CHAPTER W4

Integration and the General Linear Model

Chapter Outline

This chapter is intended to integrate and deepen your knowledge about the major statistical techniques you have learned throughout the book. Equally important, it provides a thorough review of those techniques. [### Tip for Success: To understand this chapter, you should already have covered the chapters on *t* tests, analysis of variance, correlation, and prediction. You should also have covered the first part of Chapter 15 that provides an introduction to the general linear model. We suggest that you re-read that section before reading this chapter.]

The General Linear Model

In Chapter 15, we introduced you to the general linear model, which is a general statement of the influences that make up an individual's score on a particular variable. The general linear model states that the value of a variable for any individual is the sum of a constant, plus the weighted influence of each of several other variables, plus error. Bivariate and multiple correlation and regression (and associated significance tests), the *t* test, and the analysis of variance are all special cases of the general linear model. You also learned in Chapter 15 that bivariate correlation/prediction and analysis of variance are special cases of multiple regression, and that the *t* test can be derived directly from either bivariate correlation/prediction or analysis of variance (for a summary, see Figure W4–1, which is the same as Figure 15–1 in Chapter 15). We describe these relationships in detail in this Web chapter.

The General Linear Model and Multiple Regression

The link between the general linear model and multiple regression is very intimate—they are nearly the same. Traditionally, they have not been equated because the general linear model is understood to be behind other techniques, such as bivariate correlation and the analysis of variance, in addition to multiple regression. However, in recent years psychologists have become increasingly aware that these other techniques can be derived from multiple regression as well as from the general linear model.

Bivariate prediction and Correlation as Special Cases of Multiple Regression

Bivariate prediction, prediction from one predictor variable to one criterion variable, is a special case of multiple regression, which is prediction from any number of predictor variables to one criterion variable. Similarly, bivariate correlation, the association between one predictor variable and one criterion variable, is a special case of multiple correlation, the association of any number of predictor variables and one criterion variable.

The t Test As a Special Case of the Analysis of Variance

Both the *t* test and the analysis of variance test differences between means of groups. You use the *t* test when there are only two groups.[1] You usually use the analysis of variance, with its *F* ratio, only when there are more than two groups. However, you can use the analysis of variance with just two groups. When there are only two groups, the *t* test and the analysis of variance give identical conclusions.

The strict identity of *t* and *F* applies only in this two-group case. You cannot figure an ordinary *t* test among three groups. This is why we say that the *t* test is a *spe-*

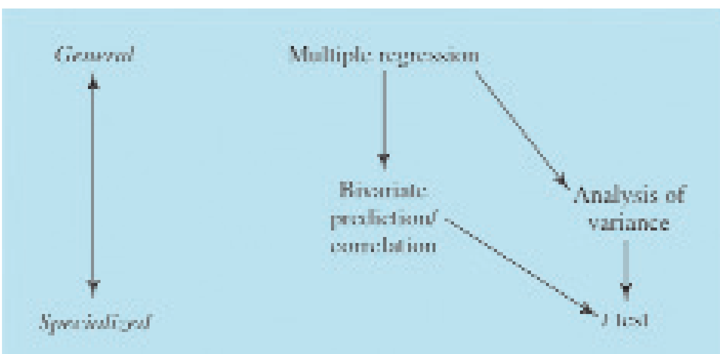


FIGURE W4–1 *The relationship among the four major statistical techniques.*

cial case of the analysis of variance. The test is mathematically identical to the analysis of variance in the particular case where there are only two groups.

Intuitive Understanding of the Relationship of the Two Procedures

One way to get a sense of the link of the two procedures is through the analogy of signal-to-noise ratio that we introduced in Chapter 9 to explain the analysis of variance. The idea is that the analysis of variance *F* ratio is a measure of how much the signal (analogous to the difference between group means) is greater than the noise (analogous to the variation within each of the groups). The same idea applies to a *t* test, which is also really about how much the signal (the difference between the two group means) is greater than the noise (the standard deviation of the distribution of differences between means, which is also based on the variation within the groups).

Parallels in the Basic Logic of the Two Procedures

The analysis of variance *F* ratio is the population variance estimate based on the variation between the means of the groups divided by the population variance estimate based on the variation within each of the groups. That is, the *F* ratio is a fraction in which the numerator is based on the differences among the groups, comparing their means, and the denominator is based on the variation within each of the groups.

The *t* score is the difference between the means of the two groups divided by the standard deviation of the distribution of differences between means (and this standard deviation is based mainly on a pooled variance estimate that is figured from the variation within each of the two groups). Thus, the *t* score is a fraction in which the numerator is the difference between the groups, comparing their means, and the denominator is based on the variation within each of the groups.

In other words, as shown in the top sections of Table W4–1, an *F* ratio and a *t* score are both fractions in which the numerator is based on the differences between the group means and the denominator is based on the variances within the groups.

Table W4–1 Some Links Between the *t* Test for Independent Means and the Analysis of Variance

t Test	Analysis of Variance
Numerator of *t* is the difference between the means of the two groups.	Numerator of *F* is partly based on variation between the means of the two or more groups.
Denominator of *t* is partly based on pooling the population variance estimates figured from each group.	Denominator of *F* is figured by pooling the population variance estimates figured from each group.
Denominator of *t* involves dividing by number of scores.	Numerator of *F* involves multiplying by number of scores. (Multiplying a numerator by a number has the same effect as dividing the denominator by that number.)
When using two groups, $t = \sqrt{F}$	When using two groups, $F = t^2$
$df = (N_1 - 1) + (N_2 - 1)$	$df_{\text{Within}} = (N_1 - 1) + (N_2 - 1) + \ldots + (N_{\text{Last}} - 1)$

Numeric Relationship of the Two Procedures

The formula for a *t* score comes out to be exactly the square root of the formula for the *F* ratio in the situation where there are just two groups. Most of you will not be interested in the precise derivation, but there is an important implication. If you figure a *t* score, it will come out to be exactly the square root of what you would get if you figured an *F* ratio for the same study. For example, if you figured a *t* of 3 and then you figured *F* for the same study, the *F* would come out to 9. Similarly, consider the cutoffs in a *t* table. These are exactly the square roots of the cutoffs in the column of an *F* table for an analysis of variance for two groups (that is, the part of the *F* table with numerator *df* = 1).

An apparent difference between the two procedures is how they are affected by sample size. In the analysis of variance, the sample size is part of the numerator. As we saw in Chapter 9, the numerator of the *F* ratio is the population variance estimate using the difference among the means multiplied by the number of scores in each group. That is, . In the *t* test, the sample size is part of the denominator. As we saw in Chapter 8, the denominator of the *t* test uses the pooled population variance estimate divided by the number of scores in each group. That is, $S_{Difference}$ = and ; ; . This apparent contradiction is resolved, however, because multiplying the numerator of a fraction by a number has exactly the same effect as dividing the denominator by that number. For example, take the fraction 3/8. Multiplying the numerator by 2 gives 6/8, or 3/4; dividing the denominator of 3/8 by 2 also gives 3/4.[2]

Worked-Out Example of the Two Procedures

An example with all the figuring makes the equivalence more vivid. Table W4–2 shows the *t* and *F* figuring for the *t* test Example Worked-Out Problem from Chapter 8. Notice the following: (a) The pooled population variance estimate in the *t* test (= 4.17) is the same as the within-group population variance estimate for the analysis of variance (= 4.17), both figured as part of the denominator. (b) The degrees of freedom for the *t* distribution (*df* = 12) is exactly the same as the denominator degrees of freedom for the *F* distribution (df_{Within} = 12). (c) The cutoff *t* for rejecting the null hypothesis (2.179) is the square root of the cutoff *F* for rejecting the null hypothesis (= 2.179). (d) The *t* for these data (2.73) is the square root of the *F* (= 2.75, the slight difference being due to rounding error). And (e) the conclusion is the same. With both methods, you reject the null hypothesis (and if you were to get an exact p value using a statistics program, both methods would give exactly the same exact p value).

how are you doing?

1. How is bivariate prediction a special case of multiple regression?
2. When can you use an analysis of variance to do the same thing as a t test?
3. How is the numerator of a t test like the numerator of an F ratio in an analysis of variance?
4. How is the denominator of a t test like the denominator of an F ratio in an analysis of variance?
5. How is like ?
6. When figured for the same scores, what is the relation of the t to the F?
7. What is the relation of the t cutoff to the F cutoff for the same study (involving two groups)?

Table W4-2 *t* Test and Analysis of Variance Computations for the Same Study (Fictional Data)

	Experimental Group			Control Group	
X_1	$X_1 - M_1$	$(X_1 - M_1)^2$	X_2	$X_2 - M_2$	$(X_2 - M_2)^2$
6	0	0	6	3	9
4	−2	4	1	−2	4
9	3	9	5	2	4
7	1	1	3	0	0
7	1	1	1	−2	4
3	−3	9	1	−2	4
6	0	0	4	1	1
Σ 42	0	24	21	0	26
$M_1 = 6$	$S_1^2 = 24/6 = 4$		$M_2 = 3$	$S_2^2 = 26/6 = 4.33$	
$N_1 = 7$	$df_1 = N_1 - 1 = 6$		$N_2 = 7$	$df_2 = N_2 - 1 = 6$	

t test	ANOVA

Numerator

t test:

Mean difference = 6.00 − 3.00 = 3.00

ANOVA:

$$df_{\text{Between}} = N_{\text{Groups}} - 1 = 2 - 1 = 1$$
$$GM = (6 + 3)/2 = 9/2 = 4.5$$
$$\Sigma(M - GM)^2 = (6 - 4.5)^2 + (3 - 4.5)^2$$
$$= 1.5^2 + -1.5^2$$
$$= 2.25 + 2.25 = 4.5$$

$$S^2_{\text{Between}} \text{ or } MS_{\text{Between}} = \left(\frac{\Sigma(M - GM)^2}{df_{\text{Between}}}\right)(n) = \left(\frac{4.5}{1}\right)(7) = 31.5$$

Denominator

t test:

$$S^2_{\text{Pooled}} = \left(\frac{df_1}{df_{\text{Total}}}\right)(S_1^2) + \left(\frac{df_2}{df_{\text{Total}}}\right)(S_2^2) = \left(\frac{6}{12}\right)(4) + \left(\frac{6}{12}\right)(4.33)$$
$$= (.5)(4) + (.5)(4.33) = 2.00 + 2.17 = 4.17$$

$$S^2_{\text{Difference}} = S^2_{M_2} + S^2_{M_1} - (S^2_{\text{Pooled}}/N_1) + (S^2_{\text{Pooled}}/N_2)$$
$$= (4.17/7) + (4.17/7)$$
$$= .60 + .60 = 1.20$$
$$S_{\text{Difference}} = \sqrt{S^2_{\text{Difference}}} = \sqrt{1.20} = 1.10$$

ANOVA:

$$S^2_{\text{Within}} \text{ or } MS_{\text{Within}} = \frac{S_1^2 + S_2^2 + \ldots + S^2_{\text{Last}}}{N_{\text{Groups}}} = \frac{4 + 4.33}{2}$$
$$= \frac{8.33}{2} = 4.17$$

Degrees of Freedom

t test:

$$df_{\text{Total}} = df_1 + df_2 = 6 + 6 = 12$$

ANOVA:

$$df_{\text{Within}} = df_1 + df_2 \ldots df_{\text{Last}} = 6 + 6 = 12$$

Cutoff

t test: Needed *t* with *df* = 12 at 5% level, two-tailed = ± 2.179

ANOVA: Needed *F* with *df* = 1, 12 at 5% level = 4.75

Score on Comparison Distribution

t test:

$$t = (M_1 - M_2)/S_{\text{Difference}} = (6.00 - 3.00)/1.10 = 3.00/1.10 = 2.73$$

ANOVA:

$$F = S^2_{\text{Between}} / S^2_{\text{Within}} \text{ or } MS_{\text{Between}}/MS_{\text{Within}} = 31.5/4.17 = 7.55$$

Conclusions

t test: Reject the null hypothesis; the research hypothesis is supported.

