Quantitative Module

Transportation Models

Module Outline

TRANSPORTATION MODELING
DEVELOPING AN INITIAL SOLUTION
   The Northwest-Corner Rule
   The Intuitive Lowest-Cost Method
THE STEPPING-STONE METHOD
SPECIAL ISSUES IN MODELING
   Demand Not Equal to Supply
   Degeneracy
SUMMARY
KEY TERMS

LEARNING OBJECTIVES

When you complete this module you should be able to

IDENTIFY OR DEFINE:
   Transportation modeling
   Facility location analysis

EXPLAIN OR BE ABLE TO USE:
   Northwest-corner rule
   Stepping-stone method

USING SOFTWARE TO SOLVE TRANSPORTATION PROBLEMS
SOLVED PROBLEMS
INTERNET AND STUDENT CD-ROM EXERCISES
DISCUSSION QUESTIONS
PROBLEMS
INTERNET HOMEWORK PROBLEMS
CASE STUDY: CUSTOM VANS, INC.
ADDITIONAL CASE STUDIES
BIBLIOGRAPHY
The problem facing rental companies like Avis, Hertz, and National is cross-country travel. Lots of it. Cars rented in New York end up in Chicago, cars from L.A. come to Philadelphia, and cars from Boston come to Miami. The scene is repeated in over 100 cities around the U.S. As a result, there are too many cars in some cities and too few in others. Operations managers have to decide how many of these rentals should be trucked (by costly auto carriers) from each city with excess capacity to each city that needs more rentals. The process requires quick action for the most economical routing; so rental car companies turn to transportation modeling.

Because location of a new factory, warehouse, or distribution center is a strategic issue with substantial cost implications, most companies consider and evaluate several locations. With a wide variety of objective and subjective factors to be considered, rational decisions are aided by a number of techniques. One of those techniques is transportation modeling.

The transportation models described in this module prove useful when considering alternative facility locations within the framework of an existing distribution system. Each new potential plant, warehouse, or distribution center will require a different allocation of shipments, depending on its own production and shipping costs and the costs of each existing facility. The choice of a new location depends on which will yield the minimum cost for the entire system.
that the 300 units required by Arizona Plumbing’s Albuquerque warehouse may be shipped in various combinations from its Des Moines, Evansville, and Fort Lauderdale factories.

The first step in the modeling process is to set up a transportation matrix. Its purpose is to summarize all relevant data and to keep track of algorithm computations. Using the information displayed in Figure C.1 and Table C.1, we can construct a transportation matrix as shown in Figure C.2.

**DEVELOPING AN INITIAL SOLUTION**

Once the data are arranged in tabular form, we must establish an initial feasible solution to the problem. A number of different methods have been developed for this step. We now discuss two of them, the northwest-corner rule and the intuitive lowest-cost method.
The Northwest-Corner Rule

The northwest-corner rule requires that we start in the upper left-hand cell (or northwest corner) of the table and allocate units to shipping routes as follows:

1. Exhaust the supply (factory capacity) of each row (e.g., Des Moines: 100) before moving down to the next row.
2. Exhaust the (warehouse) requirements of each column (e.g., Albuquerque: 300) before moving to the next column on the right.
3. Check to ensure that all supplies and demands are met.

Example C1 applies the northwest-corner rule to our Arizona Plumbing problem.

In Figure C.3 we use the northwest-corner rule to find an initial feasible solution to the Arizona Plumbing problem. To make our initial shipping assignments, we need five steps:

1. Assign 100 tubs from Des Moines to Albuquerque (exhausting Des Moines’s supply).
2. Assign 200 tubs from Evansville to Albuquerque (exhausting Albuquerque’s demand).
3. Assign 100 tubs from Evansville to Boston (exhausting Evansville’s supply).
4. Assign 100 tubs from Fort Lauderdale to Boston (exhausting Boston’s demand).
5. Assign 200 tubs from Fort Lauderdale to Cleveland (exhausting Cleveland’s demand and Fort Lauderdale’s supply).

The total cost of this shipping assignment is $4,200 (see Table C.2).

