R-Values

In heat conduction, the rate of heat flow depends on the temperature difference between sides, the thickness, and the area in contact. The greater the temperature difference, the greater the heat flow. The greater the area in contact, the greater the heat flow. The shorter the distance of conduction (the thickness), the greater the heat flow.

The connection between heat flow and these quantities is called the *thermal conductivity* or the *thermal resistance*, depending on how the relationship is written. We may write

$$\text{heat flow rate} = k\,A\,\Delta T/t,$$

or alternatively

$$R(\text{heat flow rate}) = A\,\Delta T,$$

where k is the thermal conductivity, R is thermal resistance, A the area in contact, t the thickness, and ΔT the temperature difference. These equations describe the heat conduction rate through a plate of material of face area A and thickness t, when the two sides of the plate differ in temperature by ΔT.

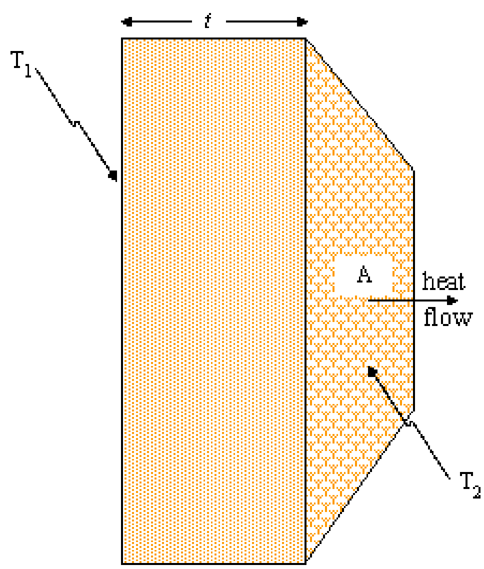


Fig. E09.4.1 A solid wall of thickness t and area A experiences heat conduction through it when there is a temperature difference $\Delta T = T_1 - T_2$.

For fixed values of *R*, *A*, and *t*, the heat flow increases if ΔT increases. Therefore, the warmer your house is kept relative to the outside temperature, the more heat you will lose by conduction. Since making the walls of a house thicker is not usually feasible, and since the area in contact with the outside is fixed, we can see that the heat flow can be reduced (according to the equation) either by increasing *R* for a fixed ΔT, decreasing ΔT for fixed *R*, or some combination of the two. You can increase *R* by adding insulation to ceiling, walls, and floor. You can decrease ΔT for at least part of the day by turning down the furnace when no one is home or at night when everyone is sleeping.

TABLE E09.4.1

R-Values of Common Construction Materials per Inch of Material

Material	*R*-value per inch of material
Air	1.44
Rock wool (batt)	3.38
Fiberglass (batt)	3.16
Rock wool (blown)	2.75
Fiberglass (blown)	2.20
Cellulose (blown)	3.67
Vermiculite	2.20
Perlite	2.75
Wood (av. pine)	1.28
Wallboard	1.0
Brick	0.11
Glass	7.2

Source: Reprinted with permission from *Handbook of Chemistry and Physics*, 32nd ed., Copyright 1950, CRC Press, Inc. BOAC Raton, FL, and Department of Energy, *Insulation* (Washington, D.C.: Government Printing Office, 1980).

The units used to measure *R*-values, as presented in Table E09.4.1, are common English units, ft^2 °F/(Btu/h). In the metric system, the unit would be m^2 °C/W.

Dry air does not conduct very well. Most insulation works by trapping air so that it cannot move. Dry, relatively still air an inch thick has an *R*-value of about 1. Completely motionless air an inch thick has an *R*-value of 3 to 4. Some selected *R*-values are listed in Table E09.4.1

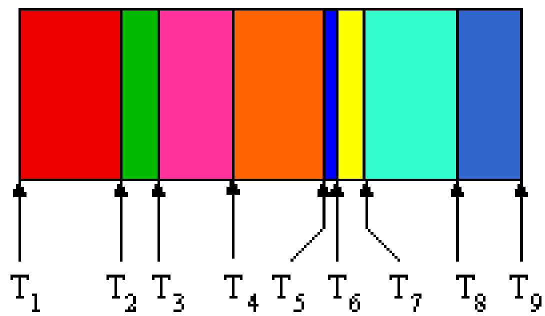


Fig. E09.4.2 A wall made up of layers of varying thickness but the same area.