ANOVA: Reject the null hypothesis; the research hypothesis is supported.

Answer

1. Multiple regression predicts the criterion variable from any number of predictor variables; bivariate prediction is the special case in which you are predicting from only one predictor variable.
2. When there are only two groups.
3. Both are about the difference or variation between the groups.
4. Both are about variation within groups.
5. The two are identical.
6. The t is the square root of the F.
7. The t cutoff is the square root of the F cutoff.

The t Test as a Special Case of the Significance Test for the Correlation Coefficient

The relationship of the correlation coefficient to the *t* test is far from obvious. The correlation coefficient is about the degree of association between two variables; the *t* test is about the significance of the difference between two population means. What is the possible connection?

As you learned in Chapter 11, one connection is that both use the *t* distribution to determine significance. As a reminder, the score for a correlation coefficient on the comparison distribution is a *t* score figured from the correlation coefficient using the formula *t* = (formula 11-2, Chapter 11, page 453). However, knowing about this procedure does not give much insight into *why* the correlation coefficient can be turned into a *t* score for purposes of hypothesis testing or of the connection between this *t* based on the correlation coefficient and the *t* test for the difference between means of two groups. It is to these issues that we now turn.

Group Differences as Associations Among Variables

We usually think of the correlation coefficient as the association between two variables, typically a predictor variable and a criterion variable. Testing the significance of a correlation coefficient asks whether you can reject the null hypothesis that in the population there is no association between the predictor and criterion variable (that in the population, $r = 0$).

The *t* test for independent means examines the difference between two population means, based on the means of two samples. The sample scores are on a measured variable that is like a criterion variable (you want to know the effect on it). The distinction between the two groups in a *t* test is like the predictor variable. In our example from the previous section (see Table W4–2), the variable that divides the two groups was whether participants were in the experimental or control group. Thus, you can think of the *t* test as about whether there is any association between the variable that divides the groups and the measured variable (see Table W4–3).

Numerical Predictor Variables Versus Two-Category Nominal Variable that Divides the Groups

"But wait!" you may say. "The predictor variable in a correlation coefficient is a numerical variable, such as number of hours sleep or high school GPA. The variable

Table W4–3 Relation between Correlation and *t* Test for Independent Means

	Correlation	*t* Test
Variable 1	Predictor Variable	Variable that Divides the Groups
Variable 2	Criterion Variable	Measured Variable
Relation tested	High scores on predictor go with high scores on criterion	Those in one group on the variable that divides the groups have higher scores on the measured variable

that divides the groups in a *t* test for independent means is a variable with exactly two values, the two categories, such as experimental group versus control group." Yes, you are quite correct. This is precisely the difference between the situations in which you use a correlation coefficient and those in which you ordinarily use a *t* test for independent means.

How can this gap be bridged? Suppose that you arbitrarily give a number to each level of the two-category nominal variable that divides the groups. For example, you could make the experimental group a 1 and the control group a 2. (Using any other two numbers will, in the end, give exactly the same result. However, which group gets the higher number does determine the plus or minus sign of the final result.) Once you change the two-category nominal variable that divides the groups to a numerical variable, you can then figure the correlation between this two-valued numeric variable and the measured variable.

Example of the Numeric Equivalence of the t Test and the Correlation Coefficient Significance Test

Table W4–4 shows the figuring for the correlation coefficient and its significance using the scores from the same *t* test example we used earlier (see Table W4–2). Notice that in this correlation setup, each individual has two scores: (a) a 1 or a 2, depending on whether the person is in the experimental group or the control group, and (b) a score on the measured variable.

The resulting correlation is –.62. Using the formula for changing a correlation to a *t* score gives a *t* of –2.74. This *t* is the same, within rounding error, that we figured earlier (2.73) using the ordinary *t*-test procedures (see Chapter 8, Table 8–8 and Table W4–2 in this chapter). The difference in sign has to do with which group gets the 1 and which group gets the 2—a decision that is arbitrary. The degrees of freedom, and thus the needed *t* for significance and the conclusion, are also the same as for the *t* test for independent means.

In sum, the significance test of the correlation coefficient gives the same result as the ordinary *t* test. We say that the *t* test is a special case of the correlation coefficient, however, because you can use the *t* test only in the situation in which the predictor variable has exactly two values.

Graphic Interpretation of the Relationship of the t Test to the Correlation Coefficient

Figure W4–2 shows the scatter diagram, including the regression line, for the scores in the example we have been following. The predictor variable (the variable that divides the groups) has just two values, so the dots all line up above these two values.

Table W4-4 Figuring of the Correlation Coefficient and a Hypothesis Test of the Correlation Coefficient Using the Data from Table W4–2 (and Table 8–8) and Changing the Variable that Divides the Groups Into a Numeric Variable Having Values of 1 (for the Experimental Group) or 2 (for the Control Group)

Variable that Divides the Groups (Experimental Versus Control) (X)			Measured Variable (Y)			
	Deviation	Deviation Squared		Deviation	Deviation Squared	Products of Deviation Scores
X	$X - M_X$	$(X - M_X)^2$	Y	$Y - M_Y$	$(Y - M_Y)^2$	$(X - M_X)(Y - M_Y)$
1	−.5	.25	6	1.5	2.25	−.75
1	−.5	.25	4	−.5	.25	.25
1	−.5	.25	9	4.5	20.25	−2.25
1	−.5	.25	7	2.5	6.25	−1.25
1	−.5	.25	7	2.5	6.25	−1.25
1	−.5	.25	3	−1.5	2.25	.75
1	−.5	.25	6	1.5	2.25	−.75
2	.5	.25	6	1.5	2.25	.75
2	.5	.25	1	−3.5	12.25	−1.75
2	.5	.25	5	.5	.25	.25
2	.5	.25	3	−1.5	2.25	−.75
2	.5	.25	1	−3.5	12.25	−1.75
2	.5	.25	1	−3.5	12.25	−1.75
2	.5	.25	4	−.5	.25	−.25
$\Sigma = 21$		$\Sigma = SS_X = 3.5$	$\Sigma = 63$		$\Sigma = SS_Y = 81.5$	$\Sigma = -10.5$
$M = 1.5$			$M = 4.5$			

$$r = \frac{\Sigma[(X - M_X)(Y - M_Y)]}{\sqrt{(SS_X)(SS_Y)}} = \frac{-10.5}{\sqrt{(3.5)(81.5)}} = \frac{-10.5}{\sqrt{285.25}} = -.62$$

$df = N - 2 = 14 - 2 = 12.$

t needed with $df = 12$ at 5% level, two-tailed $= \pm 2.179$.

$t = (r)/\sqrt{(1 - r^2)(N - 2)} = (-.62)/\sqrt{(1 - (-.62)^2)/(14 - 2)} = (-.62)/\sqrt{.0513} = -2.74$

Decision: Reject the null hypothesis; the research hypothesis is supported.

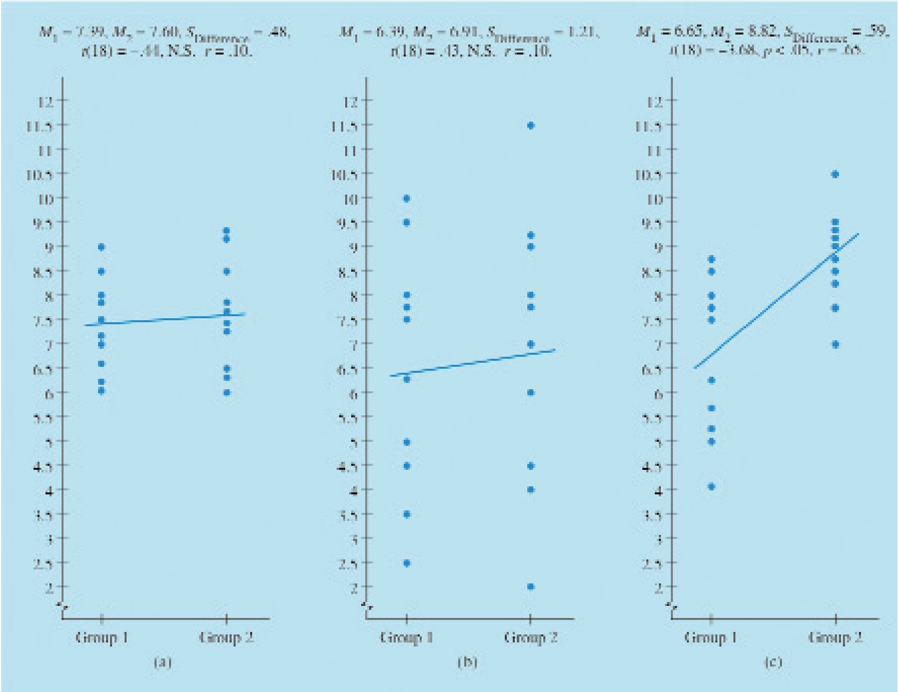


FIGURE W4–2 *Scatter diagram and regression line for the example, originally analyzed with a t test for independent means, with a value of 1 for the experimental group and 2 for the control group.*

Note that the regression line goes through the middle of each line of dots. In fact, when making a scatter diagram of the scores for a *t* test, the regression line always goes exactly through the mean of each set of dots. This is because the regression line shows the best predicted score at each level of the predictor variable, and for any group of scores, the best predicted score is always the mean.

Figure W4–3 shows some additional examples. In Figure W4–3a, the two means are nearly the same. Here, the slope of the regression line is about 0; the correlation is low and not statistically significant. The correlation is .10; thus, with 20 participants, t = = = .43. Thinking in terms of a *t* test for independent means, because there is little difference between the means of the two groups, the *t* test will not be significant. The mean difference is 7.39 – 7.60 = –.21. The standard deviation of the distribution of differences between means is .48; thus, $t = (M_1 - M_2)/S_{\text{Difference}} = (7.39 - 7.60)/.48 = -.44$. This is the same result as you get using the correlation approach (within rounding error, and ignoring sign).