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>(A) Albuquerque</th>
<th>(B) Boston</th>
<th>(C) Cleveland</th>
<th>Factory capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Des Moines</td>
<td>(D)</td>
<td>100</td>
<td>$5</td>
<td>$4</td>
<td>$3</td>
</tr>
<tr>
<td>Evansville</td>
<td>(E)</td>
<td>200</td>
<td>$8</td>
<td>100</td>
<td>$3</td>
</tr>
<tr>
<td>Fort Lauderdale</td>
<td>(F)</td>
<td>$9</td>
<td>100</td>
<td>$7</td>
<td>$5</td>
</tr>
<tr>
<td>Warehouse requirement</td>
<td>300</td>
<td>200</td>
<td>200</td>
<td>700</td>
<td></td>
</tr>
</tbody>
</table>

Means that the firm is shipping 100 bathtubs from Fort Lauderdale to Boston.

Measures the cost per unit from the remaining (not crossed out) cells.

The solution given is feasible because it satisfies all demand and supply constraints.

The Intuitive Lowest-Cost Method

The intuitive method makes initial allocations based on lowest cost. This straightforward approach uses the following steps:

1. Identify the cell with the lowest cost. Break any tie for the lowest cost arbitrarily.
2. Allocate as many units as possible to that cell without exceeding the supply or demand. Then cross out that row or column (or both) that is exhausted by this assignment.
3. Find the cell with the lowest cost from the remaining (not crossed out) cells.
4. Repeat steps 2 and 3 until all units have been allocated.
### THE STEPPING-STONE METHOD

The **stepping-stone method** will help us move from an initial feasible solution to an optimal solution. It is used to evaluate the cost effectiveness of shipping goods via transportation routes not currently in the solution. When applying it, we test each unused cell, or square, in the transportation table by asking: What would happen to total shipping costs if one unit of the product (for example, one bathtub) was tentatively shipped on an unused route? We conduct the test as follows:

1. Select any unused square to evaluate.
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used (only horizontal and vertical moves are permissible). You may, however, step over either an empty or an occupied square.
3. Beginning with a plus (+) sign at the unused square, place alternating minus signs and plus signs on each corner square of the closed path just traced.
4. Calculate an improvement index by first adding the unit-cost figures found in each square containing a plus sign and then by subtracting the unit costs in each square containing a minus sign.
5. Repeat steps 1 through 4 until you have calculated an improvement index for all unused squares. If all indices computed are greater than or equal to zero, you have reached an optimal solution. If not, the current solution can be improved further to decrease total shipping costs.

Example C3 illustrates how to use the stepping-stone method to move toward an optimal solution. We begin with the northwest-corner initial solution developed in Example 1.

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**Example C2**

The intuitive lowest-cost approach

When we use the intuitive approach on the data in Figure C.2 (rather than the northwest-corner rule) for our starting position we obtain the solution seen in Figure C.4.

The total cost of this approach = $3(100) + $3(100) + $4(200) + $9(300) = $4,100.

(D to C) (E to C) (E to B) (F to A)

While the likelihood of a minimum-cost solution does improve with the intuitive method, we would have been fortunate if the intuitive solution yielded the minimum cost. In this case, as in the northwest-corner solution, it did not. Because the northwest-corner and the intuitive lowest-cost approaches are meant only to provide us with a starting point, we often will have to employ an additional procedure to reach an optimal solution.

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**Example C3**

Checking unused routes with stepping stone

We can apply the stepping-stone method to the Arizona Plumbing data in Figure C.3 (see Example 1) to evaluate unused shipping routes. As you can see, the four currently unassigned routes are Des Moines to Boston, Des Moines to Cleveland, Evansville to Cleveland, and Fort Lauderdale to Albuquerque.

**Steps 1 and 2.** Beginning with the Des Moines–Boston route, first trace a closed path using only currently occupied squares (see Figure C.5). Place alternating plus and minus signs in the corners of this path. In the
upper left square, for example, we place a minus sign because we have subtracted 1 unit from the original 100. Note that we can use only squares currently used for shipping to turn the corners of the route we are tracing. Hence, the path Des Moines–Boston to Des Moines–Albuquerque to Fort Lauderdale–Albuquerque to Fort Lauderdale–Boston to Des Moines–Boston would not be acceptable because the Fort Lauderdale–Albuquerque square is empty. It turns out that only one closed route exists for each empty square. Once this one closed path is identified, we can begin assigning plus and minus signs to these squares in the path.