The *R*-value is in common use, rather than the conductivity *k*, because *R*-values can be added. This is easily seen from our defining equation, *R*(heat flow rate) = $A\ \Delta T$. For a given fixed heat flow rate and fixed area *A*, we have, for each layer (Fig. E09.4.2), the equation as given. If all the individual ΔTs are added up, we get expressions like

$$(T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) + \ldots .$$

Clearly the sum of all these is just the overall ΔT. If we add the ΔTs on the right-hand side, we must also add the terms *R* (heat flow rate) on the left-hand side to get

$$(R_1 + R_2 + R_3 + \ldots)(\text{heat flow rate}) = A\Delta T;$$

this shows that the *R*-values can be added.

To illustrate how this property of additivity might be used, consider a typical house wall. From the inside out, there is still air along the wall (R-0.68); wallboard (R-0.45); air space

(R-1.01); sheathing over the studs (R-1.32); wood siding (R- 0.81); and the outside air that moves, thus contributing very little.[151] Ignoring the contribution of outside air, which depends on the weather conditions, the *R*-value for the house wall is

$$R\text{-value, total} = 0.68 + 0.45 + 1.01 + 1.32 + 0.81 = 4.27.$$

Clearly, adding R-11 or R-19 insulation will decrease heat loss substantially for this house.

TABLE E09.4.2

Average Actual Savings of Retrofits in Different Climatic Regions

Measure	Percent savings	Payback period (yr)
Wall insulation		
Study 1	12	12
Study 2	20	10
Study 3	17	6
Ceiling insulation		
R-0 → *R*-19	13	6
R-0 → *R*-19	21	4
R-11 → *R*-30	13	6
? → *R*-30	13	4
Interior foundation insulation		
R-0 → *R*-11	15	11
R-0 → *R*-14	6	61
R-0 → *R*-10	10	19
R-0 → *R*-10	3	127

Source: Department of Energy, Ref. 57, Table D-1.

Table E09.4.2 gives a representative picture of the poor state of insulation in typical U.S. houses. Buildings built in the energy-cheap 1950s and 1960s use the most energy of the entire stock of buildings.[152] In the late 1970s, buildings were still being built that would generate an energy bill over their 50-year lives that would be double or triple the original construction cost.[57] Large savings from insulation remain possible today. A 1990 study on conservation potential identified technological improvements in U.S. residential buildings that would lead, even in the no-change reference case, to a 10% improvement overall; in the best case, a 65% reduction in space heat could be achieved in new construction.[57] In Germany, insulation can reduce energy use 30% to 40%.[65]

A recording method called PRISM has been developed at the Center for Energy and Environmental Studies at Princeton University. PRISM could be used to gather valid data and evaluate how well or poorly various conservation methods work.[153] Several experiments showed that it was possible to save as much as 10% of home energy use by a single visit from a “house doctor” [154,155] and that major rebuilding efforts could save as much as 20%.[154,156]

A program backed by the Bonneville Power Administration at Hood River, Oregon, demonstrated that retrofitting of houses with insulation and other conventional measures of energy saving was cost-effective,[157] as other studies had shown.[158,159] The study found that, as compared to an audit model, actual savings due to storm windows and heating ducts were greater than the predictions of the model. The actual savings due to storm doors, caulking, and weather stripping were much smaller than predicted. Overall, predicted savings were 6200 kWh/yr, while actual mean savings were 4130 kWh/yr, for an average $2100 savings.[131,141,157] Part of the reason for these results may have been that many homes that had used wood as a heating supplement (wood was not accounted for in prechange surveys) switched back to conventional heating.

The map of Fig. E09.4.3 shows the recommended insulation for each area of the country. It is divided into six climatic zones.

					Ceiling						Basement	
Zone	Gas	Heat pump	Fuel oil	Electric furnace	Attic	Cathedral	Wall (A)	Floor	Crawl space (B)	Slab edge	Interior	Exterior
1	✔	✔	✔		R-49	R-38	R-18	R-25	R-19	R-8	R-11	R-10
1				✔	R-49	R-60	R-28	R-25	R-19	R-8	R-19	R-15
2	✔	✔	✔		R-49	R-38	R-18	R-25	R-19	R-8	R-11	R-10
2				✔	R-49	R-38	R-22	R-25	R-19	R-8	R-19	R-15
3	✔	✔	✔	✔	R-49	R-38	R-18	R-25	R-19	R-8	R-11	R-10
4	✔	✔	✔		R-38	R-38	R-13	R-13	R-19	R-4	R-11	R-4
4				✔	R-49	R-38	R-18	R-25	R-19	R-8	R-11	R-10
5	✔				R-38	R-30	R-13	R-11	R-13	R-4	R-11	R-4
5		✔	✔		R-38	R-38	R-13	R-13	R-19	R-4	R-11	R-4
5				✔	R-49	R-38	R-18	R-25	R-19	R-8	R-11	R-10
6	✔				R-22	R-22	R-11	R-11	R-11	(C)	R-11	R-4
6		✔	✔		R-38	R-30	R-13	R-11	R-13	R-4	R-11	R-4
6				✔	R-49	R-38	R-18	R-25	R-19	R-8	R-11	R-10