[### Insert Figure W4–3 about here]

In Figure W4–3b the means of the two groups are somewhat different, but the dots in each group are even more widely spread out. Once again, the correlation coefficient is low and not statistically significant. In the *t* test for independent means, the spread of the dots makes a large estimated population variance for each group, creating a large pooled variance estimate and a large standard deviation of the distribution of differences between means. In a *t* test you divide the mean difference by the standard deviation of the distribution of differences between means; thus, the larger this standard deviation, the smaller the *t* score. In the example the mean difference is .52 and the standard deviation of the distribution of differences between means is 1.21. This gives a *t* of .43, which is clearly not significant.

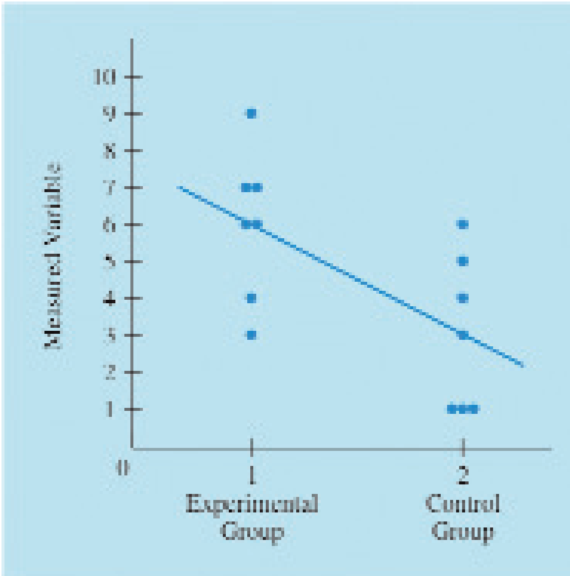


FIGURE W4–3 *Three possible scatter diagrams of scores analyzed with a t test for independent means, in which the means for the two groups are (a) nearly the same, (b) different but the scores are widely spread (large pooled variance and thus large standard deviation of the distribution of differences between means), and (c) very different, with the scores not widely spread.*

Finally, in Figure W4–3c there is a large difference between the means and less variation among the dots around each mean. Thus, the regression line is a very good predictor. Similarly, the large mean difference and small variance within each group make for a large *t* using a *t* test for independent means.

The principle that these figures illustrate is that the *t* test for independent means and the significance test for the correlation coefficient give the same results because both are largest when the difference between the two means is large and the variation among the scores in each group is small.

The Analysis of Variance as a Special Case of the Significance Test of Multiple Regression

The relationship between the analysis of variance and multiple regression parallels the relationship we just considered between the *t* test for independent means and the correlation coefficient. And in both, the solution is the same. The analysis of variance tests whether there is a difference on the measured variable between means of three or more groups. The multiple regression approach sees this as a relationship between a criterion variable (the measured variable) and a predictor variable (the different levels of the variable that divides the groups). For example, in the Hazan and Shaver (1987) study of attachment style and jealousy discussed in Chapter 9, the analysis of variance showed a significant difference in jealousy (the measured variable) among the three attachment styles (the variable that divides the groups). A correlation or regression approach, by contrast, would describe this result as a significant association between jealousy (the criterion variable) and attachment style (the predictor variable). We describe the relationship between analysis of variance and regression in more detail in an Advanced Topic section later in the chapter.

Choice of Statistical Tests

We have seen that the four major statistical procedures you have learned in this book can be considered special cases of multiple regression. You may now wonder why you don't learn just one technique, multiple regression, and do everything using it. You could. And you would get entirely correct results.

Why, then, should anyone use, say, a *t* test instead of an analysis of variance? The reason is that it is a procedure that is traditional and widely understood. Most researchers today expect to see a *t* test when two groups are compared. It seems strange, and somehow grandiose, to see an analysis of variance when a *t* test would do—though, in fact, the sense of grandiosity is simply a holdover from the days when all the figuring was done by hand and an analysis of variance was harder to do than a *t* test.

To use a correlation coefficient (and its associated significance test) in the two-group situation instead of an ordinary *t* test would confuse people who were not very statistically sophisticated. Similarly, analyzing an experiment with several groups using multiple regression instead of analysis of variance would confuse those same unsophisticated readers.[3]

There is one advantage in using correlation and regression over the *t* test or an analysis of variance: The correlational approach automatically gives you direct information on the relationship between the variable that divides the groups and the measured variable as well as permitting a significance test. The *t* test and the analysis of variance give only statistical significance. (You can figure an effect size for either of these, but with a correlation coefficient or a multiple regression, you get the effect size automatically.)

how are you doing?

1. How can you understand a difference between groups on a measured variable in terms of an association between a predictor and a criterion variable?
2. How can you make a two-level nominal variable that divides the groups into a numeric variable that you can use in correlation or regression?
3. (a) What is the effect of the scores being spread out around their mean, and (b) why, for the t test for independent means?
4. When you make a scatter diagram for the scores in a t test for independent means, (a) what does it look like, and (b) where does the regression line go?
5. How do the variables in an analysis of variance correspond to the variables in a regression?
6. (a) Why do researchers use t tests and analyses of variance when they could use correlation or regression instead? (b) What is an advantage of using regression and correlation over using analysis of variance and the t test.

4. (a) The dots are all lined up above the points for the two levels of the variable that divides the groups.
(b) It goes through the mean of each group.
5. The grouping variable in an analysis of variance is like a predictor variable in regression. The measured variable in an analysis of variance is like a criterion variable in regression.
6. (a) Researchers are familiar with t tests and analysis of variance for testing differences between groups, they are traditional for this purpose, and some researchers are unfamiliar with and would be confused by the use of correlation and regression for this purpose.
(b) Correlation and regression automatically give you estimates of effect size and not just significance.

Answers

1. A difference between groups on a measured variable is the same as an association between the variable that divides the groups (which is like the predictor variable in correlation or regression) and the measured variable (which is like the criterion variable in correlation or regression).
2. Make it in to a two-valued numeric variable by giving a score of, say, 1 on this variable to everyone in one group and a score of, say, 2 on this variable to everyone in the other group.
3. (a) It reduces the t.
   (b) The variance of each group will be greater, making the pooled estimate of the population variance greater, making the variance of the distribution of differences between means greater, making the standard deviation of the distribution of differences between means greater. You figure the t by dividing by the standard deviation of the differences between means. Thus, if it is bigger, the t is smaller.

Controversy: What Is Causality?

The general linear model itself is not very controversial; it is simply a mathematical statement of a relationship among variables. In fact, its role as the foundation of the major statistical techniques has not yet been widely realized among practicing researchers. There is, however, an area of controversy that is appropriate to mention here. It has to do with the role of statistics in science generally, but in practice it is most often raised in the context of the major general linear model-based procedures. This is the issue of causality. We have already addressed this issue at one level in Chapter 11, where we considered the problem of inferring a direction of causality from a study that does not use random assignment to groups. But there is a still deeper level to the issue: What does causality mean?

In a classic discussion of the issues, the eminent developmental psychologist, Diana Baumrind (1983) outlined two main understandings of causality that are used in science. One, which she calls the *regularity theory of causality*, has its roots in philosophers like David Hume and John Stuart Mill (as well as early scientists, including Galileo). This view holds that we recognize X as a cause of Y if (a) X and Y are regularly associated, (b) X precedes Y, and (c) there are no other causes that precede X that might cause both X and Y. In psychology, we address the (a) part by finding a significant correlation between X and Y. We address the (b) part, if possible, by our knowledge of the situation (for example, in a correlation of whether one is the firstborn in one's family with anxiety later in life, you can rule out the possibility that anxiety later in life caused the person to be firstborn) or designing the study into an experiment (by manipulating X prior to measuring Y). The (c) part has to do with the issue of a correlation between X and Y being due to some third variable causing both. Ideally, we address this by random assignment to groups. But if that is not possible, various statistical methods of equating groups on proposed third factors are used as a makeshift strategy (we explore some of these in Chapter 15).

As psychologists, we are only sometimes in a position to do the kind of rigorous experimental research that provides a strong basis for drawing conclusions about cause and effect. Thus, much of the criticism and controversy involving research of practical importance, where it is usually least easy to apply rigorous methods, often hinges on such issues. For example, if marriage correlates with happiness, does marriage make people happier, or do happy people get and stay married?

There is another view of causality, a still more stringent view that sees the regularity theory conditions as a prerequisite to calling something a cause, but that these conditions are not sufficient alone. This other view, which Baumrind calls the *generative theory of causality*, has its roots in Aristotle, Thomas Aquinas, and Immanuel Kant. The focus of this view is on just *how* X affects Y. This is the way most nonscientists (and nonphilosophers) understand causality. The very idea of causality may have its roots as a metaphor of experiences such as willing your own arm to move (Event X) and it moves (Event Y). Scientists also take this view of causality very much to heart, even if it offers much more difficult challenges. It is addressed primarily by theory and by careful analysis of mediating processes. But even those who emphasize this view would recognize that demonstrating a reliable connection between X and Y (by finding statistical significance, for example) plays an important role at least in identifying linkages that require scrutiny for determining the real causal connection.

Finally, there are also those who hold—with some good arguments—that demonstrating causality should not be a goal of scientific psychology at all. But we have already had enough controversy for one chapter.

ADVANCED TOPIC: Detailed Examination of The Analysis of Variance as a Special Case of the Significance Test of Multiple Regression

In order to follow the material in this Advanced Topic section, you must have read the Advanced Topic sections in Chapter 9 (on the structural model in the analysis of variance) and Chapter 12 (on error and proportionate reduction in error).

Earlier in the chapter we noted that the relationship between the analysis of variance and multiple regression parallels the relationship between the t test for independent means and the correlation coefficient. Here, we give a detailed analysis of the relationship between the analysis of variance and multiple regression.

Analysis of Variance for Two Groups As a Special Case of the Significance of a Bivariate Correlation

The link between the analysis of variance and multiple regression is easiest to see if we begin with a two-group situation and (a) consider the correlation coefficient in terms of its being the square root of the proportionate reduction in error (see Chapter 12), and (b) consider the analysis of variance using the structural model approach (see the Advanced Topic section of Chapter 9). Table W4–5 shows the scores for our experimental versus control group example. However, this time we show the predicted scores and the errors and squared errors, as well as the figuring for the proportionate reduction in error. Table W4–6 shows the analysis of variance figuring, using the structural model approach, for the same scores.