**Step 3.** How do we decide which squares get plus signs and which squares get minus signs? The answer is simple. Because we are testing the cost-effectiveness of the Des Moines–Boston shipping route, we try shipping 1 bathtub from Des Moines to Boston. This is 1 more unit than we were sending between the two cities, so place a plus sign in the box. However, if we ship 1 more unit than before from Des Moines to Boston, we end up sending 101 bathtubs out of the Des Moines factory. Because the Des Moines factory’s capacity is only 100 units, we must ship 1 bathtub less from Des Moines to Albuquerque. This change prevents us from violating the capacity constraint.

To indicate that we have reduced the Des Moines–Albuquerque shipment, place a minus sign in its box. As you continue along the closed path, notice that we are no longer meeting our Albuquerque warehouse requirement for 300 units. In fact, if we reduce the Des Moines–Albuquerque shipment to 99 units, we must increase the Evansville–Albuquerque load by 1 unit, to 201 bathtubs. Therefore, place a plus sign in that box to indicate the increase. You may also observe that those squares in which we turn a corner (and only those squares) will have plus or minus signs.

Finally, note that if we assign 201 bathtubs to the Evansville–Albuquerque route, then we must reduce the Evansville–Boston route by 1 unit, to 99 bathtubs, to maintain the Evansville factory’s capacity constraint of 300 units. To account for this reduction, we thus insert a minus sign in the Evansville–Boston box. By so doing we have balanced supply limitations among all four routes on the closed path.

**Step 4.** Compute an improvement index for the Des Moines–Boston route by adding unit costs in squares with plus signs and subtracting costs in squares with minus signs.

\[
\text{Des Moines–Boston index} = 4 - 5 + 8 - 4 = +3
\]

This means that for every bathtub shipped via the Des Moines–Boston route, total transportation costs will increase by $3 over their current level.

**FIGURE C.5** Stepping-Stone Evaluation of Alternative Routes for Arizona Plumbing
Let us now examine the unused Des Moines–Cleveland route, which is slightly more difficult to trace with a closed path (see Figure C.6). Again, notice that we turn each corner along the path only at squares on the existing route. Our path, for example, can go through the Evansville–Cleveland box but cannot turn a corner; thus we cannot place a plus or minus sign there. We may use occupied squares only as stepping-stones:

\[
\text{Des Moines–Cleveland index} = \frac{9}{100} + \frac{3}{100} = +\frac{4}{100}
\]

Again, opening this route fails to lower our total shipping costs.

Two other routes can be evaluated in a similar fashion:

- Evansville–Cleveland index: \(3\) - \(5\) + \(8\) - \(4\) + \(7\) - \(5\) = +\(4\)

Because this last index is negative, we can realize cost savings by using the (currently unused) Fort Lauderdale–Albuquerque route.

In Example C3, we see that a better solution is indeed possible because we can calculate a negative improvement index on one of our unused routes. Each negative index represents the amount by which total transportation costs could be decreased if one unit was shipped by the source-destination combination. The next step, then, is to choose that route (unused square) with the largest negative improvement index. We can then ship the maximum allowable number of units on that route and reduce the total cost accordingly.

What is the maximum quantity that can be shipped on our new money-saving route? That quantity is found by referring to the closed path of plus signs and minus signs drawn for the route and then selecting the smallest number found in squares containing minus signs. To obtain a new solution, we add this number to all squares on the closed path with plus signs and subtract it from all squares on the path to which we have assigned minus signs.

One iteration of the stepping-stone method is now complete. Again, of course, we must test to see if the solution is optimal or whether we can make any further improvements. We do this by evaluating each unused square, as previously described. Example C4 continues our effort to help Arizona Plumbing arrive at a final solution.