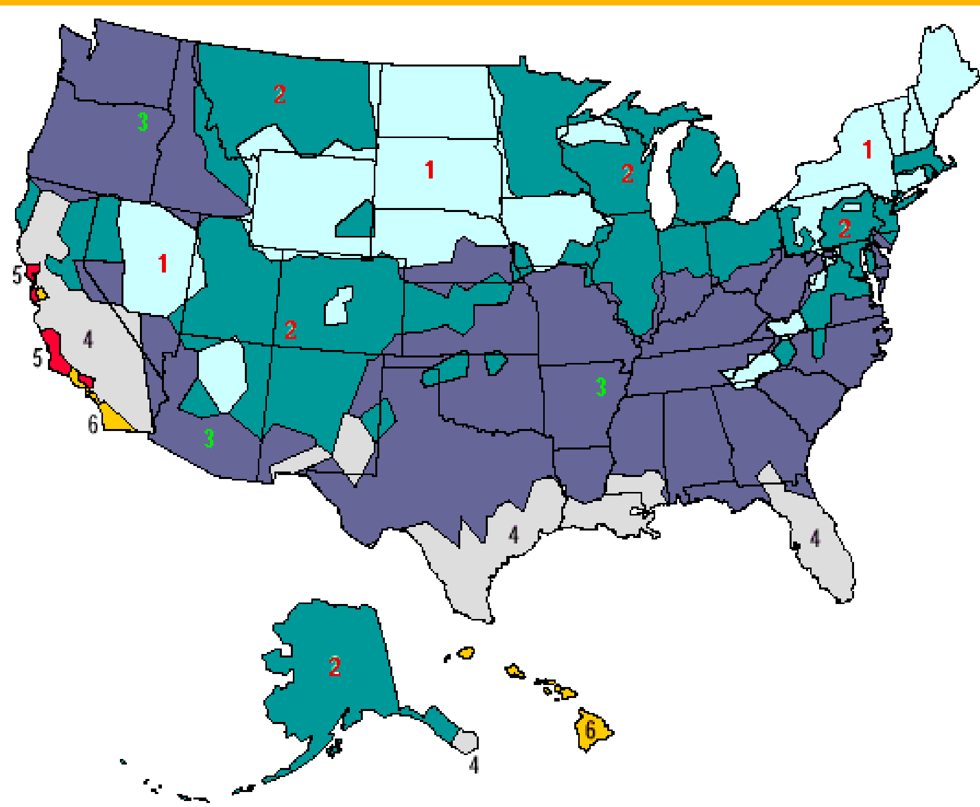


Fig. E09.4.3 Recommended insulation values.
(U.S. Department of Energy, NREL, Ref. 81)

Typical North American houses built in 1980 have an annual energy use of 122 kJ per m^2 per degree-day (°C). The energy use in the superinsulated Minnesota homes averaged 51 kJ per m^2 per degree-day (°C).[(61)] In addition, some economies with superinsulated homes are not readily apparent. For example, the heating system of such a home is much smaller than that of an ordinary home built in a similar climate.

In Uppsala, Sweden, a 130 m^2 superinsulated house uses only 110 MJ/m^2/yr as compared to the usual 800 MJ/m^2/yr.[(160)] Sweden is now marketing superinsulated homes in the United States. They are prefabricated and shipped in crates. A Swedish-built house costs about \$30 to \$35/ft^2, which is comparable to the late 1980s cost of new American housing.[(161)] The additional costs of superinsulation are probably not justified except in areas of sustained winter cold.