There are several clear links. First, the sum of squared error figured in the correlation when using the bivariate prediction rule ($SS_{\text{Error}} = 50$) is the same as the within-group sum of squared deviations (SS_{Within}) for the analysis of variance. Why are they the same? In regression, the error is a score's difference from the predicted value, and the predicted value in this situation of only two values for the predictor variable is the mean of the scores at each value (that is, the mean of each group's scores). In other words, in the regression, the sum of squared error comes from squaring and summing

Table W4-5 Figuring of the Proportionate Reduction in Error With Bivariate Prediction Using the Data From Table W4–2 (and Table 8–8)

Predictor Variable (Experimental Versus Control)	Criterion Variable			
	Actual Score	Predicted Score	Error	Error^2
	Y	$\hat{Y}$	$Y - \hat{Y}$	$(Y - \hat{Y})^2$
1	6	6	0	0
1	4	6	−2	4
1	9	6	3	9
1	7	6	1	1
1	7	6	1	1
1	3	6	−3	9
1	6	6	0	0
2	6	3	3	9
2	1	3	−2	4
2	5	3	2	4
2	3	3	0	0
2	1	3	−2	4
2	1	3	−2	4
2	4	3	1	1
				$\Sigma = SS_{\text{Error}} = 50$

Sum of squared error using the overall mean as a prediction rule (computation not shown): $SS_{\text{Total}} = 81.5$

$$\text{Proportionate reduction in error} = \frac{SS_{\text{Total}} - SS_{\text{Error}}}{SS_{\text{Total}}} = \frac{81.5 - 50}{81.5} = .39$$

$r^2 = .39; r = \sqrt{r^2} = \sqrt{.39} = \pm.62.$

the difference of each score from its group's mean. In the analysis of variance, you figure the sum of squared error within groups as precisely the same thing—the sum of the squared deviations of each score from its group's mean.

Second, the sum of squared error total (SS_{Total}) is the same in regression and analysis of variance (in this example they are both 81.5). They are the same because in regression, SS_{Total} is the sum of the squared deviations of each criterion variable score from the overall mean of all the criterion variable scores and in the analysis of variance, SS_{Total} is the sum of the squared deviations of each measured variable score from the grand mean, which is the overall mean of all the measured variable scores.

Third, the reduction in squared error in regression—the sum of squared error using the mean to predict (that is, 81.5) minus the sum of squared error using the bivariate prediction rule (that is, 50)—comes out to 31.5. This is the same as the analysis of variance sum of squared error between groups (that is, $SS_{\text{Between}} = 31.5$). The reduction in error in regression is what the prediction rule adds over knowing just the mean. In this example, the prediction rule estimates the mean of each group, so the

Table W4-6 Figuring of the Proportional Reduction in Error with the One-Way Analysis of Variance Structural Model Approach Using the Data From Table W4–2 (and Table 8–8)

One-way analysis of variance structural model calculation
***GM* = 4.5**

	Experimental Group							Control Group						
X_1	$X-GM$		$X-M$		$M-GM$		X	$X-GM$		$X-M$		$M-GM$	
	Dev	*Dev*^{2}	*Dev*	*Dev*^{2}	*Dev*	*Dev*^{2}		*Dev*	*Dev*^{2}	*Dev*	*Dev*^{2}	*Dev*	*Dev*^{2}
6	1.5	2.25	0	0	1.5	2.25	6	1.5	2.25	3	9	−1.5	2.25
4	−.5	.25	−2	4	1.5	2.25	1	−3.5	12.25	−2	4	−1.5	2.25
9	4.5	20.25	3	9	1.5	2.25	5	.5	.25	2	4	−1.5	2.25
7	2.5	6.25	1	1	1.5	2.25	3	−1.5	2.25	0	0	−1.5	2.25
7	2.5	6.25	1	1	1.5	2.25	1	−3.5	12.25	−2	4	−1.5	2.25
3	−1.5	2.25	−3	9	1.5	2.25	1	−3.5	12.25	−2	4	−1.5	2.25
6	1.5	2.25	0	0	1.5	2.25	4	−.5	.25	1	1	−1.5	2.25
Σ:		39.75		24		15.75			41.75		26		15.75

Note: *Dev* = Deviation; *Dev*^{2} = Squared deviation

Sums of squared deviations:

$\Sigma(X - GM)^2$ or $SS_{\text{Total}} = 39.75 + 41.75 = 81.5$

$\Sigma(X - M)^2$ or $SS_{\text{Within}} = 24 + 26 = 50$

$\Sigma(M - GM)^2$ or $SS_{\text{Between}} = 15.75 + 15.75 = 31.5$

Degrees of freedom:

$df_{\text{Total}} = N - 1 = 14 - 1 = 13$

$df_{\text{Within}} = df_1 + df_2 + \ldots + df_{\text{Last}} = 6 + 6 + 12$

$df_{\text{Between}} = N_{\text{Groups}} - 1 = 2 - 1 = 1$

Check $(df_{\text{Total}} - df_{\text{Within}} + df_{\text{Between}})$: $13 - 12 + 1$

Population variance estimates:

S^2_{Total} or $MS_{\text{Total}} = SS_{\text{Total}}/df_{\text{Total}} = 81.5/13 = 6.27$

S^2_{Within} or $MS_{\text{Within}} = SS_{\text{Within}}/df_{\text{Within}} = 50/12 = 4.17$

S^2_{Between} or $MS_{\text{Between}} = SS_{\text{Between}}/df_{\text{Between}} = 31.5/1 = 31.5$

F ratio: $F = S^2_{\text{Between}}/S^2_{\text{Within}}$ or $MS_{\text{Between}}/MS_{\text{Within}} = 31.5/4.17 = 7.55$

$R^2 = \text{eta}^2 = SS_{\text{Between}}/SS_{\text{Total}} = 31.5/81.5 = .39$

reduction in squared error for each score is the squared difference between the mean of that score's group and the overall mean. In analysis of variance, you figure SS_{Between} by adding up, for each participant, the squared differences between the participant's group's mean and the grand mean.

Finally, the proportionate reduction in error in the regression ($r^2 = .39$) comes out to exactly the same as the proportionate reduction in error used as an effect size in analysis of variance (R^2 or eta^2 = .39). Both tell us the proportion of the total variation in the criterion (or measured) variable that is accounted for by its association with the predictor variable (the variable that divides the groups). That these numbers come out the same should be no surprise by now; we have already seen that the numerator and the proportionate reduction in error are the same for both.

Thus, the links between regression and the analysis of variance are quite deep. In fact, some researchers figure the significance of a correlation coefficient by laying it out as a regression analysis and plugging the various sums of squared error into an analysis of variance table and figuring *F*. The result is identical to any other way of figuring the significance of the correlation coefficient. If you figure the *t* for the correlation, it comes out to the square root of the *F* you would get using this procedure.

Analysis of Variance for More Than Two Groups as a Special Case of Multiple Correlation

When considering the *t* test for independent means or the analysis of variance for two groups, we could carry out a correlation or regression analysis by changing the two categories of the nominal variable that divides the groups into any two different numbers (in the example, we used 1 for the experimental group and 2 for the control group). The problem is more difficult with an analysis of variance with more than two groups because the variable that divides the groups has more than two categories.

In the two-category situation, the particular two numbers you use do not matter (except for the sign). However, when there are three or more groups, making up a predictor variable with arbitrary numbers for the different groups will not work. Whatever three numbers you pick imply some particular relation among the groups, and not all relations will be the same. For example, with three groups, making a predictor variable with 1s, 2s, and 3s gives a different result depending on which groups gets put in the middle. It also gives a different result than using 1s, 2s, and 4s.

Recall the example from Chapter 9 comparing ratings of a defendant's degree of guilt for participants who believed the defendant had either a criminal record or a clean record or in which nothing was said about the defendant's record. Suppose that we arbitrarily give a 1 to the first group, a 2 to the second, and a 3 to the third. This would imply that we consider these three levels to be equally spaced values on a numerical variable of knowledge about the criminal record. For this particular example, we might want to think of the three groups as ordered from criminal record to clean record, with the no information group in between. However, even then it would not be clear that the groups are evenly spaced on this dimension.

More generally, when you have several groups, you may have no basis in advance for putting the groups in a particular order, let alone for deciding how they should be spaced. For example, in a study comparing attitudes of four different Central American nationalities, nationality is the nominal variable that divides the groups. But you can't make these four nationalities into any meaningful four values of a single numerical variable.

There is a clever solution to this problem. When there are more than two groups, instead of trying to make the nominal variable that divides the groups into a single numerical variable, you can make it into several numerical predictor variables with two levels each.

Here is how this is done: Suppose that the variable that divides the groups has four categories—for example, four Central American nationalities: Costa Rican, Guatemalan, Nicaraguan, and Salvadoran. You can make one predictor variable for whether the participant is Costa Rican—1 if Costa Rican, 0 if not. You can then make a second predictor variable for whether the participant is Guatemalan, 1 or 0; and a third for whether the participant is Nicaraguan, 1 or 0. You could make a fourth for whether the participant is Salvadoran. However, if a participant has 0s on the first three variables, the participant has to be Salvadoran (because there are only the four possibilities).

Table W4–7 Example of Nominal Coding for Participants of Four Central American Nationalities

Participant	Nationality	Variable 1 Costa Rican or Not	Variable 2 Guatemalan or Not	Variable 3 Nicaraguan or Not
1	Guatemalan	0	1	0
2	Nicaraguan	0	0	1
3	Salvadoran	0	0	0
4	Nicaraguan	0	0	1
5	Costa Rican	1	0	0
6	Costa Rican	1	0	0
7	Salvadoran	0	0	0
8	Nicaraguan	0	0	1
9	Costa Rican	1	0	0
10	Guatemalan	0	1	0

In this example, you know any participant's nationality by the scores on the combination of the three two-value numerical variables. For example, a Costa Rican participant would have a 1 for Costa Rican and 0s for Guatemalan and Nicaraguan. Each Guatemalan participant would have a 1 for Guatemalan but 0s for Costa Rican and Nicaraguan. Each Nicaraguan participant would have 0s for Costa Rican and Guatemalan. Each Salvadoran participant would have 0s on all three variables. (Incidentally, you can use any two numbers for each two-valued nominal variable; we just used 1 and 0 for convenience.) Table W4–7 shows this coding for 10 participants.

Table W4–8 Example of Nominal Coding for the Criminal Record Example

		Predictor Variable		Criterion Variable
Participant	Experimental Condition	Variable 1: Criminal Record or Not	Variable 2: Clean Record or Not	Participant's Rating of Defendant's Guilt
1	Criminal record	1	0	10
2	Criminal record	1	0	7
3	Criminal record	1	0	5
4	Criminal record	1	0	10
5	Criminal record	1	0	8
6	Clean record	0	1	5
7	Clean record	0	1	1
8	Clean record	0	1	3
9	Clean record	0	1	7
10	Clean record	0	1	4
11	No information	0	0	4
12	No information	0	0	6
13	No information	0	0	9
14	No information	0	0	3
15	No information	0	0	3

This entire procedure is called **nominal coding.** The result in this example is that the variable that divides the groups, instead of being a nominal variable with four categories, is now three numerical variables but with only two values each. Creating several two-valued numerical variables in this way avoids the problem of creating an arbitrary ranking and distancing of the four levels.

Table W4–8 shows another example, this time for the criminal record study from Chapters 9 and 10. The variable that divides the groups, instead of being a nominal variable with three categories, is now two numerical variables (each with values of 1 or 0). More generally, you can code the nominal variable that divides the groups in an analysis of variance into several two-value numerical variables, exactly one less such two-valued numerical variables than there groups. (Not coincidentally, this comes out the same as the degrees of freedom for the between-group population variance estimate.)