To improve our Arizona Plumbing solution, we can use the improvement indices calculated in Example C3. We found in Example C3 that the largest (and only) negative index is on the Fort Lauderdale–Albuquerque route (which is the route depicted in Figure C.7).

The maximum quantity that may be shipped on the newly opened route, Fort Lauderdale–Albuquerque (FA), is the smallest number found in squares containing minus signs—in this case, 100 units. Why 100 units? Because the total cost decreases by $2 per unit shipped, we know we would like to ship the maximum
possible number of units. Previous stepping-stone calculations indicate that each unit shipped over the FA route results in an increase of 1 unit shipped from Evansville (E) to Boston (B) and a decrease of 1 unit in amounts shipped both from F to B (now 100 units) and from E to A (now 200 units). Hence, the maximum we can ship over the FA route is 100 units. This solution results in zero units being shipped from F to B. Now we take the following four steps:

1. Add 100 units (to the zero currently being shipped) on route FA.
2. Subtract 100 from route FB, leaving zero in that square (though still balancing the row total for F).
3. Add 100 to route EB, yielding 200.
4. Finally, subtract 100 from route EA, leaving 100 units shipped.

Note that the new numbers still produce the correct row and column totals as required. The new solution is shown in Figure C.8.

Total shipping cost has been reduced by (100 units) × ($2 saved per unit) = $200 and is now $4,000. This cost figure, of course, can also be derived by multiplying the cost of shipping each unit by the number of units transported on its respective route, namely: 100($5) + 100($8) + 200($4) + 100($9) + 200($5) = $4,000.

Looking carefully at Figure C.8, however, you can see that it, too, is not yet optimal. Route EC (Evansville–Cleveland) has a negative cost improvement index. See if you can find the final solution for this route on your own. (Programs C.1 and C.2, at the end of this module, provide an Excel OM solution.)

**SPECIAL ISSUES IN MODELING**

**Demand Not Equal to Supply**

A common situation in real-world problems is the case in which total demand is not equal to total supply. We can easily handle these so-called unbalanced problems with the solution procedures that we have just discussed by introducing **dummy sources** or **dummy destinations**. If total supply is
Example C5
Adjusting for unequal supply and demand with a dummy column

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>(A) Albuquerque</th>
<th>(B) Boston</th>
<th>(C) Cleveland</th>
<th>Dummy</th>
<th>Factory capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D) Des Moines</td>
<td>250</td>
<td>$5</td>
<td>$4</td>
<td>$3</td>
<td>0</td>
<td>250</td>
</tr>
<tr>
<td>(E) Evansville</td>
<td>50</td>
<td>$8</td>
<td>$4</td>
<td>$3</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>(F) Fort Lauderdale</td>
<td>150</td>
<td>$9</td>
<td>$7</td>
<td>$5</td>
<td>0</td>
<td>300</td>
</tr>
</tbody>
</table>

Warehouse requirement
300 200 200 150

New Des Moines capacity
850

Total cost = 250($5) + 50($8) + 200($4) + 50($3) + 150($5) + 150(0) = $3,350

Degeneracy
An occurrence in transportation models in which too few squares or shipping routes are being used, so that tracing a closed path for each unused square becomes impossible.

Degeneracy
To apply the stepping-stone method to a transportation problem, we must observe a rule about the number of shipping routes being used: The number of occupied squares in any solution (initial or later) must be equal to the number of rows in the table plus the number of columns minus 1. Solutions that do not satisfy this rule are called degenerate.

Degeneracy occurs when too few squares or shipping routes are being used. As a result, it becomes impossible to trace a closed path for one or more unused squares. The Arizona Plumbing problem we just examined was not degenerate, as it had 5 assigned routes (3 rows or factories + 3 columns or warehouses – 1).

When the navy in Thailand drafts a young man, he first reports to the induction center closest to his home. From one of 36 centers, he is transported by truck to one of four naval bases. The problem of deciding how many men should be assigned and transported from each center to each base is solved using the transportation model. Each base gets the number of recruits it needs, and costly extra trips are avoided.
To handle degenerate problems, we must artificially create an occupied cell: That is, we place a zero or a very small amount (representing a fake shipment) in one of the unused squares and then treat that square as if it were occupied. Remember that the chosen square must be in such a position as to allow all stepping-stone paths to be closed. We illustrate this procedure in Example C6.