Convection and heat transfer

Of course, insulation does not necessarily prevent or even minimize heat flow to the outside. Air infiltrates a house through cracks and when a door is opened for entry or egress (convective losses). As a result, there is some approximately steady rate at which heat must be supplied to a house, in addition to that which would be necessary to maintain a temperature difference between inside and outside because of conduction through exterior surfaces. The added heat is used to warm up the outside air to interior room temperature. The amount of energy transfer needed depends to some extent on weather conditions outside.[(161)] In times of high wind, transfer of air will be greater than at other times.

TABLE E09.4.3

Conduction-Convection Parameter

Orientation	h [W/(m^2 °C)]
Upward-facing horizontal surface	$2.5\ (\Delta T)^{1/4}$
Downward-facing horizontal surface	$1.3\ (\Delta T)^{1/4}$
Vertical surface	$1.8\ (\Delta T)^{1/4}$

Source: Based on Ref. 162, Table 5-2

As mentioned in Chapter 8, if we look at the temperature profile of a window exposed to cold outdoors with a heated inside, there is a boundary layer of air of intermediate temperature on both sides of the window. If the air outside is in motion, some of the heated outside air is moved away rapidly, increasing the rate of heat flow (or, alternatively, effectively decreasing the *R*-value). The same principle applies to a house. When a cold wind blows, the boundary layer is much smaller, and the effective *R*-value decreases. The rate of heat transfer for reasonable-size temperature differences given the conduction-convection parameter of Table E09.4.3, is described by

$$\text{heat flow rate} = h\,A\,\Delta T.$$

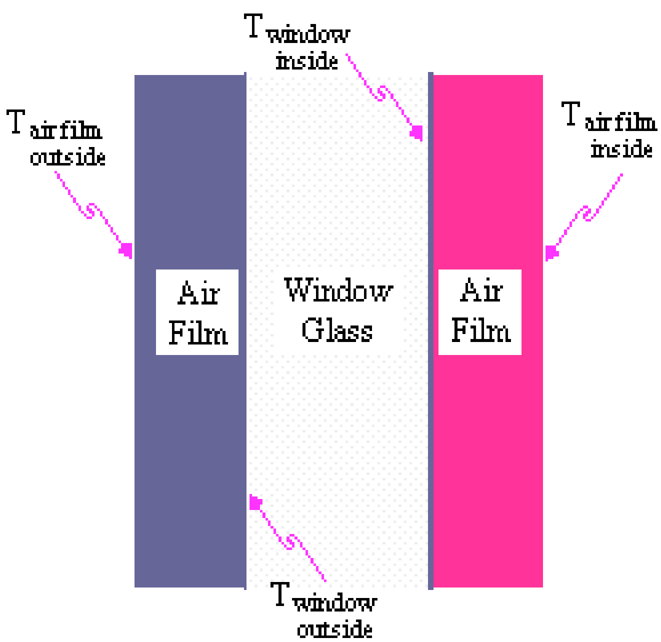


Fig. E09.4.4 A window with convective layers both inside and outside.

To understand what happens at a window (Fig. E09.4.4), we must take into account both the conduction through the window glass and the convection at each surface of the window.

For specificity, suppose the outside air is at 5 °C and the inside air is at 20 °C and assume the air is quiescent both inside and outside, and the glass is 3.0 mm thick. We will also suppose the glass is 1 m by 1 m. We may then find the total temperature difference ΔT by adding the pieces:

$$\Delta T = \left(T_{\substack{\text{air film}\\\text{inside}}} - T_{\substack{\text{window}\\\text{inside}}}\right) + \left(T_{\substack{\text{window}\\\text{inside}}} - T_{\substack{\text{window}\\\text{outside}}}\right) + \left(T_{\substack{\text{window}\\\text{outside}}} - T_{\substack{\text{air film}\\\text{outside}}}\right)$$
$$= T_{\substack{\text{air film}\\\text{inside}}} - T_{\substack{\text{air film}\\\text{outside}}} = 20\text{ °C} - 5\text{ °C} = 15\text{ °C}.$$

Looking at each of the pieces,

$$T_{\substack{\text{air film}\\\text{inside}}} - T_{\substack{\text{window}\\\text{inside}}} = (\text{heat flow rate})/h\,A,$$
$$T_{\substack{\text{window}\\\text{inside}}} - T_{\substack{\text{window}\\\text{outside}}} = (\text{heat flow rate})(\text{thickness of window})/k\,A, \text{ and}$$
$$T_{\substack{\text{window}\\\text{outside}}} - T_{\substack{\text{air film}\\\text{outside}}} = (\text{heat flow rate})/h\,A.$$