Once you have done the nominal coding (changed the variable that divides the groups into two-value numerical variables), you then want to know the relation of this set of variables to the measured variable. You do this with multiple regression, using the set of two-value numerical variables as predictors and the measured variable as the criterion variable. Consider again the criminal record example. Having done the nominal coding, you can now figure the multiple regression of the two numerical predictor variables taken together with what you now think of as the criterion variable, rating of guilt. The result (in terms of significance level and R^2) comes out exactly the same as the analysis of variance.

The nominal coding procedure is extremely flexible and can be extended to the most complex factorial analysis of variance situations. In practice, researchers rarely actually do nominal coding—usually, a computer does it for you. We wanted you to see the principle so that you can understand how it is possible to make an analysis of variance problem into a multiple regression problem. There are, however, a number of analysis of variance research situations in which there are advantages to using the multiple regression approach (such as in a factorial analysis with unequal cell sizes). In fact, many analysis of variance computer programs do the actual computations not using the analysis of variance formulas, but by doing nominal coding and multiple regression.

how are you doing?

1. Under what conditions can you use the analysis of variance to find the significance of a bivariate prediction or correlation?
2. When there are only two groups, explain the similarity between the analysis of variance structural model approach and regression in terms of (a) SSTotal, (b) SSWithin and SSError, (c) SSBetween and SSTotal – SSError, and (d) proportionate reduction in error.
3. Based on what you have learned in previous sections, give an argument for why, when there are only two groups, the analysis of variance and correlation should give the same significance.
4. (a) What is nominal coding? (b) How is it done? (c) Why is it done? (d) Why can't you just use a single numeric variable with more than two values? (e) In a particular study, participants 1 and 2 are in Group A, participants 3 and 4 are in Group B, and participants 5 and 6 in Group C. Make a table showing nominal coding for these six participants.

Answers

1. When the predictor variable has only two values.
2. (a) In analysis of variance, SSTotal is the sum of squared deviations of each measured variable score from the grand mean, which is the mean of all measured variable scores; in regression, SSTotal is the sum of squared deviations of each criterion variable score from the mean of all criterion variable scores. The measured variable in analysis of variance is the same as the criterion variable in regression. Thus, for the same study, SSTotal is the same in both.
   (b) SSWithin in analysis of variance is the sum of squared deviations of each measured variable score from the mean of the measured variable scores of its group. SSError in regression is the sum of squared deviations of each criterion variable score from the predicted criterion variable score. The mean of the measured variable scores of a particular group in analysis of variance is exactly what would be the predicted score for the criterion variable in regression if there are only two groups. Thus, for the same study, SSWithin and SSError is the same.
   (c) In analysis of variance, SSTotal = SSBetween + SSWithin. Thus, SSBetween has to equal SSTotal – SSWithin. We have already seen that SSTotal is the same in analysis of variance and regression, and that SSWithin in analysis of variance is the same as SSError in regression. Thus, for the same study, SSBetween and SSTotal – SSError are the same.
   (d) Proportionate reduction in error in analysis of variance is SSBetween/SSTotal. The proportionate reduction in error in regression is (SSTotal – SSError)/SSTotal. We have already seen that the terms that make up these numerators and denominators are the same in analysis of variance and regression. Thus, in the same study, the proportionate reduction in error is the same.
3. In this situation, both the analysis of variance and the significance test of the correlation give the same results as the t test for independent means, thus they must give the same result as each other.
4. **(a)** Changing a nominal variable that divides groups into several two-value numeric variables.
   (b) Participants in the first group are given a 1 on the first two-value numeric variable and a 0 on all others; participants in the second group are given a 1 on the second two-value numeric variable and a 0 on the rest; this continues up to participants in the last group, who are given a 0 on all the two-value numeric variables.
   (c) It allows you to figure an analysis of variance using the two-value numeric variables as predictors in a multiple regression.
   (d) The order of those values and the distance between them would influence the results.
   (e)

Participant	Score on Numeric Variable 1	Score on Numeric Variable 2
1	1	0
2	1	0
3	0	1
4	0	1
5	0	0
6	0	0

Summary

1. The general linear model states that the value of a variable for any individual is the sum of a constant, plus the weighted influence of each of several other variables, plus error. Bivariate and multiple correlation and regression (and associated significance tests), the *t* test, and the analysis of variance are all special cases of the general linear model.
2. Multiple regression is almost identical to the general linear model, and bivariate correlation and regression are the special cases of multiple regression/correlation in which there is only one predictor variable.
3. The *t* test for independent means can be mathematically derived from the analysis of variance. It is a special case of the analysis of variance in which there are only two groups. The *t* score for the same data is the square root of the *F* ratio. The numerators of both *t* and *F* are based on the differences between group means; the denominators of both are based on the variance within the groups; the denominator of *t* involves dividing by the number of participants, and the numerator of *F* involves multiplying by the number of participants; and the *t* degrees of freedom are the same as the *F* denominator degrees of freedom.
4. The *t* test for independent means is also a special case of the significance test for the correlation coefficient. A correlation is about the association of a predictor variable with a criterion variable. In the same way, by showing a difference between group means, the *t* test is about an association of the variable that divides the groups with the measured variable. If you give a score of 1 to each participant in one of the two groups and a 2 to each participant in the other group (or any two different numbers), then figure a correlation of these scores with the measured variable, the significance of that correlation will be the same as the *t* test. Drawing a scatter diagram of these data makes a column of scores for each group, with the regression line passing through the mean of each group. The more the means are different, the greater the proportionate reduction in error over using the grand mean and the greater the *t* score based on a comparison of the two groups' means.
5. The relationship between the analysis of variance and multiple regression parallels the relationship between the *t* test for independent means and the correlation coefficient. The grouping variable in an analysis of variance is like a predictor variable in regression. The measured variable in an analysis of variance is like a criterion variable in regression.
6. The *t* test, analysis of variance, and correlation can all be done as multiple regression. However, conventional practice leads to these procedures being used in different research contexts, as if they were actually different.
7. The regularity view identifies *X* as a cause of *Y* if *X* and *Y* are associated, *X* precedes *Y,* and no other third factors precede *X* that could cause them both. The generative view argues that in addition there must be a clear understanding of the mechanism by which *X* affects *Y.*
8. ADVANCED TOPIC: The analysis of variance and regression also have many similarities. SS_{Total} in regression and in the analysis of variance are both about the deviations of each score from the mean of all the criterion or measured variable scores. The group means in an analysis of variance are the predicted scores for each individual in regression; thus, SS_{Error} and SS_{Within} are the same. The reduction in squared error ($SS_{\text{Total}} - SS_{\text{Error}}$) in regression is the same as the sum of squared deviations of scores' group's means from the grand mean (SS_{Between}) in the analysis of variance. Finally, regression's proportionate reduction in error (r^2

or R^2) is the same as the proportion of variance accounted for (R^2 or eta^2) effect size in analysis of variance.

9. ADVANCED TOPIC: An analysis of variance can be set up as a multiple regression using nominal coding to make the categories for the different groups into two-value numerical variables. The analysis of variance is a special case of multiple regression in which the predictor variables are set up in this way.

Key Term

nominal coding

Practice Problems

These problems involve figuring. Most real-life statistics problems are done on a computer with special statistical software. Even if you have such software, do these problems by hand to ingrain the method in your mind. To learn how to use a computer to solve statistics problems like those in this chapter, refer to the Using SPSS section at the end of this chapter and the *Student's Study Guide and Computer Workbook* that accompanies this text.

All data are fictional unless an actual citation is given.

SET I (for Answers to Set I problems, see the end of this Web chapter)

1. (a) Look up and write down the *t* cutoff at the .05 level (two-tailed) for 5, 10, 15, and 20 degrees of freedom. (b) Square each *t* cutoff and write it down next to the *t*. (c) Look up and write down, next to the squared *t*s, the cutoffs for *F* distributions with 1 degree of freedom in the numerator and 5, 10, 15, and 20 degrees of freedom as the denominators. (The results should be identical, within rounding error.)

2. Below are two data sets. For the first data set, in addition to the means and estimated population variances, we have shown the *t* test information. You should figure the second yourself. Also, for each, figure a one-way analysis of variance using the regular Chapter 9 method (not the structural model approach shown in the Advanced Topic section of that chapter). Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio. (Use the .05 level throughout; *t* tests are two-tailed.)

	Experimental Group			Control Group			t test			
	N	M	S^2	N	M	S^2	df	t needed		t
(i)	36	100	40	36	104	48	70	1.995	44	2.56
(ii)	16	73	8	16	75	6				

3. Below is a data set from practice problem 3 in Chapter 8. If you did not figure the *t* test for this problem with Chapter 8, do so now. Then, also figure a one-way analysis of variance using the regular Chapter 9 method (not the structural model approach shown in the Advanced Topic section of that chapter). Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio.

Experimental Group			Control Group		
N	*M*	S^2	*N*	*M*	S^2
30	12.0	2.4	30	11.1	2.8

4. Group A includes 10 people whose scores have a mean of 170 and a population variance estimate of 48. Group B also includes 10 people: $M = 150$, $S^2 = 32$. Carry out a *t* test for independent means (two-tailed) and an analysis of variance (using the regular Chapter 9 method, not the structural model approach shown in the Advanced Topic section of that chapter). Do your figuring on the two halves of the same page, with parallel computations next to each other. (That is, make a table similar in layout to the lower part of Table W4–2.) Use the .05 level for both.

5. Do the following for the scores in practice problems (a) 6, (b) 7, and (c) 8: (i) Figure a *t* test for independent means, (ii) figure the correlation coefficient (between the group that participants are in and their scores on the measured variable), (iii) figure the *t* for significance of the correlation coefficient (using the formula *t* =) and note explicitly the similarity of results, and (iv) make a scatter diagram. For (a), also (v) explain the relation of the spread of the means and the spread of the scores around the means to the *t* test result.

6. ADVANCED TOPIC: For the scores listed below, figure a *t* test for independent means (two-tailed) if you have not already done so and then figure an analysis of variance using the structural model approach from Chapter 9 (use the .05 level for both). Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio.

Group A	Group B
13	11
16	7
19	9
18	
19	

7. ADVANCED TOPIC: Below we list scores from practice problem 5 in Chapter 8. If you did not figure the *t* test for these with Chapter 8, do so now, using the .05 level, two-tailed. Then figure a one-way analysis of variance (also .05 level) using the structural model method from Chapter 9. Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio.

Ordinary Story		Own-Name Story	
Student	*Reading Time*	*Student*	*Reading Time*
A	2	G	4
B	5	H	16
C	7	I	11
D	9	J	9
E	6	K	8

F 7

8. ADVANCED TOPIC: For the scores listed below, figure a *t* test for independent means if you have not already done so and then figure an analysis of variance using the structural model approach from Chapter 9. Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio. (Use the .05 level throughout; the *t* test is two-tailed.)