Martin Shipping Company has three warehouses from which it supplies its three major retail customers in San Jose. Martin’s shipping costs, warehouse supplies, and customer demands are presented in the transportation table in Figure C.10. To make the initial shipping assignments in that table, we apply the northwest-corner rule.

The initial solution is degenerate because it violates the rule that the number of used squares must equal the number of rows plus the number of columns minus 1. To correct the problem, we may place a zero in the unused square, which permits evaluation of all empty cells. Some experimenting may be needed because not every cell will allow tracing a closed path for the remaining cells. Also, we want to avoid placing the 0 in a cell that has a negative sign in a closed path. No reallocation will be possible if we do this.

For this example, we try the empty square that represents the shipping route from Warehouse 2 to Customer 1. Now we can close all stepping-stone paths and compute improvement indices.

Example C6
Dealing with degeneracy

Check the unused squares to be sure that number of rows + number of columns − 1 equals the number of filled squares.

**FIGURE C.10** Martin’s Northwest-Corner Rule

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**SUMMARY**

The transportation model, a form of linear programming, is used to help find the least-cost solutions to systemwide shipping problems. The northwest-corner method (which begins in the upper-left corner of the transportation table) or the intuitive lowest-cost method may be used for finding an initial feasible solution. The stepping-stone algorithm is then used for finding optimal solutions.

Unbalanced problems are those in which the total demand and total supply are not equal. Degeneracy refers to the case in which the number of rows + the number of columns = 1 is not equal to the number of occupied squares. The transportation model approach is one of the four location models described earlier in Chapter 8. Additional solution techniques are presented on your CD in Tutorial 4.

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**KEY TERMS**

<table>
<thead>
<tr>
<th>Transportation modeling (p. 724)</th>
<th>Dummy sources (p. 730)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northwest-corner rule (p. 726)</td>
<td>Dummy destinations (p. 730)</td>
</tr>
<tr>
<td>Intuitive method (p. 726)</td>
<td>Degeneracy (p. 731)</td>
</tr>
<tr>
<td>Stepping-stone method (p. 727)</td>
<td></td>
</tr>
</tbody>
</table>

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**USING SOFTWARE TO SOLVE TRANSPORTATION PROBLEMS**

Excel, Excel OM, and POM for Windows may all be used to solve transportation problems. Excel uses Solver, which requires that you enter your own constraints. Excel OM also uses Solver but is prestructured so that you need enter only the actual data. POM for Windows similarly requires that only demand data, supply data, and shipping costs be entered.

**Using Excel OM**

Excel OM’s Transportation module uses Excel’s built-in Solver routine to find optimal solutions to transportation problems. Program C.1 illustrates the input data (from Arizona Plumbing) and total-cost formulas. To reach an optimal solution, we must go to Excel’s *Tools* bar, request *Solver*, then select *Solve*. The output appears in Program C.2.
The total cost is created here by multiplying the data table by the shipment table using the SUMPRODUCT function.

Enter the origin and destination names, the shipping costs, and the total supply and demand figures.

Our target cell is the total cost cell (B21), which we wish to minimize by changing the shipment cells (B16 through D18). The constraints ensure that the number shipped is equal to the number demanded, that the shipments are integer and nonnegative, and that we don’t ship more than we have on hand.

These are the cells in which Solver will place the shipments. The total shipments to and from each location are calculated here.

The total cost is created here by multiplying the data table by the shipment table using the SUMPRODUCT function.

**Program C.1** Excel OM Input Screen and Formulas, Using Arizona Plumbing Data

**Program C.2** Output from Excel OM with Optimal Solution to Arizona Plumbing Problem

**Using POM for Windows**

POM for Windows’ Transportation module can solve both maximization and minimization problems by a variety of methods. Input data are the demand data, supply data, and unit shipping costs. See Appendix IV for further details.
Solved Problem C.1

Williams Auto Top Carriers currently maintains plants in Atlanta and Tulsa to supply auto top carriers to distribution centers in Los Angeles and New York. Because of expanding demand, Williams has decided to open a third plant and has narrowed the choice to one of two cities—New Orleans and Houston. Table C.3 provides pertinent production and distribution costs as well as plant capacities and distribution demands. Which of the new locations, in combination with the existing plants and distribution centers, yields a lower cost for the firm?