Overall, then,

$$15\text{ °C} = (1/A)(\text{heat flow rate})(1/h + [\text{thickess of window}]/k + 1/h).$$

Using our values and the thermal conductivity of glass, 0.8 W/(m °C) and the conduction-convection parameter for vertical surfaces,

$$\text{heat flow rate} = (15\text{ °C})\ (1\text{ m}^2)/\text{D},$$

where

$$\text{D} = 1/1.8\text{ W/(m}^2\text{ °C)}\left(T_{\substack{\text{air film}\\\text{inside}}} - T_{\substack{\text{window}\\\text{inside}}}\right)^{1/4}$$
$$+ 1/1.8\text{ W/(m}^2\text{ °C)}\left(T_{\substack{\text{window}\\\text{outside}}} - T_{\substack{\text{air film}\\\text{outside}}}\right)^{1/4}$$
$$+ 0.0030\text{ m}/\{0.080\text{ W/(m °C)}\}.$$

To make any progress, we should guess and repeat until a consistent answer is obtained.

Step 1: Let's suppose that because the glass is so thin, we say there is only a 1° temperature difference across the window. Then, plausibly, each of the temperature differences is (15 °C - 1 °C)/2 = 7 °C and the temperature difference to the one-fourth power is 1.63, giving

$$D = \{[2/(1.8)(1.63)] + 0.00375\}\ (\mathrm{m}^2\ °\mathrm{C})/\mathrm{W} = 0.687\ (\mathrm{m}^2\ °\mathrm{C})/\mathrm{W}.$$

Then

$$\text{heat flow rate} = (15\ °\mathrm{C})\ (1\ \mathrm{m}^2)/[0.687\ (\mathrm{m}^2\ °\mathrm{C})/\mathrm{W}] = 21.8\ \mathrm{W}.$$

Putting this back into

$$T_{\substack{\text{window}\\ \text{inside}}} - T_{\substack{\text{window}\\ \text{outside}}} = (\text{heat flow rate})(\text{thickness of window})/k\,A$$

$$= (21.8\ \mathrm{W})(0.003\ \mathrm{m})/\{[0.8\ \mathrm{W}/(\mathrm{m}\ °\mathrm{C})][1\ \mathrm{m}^2]\}$$

$$= 0.082\ °\mathrm{C},$$

which is quite close to zero. We were obviously too optimistic about the insulating properties of the windows.

Step 2: Feed this back and try again. Each temperature difference is (15 °C - 0.082 °C)/2 = 7.46 °C, the temperature difference to the one-fourth power is 1.65, giving

$$D = \{[2/(1.8)(1.65)] + 0.00375\}\ (\mathrm{m}^2\ °\mathrm{C})/\mathrm{W} = 0.676\ (\mathrm{m}^2\ °\mathrm{C})/\mathrm{W},$$

and so the heat flow rate is

$$\text{heat flow rate} = (15\ °\mathrm{C})\ (1\ \mathrm{m}^2)/[0.676\ (\mathrm{m}^2\ °\mathrm{C})/\mathrm{W}] = 22.2\ \mathrm{W}.$$

The temperature difference across the glass is now

$$T_{\substack{\text{window}\\ \text{inside}}} - T_{\substack{\text{window}\\ \text{outside}}} = (\text{heat flow rate})(\text{thickness of window})/k\,A$$

$$= (22.2\ \mathrm{W})(0.003\ \mathrm{m})/\{[0.8\ \mathrm{W}/(\mathrm{m}\ °\mathrm{C})][1\ \mathrm{m}^2]\}$$