Group A	Group B
.7	.6
.9	.4
.8	.2

9. ADVANCED TOPIC: Do the following for the scores in practice problems (a) 6, (b) 7, and (c) 8: (i) Figure the analysis of variance using the structural model approach from Chapter 9 if you have not done so already; (ii) figure the proportionate reduction in error based on the analysis of variance results; (iii) carry out a regression analysis (predicting the measured variable score from the group that participants are in); (iv) figure the proportionate reduction in error using the long method of figuring predicted scores, and finding the average squared error using them; and (v) make a chart showing the parallels in the results; for (a), also (vi) explain the major similarities. (Use the .05 level throughout.)

10. ADVANCED TOPIC: Participants 1, 2, and 3 are in Group I; participants 4 and 5 are in Group II; participants 6, 7, and 8 are in Group III; and participants 9 and 10 are in Group IV. Make a table showing nominal coding for these ten participants.

SET II

11. (a) Look up and write down the *F* cutoff at the .01 level for distributions with 1 degree of freedom in the numerator and 10, 20, 30, and 60 degrees of freedom in the denominator. (b) Take the square root of each and write it down next to it. (c) Look up the cutoffs on the *t* distribution at the .01 level (two-tailed) using 10, 20, 30, and 60 degrees of freedom, and write it down next to the corresponding *F* square root. (The results should be identical, within rounding error.)

12. Below are three data sets. For the first two data sets, in addition to the means and estimated population variances, we have shown the *t* test information. You should figure the third yourself. Also, for each, figure a one-way analysis of variance using the regular Chapter 9 method (not the structural model approach shown in the Advanced Topic section of that chapter). Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio. (Use the .01 level throughout; *t* tests are two-tailed.)

	Experimental Group			Control Group			*t* test			
	N	*M*	S^2	*N*	*M*	S^2	*df*	*t* needed		*t*
(i)	20	10	3	20	12	2	38	2.724	2.5	4
(ii)	25	7.54	25	4.52	48	2.690	3.0	6.12		
(iii)	10	48	8	10	55	4				

13. Below we list scores from two data sets, both from practice problem 16 in Chapter 8. If you did not figure the *t* tests for these with Chapter 8, do so now, this time using the .01 level, two-tailed. Then, for each, also figure a one-way analysis of variance (also .01 level) using the regular Chapter 9 method (not the structural model approach shown in the Advanced Topic section of that chapter). Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio.

	Experimental Group			Control Group		
	N	*M*	S^2	*N*	*M*	S^2
(i)	10	604	60	10	607	50
(ii)	40	604	60	40	607	50
(iii)	10	604	20	10	607	16

14. Group I consists of 12 people whose scores have a mean of 15.5 and a population variance estimate of 4.5. Group B also consists of 12 people: $M = 18.3$, $S^2 = 3.5$. Carry out a *t* test for independent means (two-tailed) and an analysis of variance (using the regular Chapter 9 method, not the structural model approach shown in the Advanced Topic section of that chapter), figuring the two on two halves of the same page, with parallel computations next to each other. (That is, make a table similar in layout to the lower part of Table W4–2.) Use the .05 level.

15. Do the following for the scores in practice problems (a) 16, (b) 17, and (c) 18: (i) Figure a *t* test for independent means, (ii) figure the correlation coefficient (between the group that participants are in and their scores on the measured variable), (iii) figure the *t* for significance of the correlation coefficient (using the formula *t* =) and note explicitly the similarity of results, (iv) make a scatter diagram, and (v) explain the relation of the spread of the means and the spread of the scores around the means to the *t* test results.

16. ADVANCED TOPIC: For the scores listed below, carry out a *t* test for independent means (two-tailed) if you have not already done so and an analysis of variance using the structural model method from Chapter 9. (Use the .05 level for both.) Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio.

Group A	Group B
0	4
1	5
0	6
	5

17. ADVANCED TOPIC: For the scores below, figure a *t* test for independent means if you have not already done so (.05 level, two-tailed) and an analysis of variance (.05 level) using the structural model method from Chapter 9. Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio.

Group A	Group B
0	0
0	0
0	0
0	0
0	0
0	0
0	1
0	1
0	1

0	1
0	1
0	1
1	1
1	1
1	1
1	1

18. ADVANCED TOPIC: Below we list scores from practice problem 17 in Chapter 8. If you did not figure the *t* test for these with Chapter 8 (or for practice problem 17 in this chapter), do so now, using the .05 level, two-tailed. Then figure a one-way analysis of variance (also .05 level) using the structural model method from Chapter 9. Make a chart of the similarities of (a) *t df* to *F* denominator *df,* (b) *t* cutoff to square root of *F* cutoff, (c) to , and (d) the *t* score to the square root of the *F* ratio.

Big Meal Group		Small Meal Group	
Subject	*Hearing*	*Subject*	*Hearing*
A	22	D	19
B	25	E	23
C	25	F	21

19. ADVANCED TOPIC: Do the following for the scores in practice problems (a) 16, (b) 17, and (c) 18: (i) Figure the analysis of variance using the structural model approach from Chapter 9 if you have not already done so; (ii) figure the proportionate reduction in error based on the analysis of variance results; (iii) carry out a regression analysis (predicting the measured variable score from the group that participants are in); (iv) figure the proportionate reduction in error using the long method of figuring predicted scores, and finding the average squared error using them; and (v) make a chart showing the parallels in the results.

20. ADVANCED TOPIC: Participants 1 and 2 are in Group A; participants 3, 4, 5, and 6 are in Group B; and participants 7, 8, and 9 are in Group C. Make a table showing nominal coding for these nine participants.

The 8 in the following steps indicates a mouse click. (We used SPSS version 15.0 for Windows to carry out these analyses. The steps and output may be slightly different for other versions of SPSS.)

For each SPSS analysis below, we use the scores from the Exampled Worked-Out Problem for the *t* test for independent means from Chapter 8. This is also the main example we used in this chapter (see Tables W4–2 and W4–4, and also Table W4–6 if you read this chapter's Advanced Topic section). First, we use SPSS to figure a *t* test for independent means for the example. We compare the results of this *t* test with the results of a one-way analysis of variance. Next, we figure the correlation coefficient for the example and compare it with the results for the *t* test. Finally, in an Advanced Topic Section, we figure the bivariate prediction (regression) for the example and compare the results to the analysis of variance results.

For the results of each test below, we highlight the most important parts of the SPSS output. For additional information on the SPSS steps for each test and a more detailed description of the SPSS output, see the Using SPSS sections in the relevant chapters (Chapters 8, 9, 11, and 12).

t TEST FOR INDEPENDENT MEANS

❼ Enter the scores into SPSS as shown in Figure W4–4. In the first column (labeled “group”), we used the number “1” to indicate that a person is in the experimental group and the number “2” to indicate that a person is in the control group.

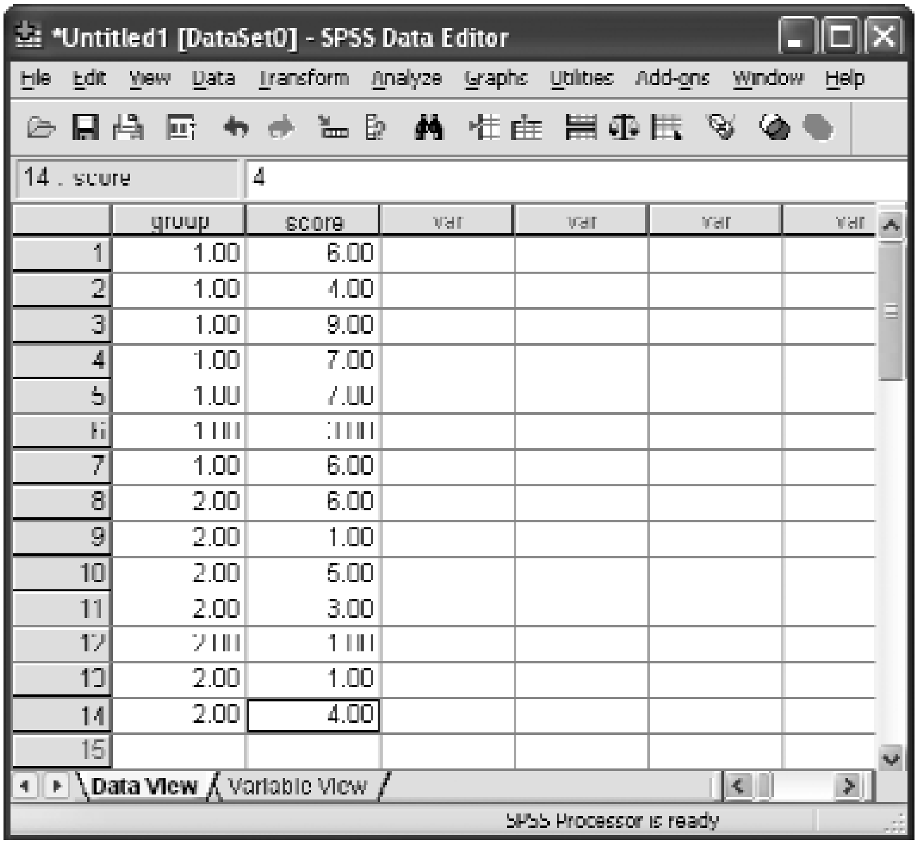


FIGURE W4–4 *SPSS data editor window for the Web Chapter W4 Using SPSS examples, showing scores on an equal-interval variable for individuals in an experimental and a control group.*

➡ ✐ *Analyze*.

➪ ✐ *Compare means*.

➶ ✐ *Independent-Samples T Test*.

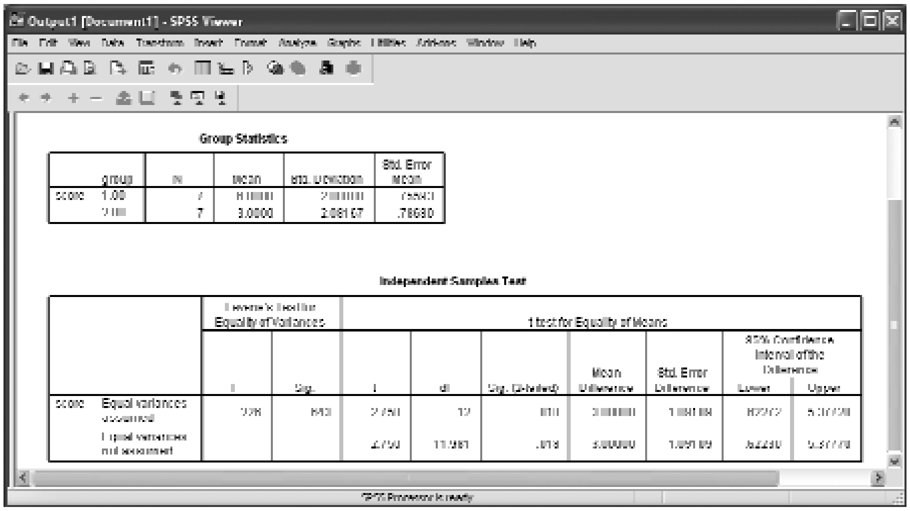
➶ ✐ on the variable called “score” and then 8 the arrow next to the box labeled “Test Variable(s)”.

✐ the variable called “group” and then 8 the arrow next to the the box labeled “Grouping Variable.” 8 *Define Groups*. Put “1” in the Group 1 box and put “2” in the Group 2 box. 8 *Continue*.

