollar workers’ contributions up to 5 percent of salary. However, Jenny wants to buy a new $25,000 car in 3 years and she wants to save enough money to make a $7,000 down payment and finance the balance.

Also in her plans is a wedding. Jenny and her boyfriend, Paul, have set a wedding date 2 years in the future, after he finishes medical school. Paul will have $100,000 of student loans to repay after graduation. But, both Jenny and Paul want to buy a home of their own as soon as possible. This might be possible because at age 30, Jenny will be eligible to access a $50,000 trust fund left as an inheritance by her late grandfather. Her trust fund is invested in 7% government bonds.

Questions

1. Justify Jenny’s participation in her employer’s 401(k) plan using time value of money concepts.

2. Calculate the amount that Jenny needs to save each year for the down payment on a new car, assuming she can earn 6 percent on her savings. Calculate how much she will need to save on a monthly basis assuming monthly compounding. For each scenario, how much of her down payment will come from interest earned?

3. What will be the value of Jenny’s trust fund at age 60, assuming she takes possession of half of the money at age 30 for a house down payment, and leaves the other half of the money untouched where it is currently invested?

4. What is Paul’s annual payment if he wants to completely repay his student loans within 10 years and pays a 4.75 percent interest rate? How much more or less would Paul pay if the loans compounded interest on a monthly basis and Paul also paid the loans on a monthly basis?

5. List at least three actions that Jenny and Paul could take to make the time value of money work in their favor.

Discussion Case 2

Doug Klock, 56, just retired after 31 years of teaching. He is a husband and father of three, two of whom are still dependent. He received a $150,000 lump-sum retirement bonus and will receive $2,800 per month from his retirement annuity. He has saved $150,000 in a 403(b) retirement plan and another $100,000 in other accounts. His 403(b) plan is invested in mutual funds, but most of his other investments are in bank accounts earning 2 or 3 percent annually. Doug has asked your advice in deciding where to invest his lump-sum settlement and other accounts now that he has retired. He also wants to know how much he can withdraw per month considering he has two children in college and a nonworking spouse. Because Rachel and Ronda are still in college, his current monthly expenses total $5,800. He is not eligible for Social Security until age 62, when he will draw approximately $1,200 per month; however, he would rather defer until age 67 to increase his monthly amount to $1,550. He has grown accustomed to some risk but wants most of his money in FDIC-insured accounts.

Questions

1. Assuming Doug has another account set aside for emergencies; how much can he withdraw on a monthly basis to supplement his retirement annuity, if his investments return 5 percent annually and he expects to live 30 more years?
2. Ignoring his Social Security benefit, is the amount determined in question 1 sufficient to meet his current monthly expenses? If not, how long will his retirement last if his current expenses remain the same? If his expenses are reduced to $4,500 per month?

3. Considering the information obtained in question 2, should he wait until age 67 for his Social Security benefits? If he waits until age 67, how will his Social Security benefit change the answers to question 2? (Hint: Calculate his portfolio value as of age 67 and then recalculate the future value formula reflecting the increased current income.)

4. If the inflation rate averages 3.5 percent during Doug’s retirement, how old will he be when prices have doubled from current levels? How much will a soda cost when Doug dies, if he lives the full 30 years and the soda costs $1 today?