$$= 0.083\ °\mathrm{C},$$

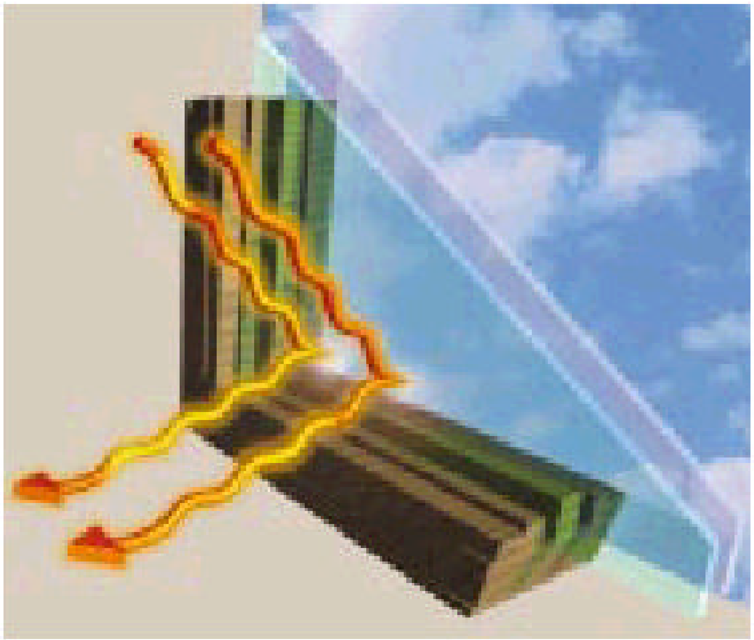
and we can't gain anything by doing it again, so there need be no Step 3.

Most of the temperature difference exists across the air layers. If the air is not still, the thermal energy loss through the window will be much higher. That's why windows feel so cold on the inside when we are near them on a windy day. Adding storm windows serves two purposes—it will add a space with still air, and, more important, more layers of air at each window surface.

Low-e windows

Low-emissivity windows, mentioned in the Chapter, are windows with a plastic coating that reflects thermal energy while letting most visible light through. Windows treated with low-e film help in both the winter and summer, with little leakage of thermal energy to the inside on a hot summer day, or little leakage to the outside in winter. The windows are usually also double-paned, and may have a filling of an inert gas such as argon or nitrogen. The idea of a low-e window is shown in Fig. E09.4 5. Figure E09.4.5 a shows the situation in winter, while Fig. E09.4.5 b shows the effect during summer.

A similar sort of idea is the vacuum window. The window has the same insulating value as 35 cm of wall. The thermal conductivity is a mere 0.2 W/(m^2 °C). The basic difference from the windows of Fig. E09.4.5 is that the space between the two glass sheets contains nothing, or very close to nothing. The Swiss firm that makes these windows, Dörig (St. Gallen-Mörschwil, Switzerland), actually makes them with four layers; a pair with a vacuum, a space filled with krypton, then another pair with a vacuum. The double vacuum design is quite similar to that of a thermos, and the window works in a similar way.[(163)]


a.

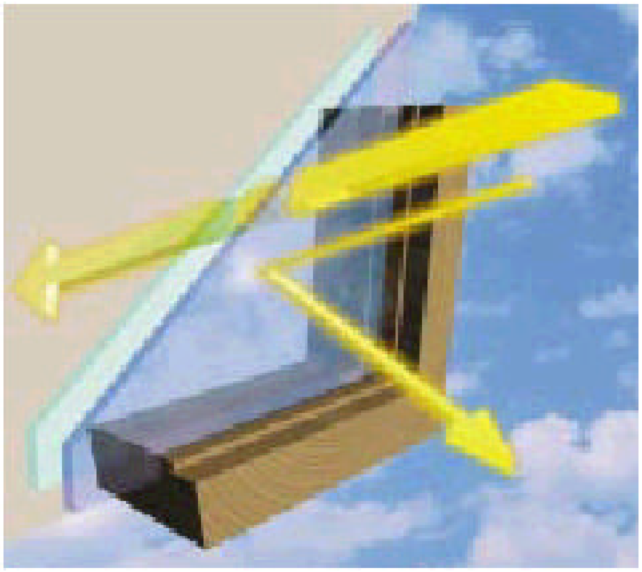

b.

Fig. E09.4.5 A window with coated glass reflects thermal energy, keeping it in in winter and out in summer.
(U.S. Department of Energy, NREL, Ref. 81)

Of special interest is the coating used on glass to reduce both light transmissions and heat transfer. These coatings are used to keep air conditioning costs relatively low. A “magic coating” that could allow light through but not transfer heat when outdoor temperatures are high and transfer heat when the outdoor temperature is low would be a great boon. It appears that such a coating may have been found! It is a mixture of tungsten and vanadium dioxide that switches at just above room temperature (29 °C).[164] That a transition occurs had been known about for a long time, but it had occurred at around 70 °C, too high to be of practical use. The researchers found a way to reduce it to 51 °C by adding tungsten, and succeeded in reaching 29 °C. The coating can be put on molten glass at atmospheric pressure, and so manufacturing should not be overly difficult. Such glass may be available commercially before 2010.