✐ *OK*. Your SPSS output window should look like Figure W4–5.

The first table in the SPSS output provides information about the two variables (see Chapter 8 for a detailed description of this information). The second table shows the actual results of the *t* test for independent means. Note the *t* value of 2.750 in the SPSS output in Figure W4–5 is consistent (within rounding error) with the value of *t* (of 2.73) shown in Table W4–2 earlier in the chapter. The result of the *t* test is statistically significant, as the significance level of .018 is less than our .05 cutoff significance level.

FIGURE W4–5 *SPSS output window for a t test for independent means using the scores shown in Figure W4–4.*



One-way analysis of variance

We will carry out the analysis of variance using the same set of scores as shown in Figure W4–4.

❼ ✍ *Analyze*.

➡ ✍ *Compare means*.

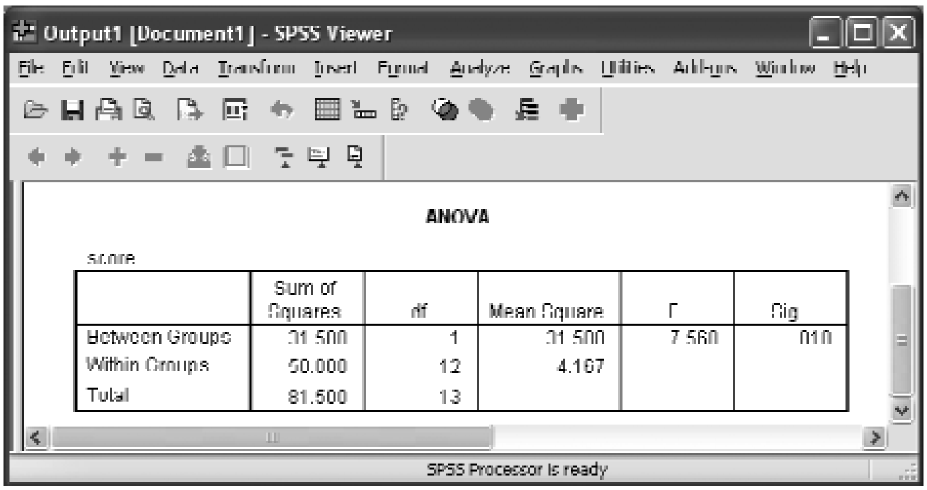
➪ ✍ *One-Way ANOVA*.

➶ ✍ on the variable called "score" and then 8 the arrow next to the box labeled "Dependent List".

➶ ✍ the variable called "group" and then 8 the arrow next to the the box labeled "Factor."

✍ *OK*. Your SPSS output window should look like Figure W4–6.

FIGURE W4–6 *SPSS output window for a one-way analysis of variance using the scores shown in Figure W4–4.*



Note the *F* value of 7.560 in the SPSS output in Figure W4–6 is consistent (within rounding error) with the value of *F* (of 7.55) shown in Table W4–2. Also, note that if we figure the square root of the *F* value of 7.560 from the SPSS output, the result is 2.750. As we would expect, this is exactly the same value as the value of *t* from the SPSS output shown in Figure W4–5. Notice also that the *F* test is statistically significant, as the significance level of .018 is less than our .05 cutoff significance level. The fact that the square root of the *F* value from this analysis of variance is exactly the same as the *t* value from the *t* test for independent means, and the fact that the significance levels of both tests were exactly the same (.018), show that the *t* test is a special case of the analysis of variance.

Finding The Correlation Coefficient

We will find the correlation coefficient using the same set of scores as shown in Figure W4–4.

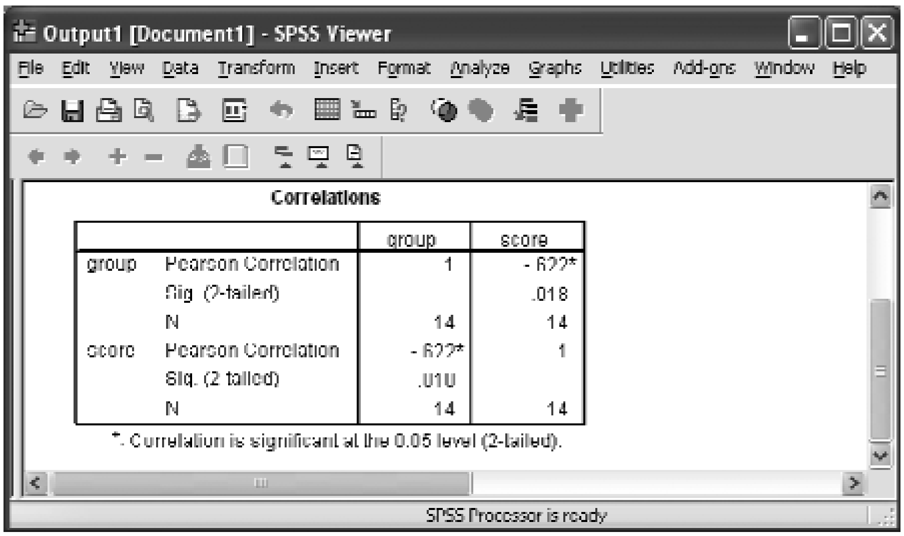
❼ *Analyze*

➡ *Correlate*

➡ *Bivariate*

➚ on the variable called "group" and then 8 the arrow next to the box labeled "Variables". 8 on the variable called "score" and then 8 the arrow next to the box labeled "Variables".

➚ *OK*. Your SPSS output window should look like Figure W4–7.



		group	score
group	Pearson Correlation	1	-.622*
	Sig. (2-tailed)		.018
	N	14	14
score	Pearson Correlation	-.622*	1
	Sig. (2-tailed)	.018	
	N	14	14

*. Correlation is significant at the 0.05 level (2-tailed).

FIGURE W4–7 *SPSS output window for a correlation coefficient using the scores shown in Figure W4–4.*

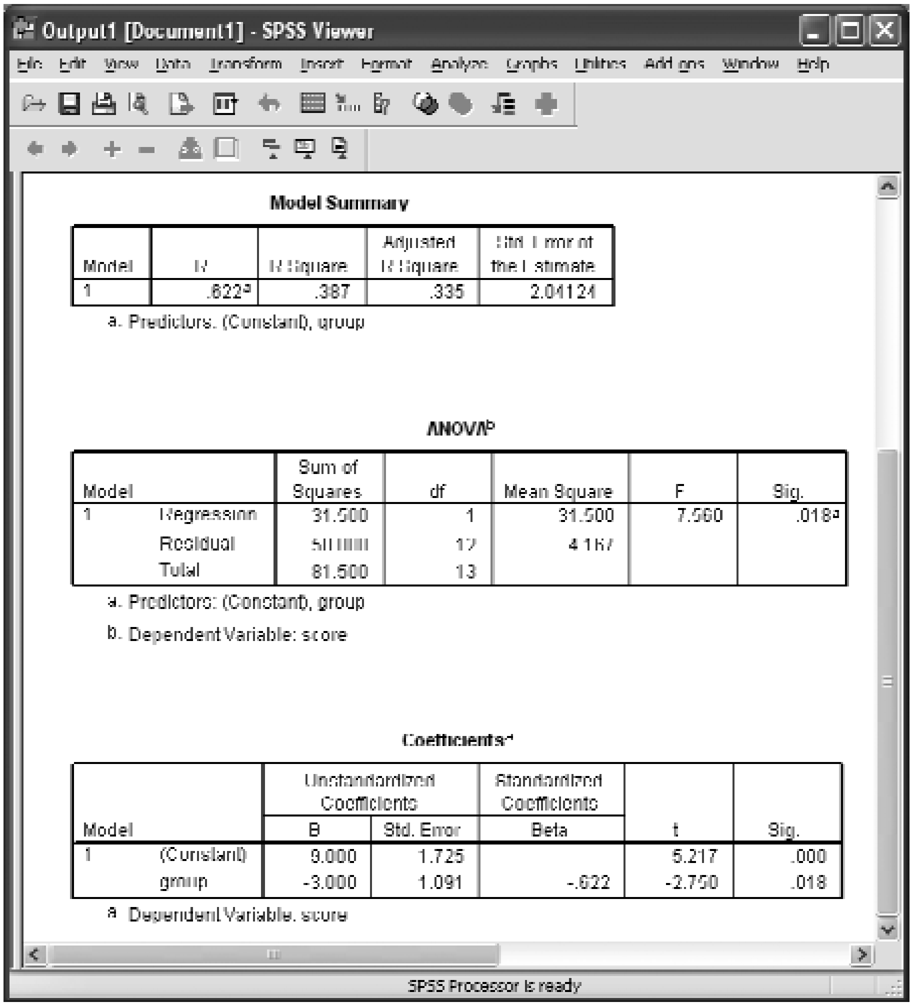
Note that the correlation coefficient (*r*) of –.622 shown in the SPSS output in Figure W4–7 is consistent with the correlation coefficient of –.62 shown in Table W4–4 earlier in the chapter. As with the *t* test (and analysis of variance), the correlation coefficient is statistically significant, as the significance level of .018 is less than our .05 cutoff level. Again, the .018 significance level is identical to the .018 significance level found for the *t* test (and the analysis of variance) SPSS output. This demonstrates that the *t* test is a special case of the significance test for the correlation coefficient.

Advanced Topic: Bivariate Prediction

We will figure the bivariate prediction using the same set of scores as shown in Figure W4–4, using "group" as the predictor variable and "score" as the criterion variable.

- *Analyze*.
- *Regression*. 8 *Linear*.
- the variable called "score" and then 8 the arrow next to the the box labeled "Dependent". 8 the variable called "group" and then 8 the arrow next to the the box labeled "Independent(s)".
- *OK*. Your SPSS output window should look like Figure W4–8.

FIGURE W4–8 *SPSS output window for a bivariate prediction using the scores shown in Figure W4–4.*



Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.622ª	.387	.335	2.04124

a. Predictors: (Constant), group

ANOVAᵇ

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	31.500	1	31.500	7.560	.018ª
	Residual	50.000	12	4.167		
	Total	81.500	13			

a. Predictors: (Constant), group

b. Dependent Variable: score

Coefficientsª

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	9.000	1.725		5.217	.000
	group	-3.000	1.091	-.622	-2.750	.018

a. Dependent Variable: score

Note that the values of SS_{Error}, SS_{Total}, R Square, and R in the model summary table of the SPSS output in Figure W4–8 are the same as the equivalent values in Table W4–5 earlier in the chapter. Notice also that the values in the "ANOVA" table for the bivariate prediction shown in Figure W4–8 are identical to the values in the "ANOVA" table for the one-way analysis of variance shown in Figure W4–6. (The only differences between the two "ANOVA" tables is in their terminology: The "Regression Sums of Squares" and "Residual Sums of Squares" for the table for bivariate prediction in Figure W4–8 are called "Between Groups Sums of Squares" and "Within Groups Sums of Squares" for one-way analysis of variance in Figure W4–6.) This shows that analysis of variance is a special case of prediction (regression). This particular example shows the equivalence of analysis of variance and bivariate prediction, which is an example of the more general principle that analysis of variance is a special case of multiple regression.

Overall, the series of analyses in this Using SPSS section show that *t* tests, analysis of variance, correlation, and regression (bivariate prediction and multiple regression) are all based on the same underlying formula provided by the general linear model. Your knowledge and understanding of this concept will provide a solid foundation for learning additional statistical procedures in intermediate and advanced statistics courses.

ANSWERS TO SET I PRACTICE PROBLEMS

1.

df		5	10	15	20
(a)	t	2.571	2.228	2.132	2.086
(b)	r^2	6.61	4.96	4.55	4.35
(c)	F	6.61	4.97	4.54	4.35

2. (i)

	(a)	(b)	(c)	(d)
	df	Cutoff	Within-Group Variance	*t* or *F*
t	70	1.995	$S^2_{\text{Pooled}} = 44$	2.56
F	70	3.980	$S^2_{\text{Within}} = 44$	6.55
		$(\surd = 1.995)$		$(\surd = 2.56)$

(ii)

	(a)	(b)	(c)	(d)
	df	Cutoff	Within-Group Variance	t or F
t	30	2.043	$S^2_{\text{Pooled}} = 7$	−2.13
F	30	4.170	$S^2_{\text{Within}} = 7$	−4.57
		$(\surd = 2.042)$		$(\surd = 2.14)$

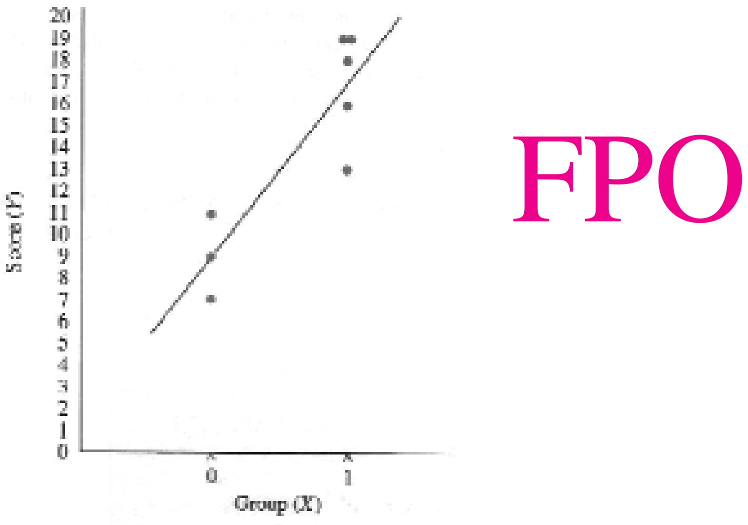
3.

	(a)	(b)	(c)	(d)
	df	Cutoff	Within-Group Variance	*t* or *F*
t	58	2.004	$S^2_{\text{Pooled}} = 2.6$	2.16
F	58	4.020	$S^2_{\text{Within}} = 2.6$	4.67
		$(\surd = 2.005)$		$(\surd = 2.16)$

***t*-Test**	**ANOVA**
Numerator	
Mean difference $= 170 - 150$ $= 20$	$GM = (170 + 150)/2$ $= 160$ $\Sigma(M - GM)^2 = (170 - 160)^2$ $= (150 - 160)^2$ $= 200$ $S^2_{\text{Between}} = \dfrac{\Sigma(M - GM)^2}{df_{\text{Between}}}(n)$ $= (200/1)(10)$ $= 2{,}000$
Denominator	
$S^2_{\text{Pooled}} = (df/df_{\text{Total}})(S^2_1)$ $+ (df_2/df_{\text{Total}})(S^2_2)$ $= (.5)(48) + (.5)(32)$ $= 40$ $S^2_{\text{Difference}}$ $= (S^2_{M1} + S^2_{M2}$ $= (S^2_{\text{Pooled}}/N_1) + S^2_{\text{Pooled}}/N_2)$ $= (40/10) + (40/10) = 8$ $S^2_{\text{Difference}}$ $= \sqrt{S^2_{\text{Difference}}}(S^2$ $= \sqrt{8} = 2.83.$	S^2_{Within} *or* $MS_{\text{Within}} =$ $(S^2_1 + S^2_2 + \ldots + S^2_{\text{Last}})/(N_{\text{Groups}})$ $= (48 + 32)/2 = 40$
Degrees of Freedom	
$df_{Total} = df_1 + df_2 = 9 + 9 = 18$	$df_{\text{Within}}\ df_1 + df_2 + \ldots + df_{Last} = 9 + 9 = 18$
Cutoff	
Needed t ($df = 18$, $p < .05$,) two-tailed): 2.101	Needed t ($df = 1, 18$; $p < .05$): 4.41 $(\sqrt{\ } = 2.1)$
Score on Comparison Distribution	
$t = (M_1 - M_2)/S_{\text{Difference}}$ $= 20/2.83$ $= 7.07$	$F = S^2_{\text{Between}}/S^2_{\text{Within}}$ or $MS_{\text{Between}}/MS_{\text{Within}}$ $= 2{,}000/40 = 50\ \sqrt{\ } = 7.07$
Conclusions	
Reject the null hypothesis.	Reject the null hypothesis.

5. (a)(i) $t = 4.60$ (see question 6 below); (ii) $r = 15/16.99 = .88$; (iii) $t =$ (result is same as 4.60 within rounding error).

(iv)



(v) A t test is the difference between the means divided by the standard deviation of the distribution of differences between means. So, the larger the difference between the means, the larger the t test result. The standard deviation of the distribution of differences between means is largely based on the variances in each sample, and the variance in each sample is an indication of how spread out the scores in each sample are around the mean. So, the smaller the variance in each sample (that is, the closer the scores are to the mean of the sample), the larger the t test result (since you will be dividing by a smaller number when figuring the value of t).

(b)(i) $t = -1.73$ (see question 7 below); (ii) $r = -.50$; (iii) $t = -1.73$; (iv) similar to 8 (a)(iv) above.

(c)(i) $t = 3.1$ (see question 8 below); (ii) $r = .84$; (iii) $t = 3.10$; (iv) similar to 8 (a)(iv) above.

6.

	(a)	(b)	(c)	(d)
	df	**Cutoff**	**Within-Group Variance**	***t* or *F***
t	6	2.447	$S^2_{\text{Pooled}} = 5.67$	4.60
F	6	5.99	$MS^2_{\text{Within}} = 5.67$	21.16

7

	(a)	(b)	(c)	(d)
	df	**Cutoff**	**Within-Group Variance**	***t* or *F***
t	9	2.262	$S^2_{\text{Pooled}} = 11.69$	−1.73
F	9	5.99	$MS_{\text{Within}} = 11.69$	3.02

8.

	(a)	(b)	(c)	(d)
	df	Cutoff	Within-Group Variance	*t* or *F*
t	4	2.776	$S^2_{Pooled} = 0.25$	3.1
F	4	7.71	$MS_{Within} = 0.25$	9.6

9. (a)(v)

Regression	ANOVA
Mean of $Y = 14$	Grand Mean = 14
$SS_{Total} = 154$	$SS_{Total} = 154$
Predicted Y for Group A = 17	Mean of Group A = 17
Predicted Y for Group B = 9	Mean of Group A = 9
$SS_{Error} = 34$	$SS_{Within} = 34$
Reduction in Error = 120	$SS_{Between} = 120$

(vi) You can think of the analysis of variance as about the relationship between the variable on which the groups differ (Group A versus Group B in this problem) and the measured variable. If you think of the variable on which the groups differ as a predictor variable, regression is also about the same thing. In this problem you can consider those in Group A to have a score of 2 on this predictor and those in Group B to have a score of 1 on it. (Any two numbers would do; these are just examples.) In fact, the underlying mathematics is the same. Here are some of the parallels. In both regression and the analysis of variance, you figure the total squared deviations from the overall mean (in both, this is SS_{Total}). A deeper link is that the best predictor for those in either group is the group's mean, so the linear prediction rule predicts the mean for each group. The result is that the errors of predictions are deviations of the scores from the mean. If you square these and add them up, they are called SS_{Error} in regression and SS_{Within} in ANOVA. In regression, before figuring the proportionate reduction in error, you figure the reduction in error ($SS_{Total} - SS_{Error}$)—the amount of squared error that the prediction rule saves over predicting from the overall mean of the criterion variable. This is the same as $SS_{Between}$ in ANOVA, because when there are only two group means, regression improves on prediction only to the extent that the means of the two groups are different. Finally, because SS_{Total} is the same in both regression and analysis of variance, and because reduction in error = $SS_{Between}$, r^2 in regression has to come out the same as R^2 figured as an effect size in ANOVA.

(b)

Regression	ANOVA
Mean reading time = 7.64	Grand Mean = 7.64
$SS_{\text{Total}} = 140.55$	$SS_{\text{Total}} = 140.55$
Predicted reading time ordinary story = 6	Mean reading time ordinary story = 6
Predicted reading time own − name story = 9.6	Mean reading time own − name story = 9.6
$SS_{\text{Error}} = 105.2$	$SS_{\text{Within}} = 105.2$
Reduction in Error = 35.35	$SS_{\text{Between}} = 35.35$

(c)

Regression	ANOVA
Mean of criterion variable = .6	Grand Mean = .6
$SS_{\text{Total}} = .34$	$SS_{\text{Total}} = .34$
Predicted score for Group A = .8	Mean of Group A = .8
Predicted score for Group B = .4	Mean of Group A = .4
$SS_{\text{Error}} = .1$	$SS_{\text{Within}} = .1$
Reduction in Error = .24	$SS_{\text{Between}} = .24$

10.

Participant	Group	Variable 1 (Group I or not)	Variable 2 (Group II or not)	Variable 3 (Group III or not)
1	I	1	0	0
2	I	1	0	0
3	I	1	0	0
4	II	0	1	0
5	II	0	1	0
6	III	0	0	1
7	III	0	0	1
8	III	0	0	1
9	IV	0	0	0
10	IV	0	0	0

1. In this chapter, we focus on the *t* test for independent means (and also the analysis of variance for between-subject designs). However, the conclusions are all the same for the *t* test for dependent means. It is a special case of the repeated-measures analysis of variance. Also, both the *t* test for dependent means and the repeated-measures

analysis of variance are special cases of multiple regression/correlation. However, the link between these methods and multiple regression involves some extra steps of logic that we do not consider here to keep the chapter focused on the main ideas.

2. Other apparent differences (such as the seeming difference that the *F*-ratio numerator is based on a variance estimate and the *t* score numerator is a simple difference between means) are also actually the same when you go into them in detail.
3. Another reason for the use of different procedures is that the *t* test and analysis of variance have traditionally been used to analyze results of true experiments with random assignment to levels of the variables that divide the groups, while correlation and regression have been used mainly to analyze results of studies in which the predictor variable was measured in people as it exists, what is called a correlational research design. Thus, using a correlation or regression approach to analyze a true experiment, while correct, might imply to the not-very-careful reader that the study was not a true experiment.