Table C.3: Production Costs, Distribution Costs, Plant Capabilities, and Market Demands for Williams Auto Top Carriers

<table>
<thead>
<tr>
<th>FROM PLANTS</th>
<th>TO DISTRIBUTION CENTERS</th>
<th>NORMAL PRODUCTION</th>
<th>UNIT PRODUCTION COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing plants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atlanta</td>
<td>$8</td>
<td>$5</td>
<td>600</td>
</tr>
<tr>
<td>Tulsa</td>
<td>$4</td>
<td>$7</td>
<td>900</td>
</tr>
<tr>
<td>Proposed locations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Orleans</td>
<td>$5</td>
<td>$6</td>
<td>500</td>
</tr>
<tr>
<td>Houston</td>
<td>$4</td>
<td>$6</td>
<td>500</td>
</tr>
<tr>
<td>Forecast demand</td>
<td>800</td>
<td>1,200</td>
<td>2,000</td>
</tr>
</tbody>
</table>

*a* Indicates distribution cost (shipping, handling, storage) will be $6 per carrier between Houston and New York.

SOLUTION

To answer this question, we must solve two transportation problems, one for each combination. We will recommend the location that yields a lower total cost of distribution and production in combination with the existing system.

We begin by setting up a transportation table that represents the opening of a third plant in New Orleans (see Figure C.11). Then we use the northwest-corner method to find an initial solution. The total cost of this first solution is $23,600. Note that the cost of each individual “plant-to-distribution-center” route is found by adding the distribution costs (in the body of Table C.3) to the respective unit production costs (in the right-hand column of Table C.3). Thus, the total production-plus-shipping cost of one auto top carrier from Atlanta to Los Angeles is $14 ($8 for shipping plus $6 for production).

Total cost = (600 units × $14) + (200 units × $9) + (700 units × $12) + (500 units × $10)
= $8,400 + $1,800 + $8,400 + $5,000
= $23,600

Is this initial solution (in Figure C.11) optimal? We can use the stepping-stone method to test it and compute improvement indices for unused routes:

Improvement index for Atlanta–New York route
= + $11 (Atlanta–New York) – $14 (Atlanta–Los Angeles)
+ $9 (Tulsa–Los Angeles) – $12 (Tulsa–New York)
= $-6

Improvement index for New Orleans–Los Angeles route
= + $9 (New Orleans–Los Angeles)
– $10 (New Orleans–New York)
+ $12 (Tulsa–New York)
– $9 (Tulsa–Los Angeles)
= $2

Because the firm can save $6 for every unit shipped from Atlanta to New York, it will want to improve the initial solution and send as many units as possible (600, in this case) on this currently unused route.
route (see Figure C.12). You may also want to confirm that the total cost is now $20,000, a savings of $3,600 over the initial solution.

Next, we must test the two unused routes to see if their improvement indices are also negative numbers:

Index for Atlanta–Los Angeles
\[ = \frac{14 - 11 + 12 - 9}{600 + 100} = 6 \]

Index for New Orleans–Los Angeles
\[ = \frac{9 - 10 + 12 - 9}{500 + 500} = 2 \]

Because both indices are greater than zero, we have already reached our optimal solution using the New Orleans plant. If Williams elects to open the New Orleans plant, the firm’s total production and distribution cost will be $20,000.

This analysis, however, provides only half the answer to Williams’s problem. The same procedure must still be followed to determine the minimum cost if the new plant is built in Houston. Determining this cost is left as a homework problem. You can help provide complete information and recommend a solution by solving Problem C.8 (on p. 737).

Solved Problem C.2

In Solved Problem C.1, we examined the Williams Auto Top Carriers problem by using a transportation table. An alternative approach is to structure the same decision analysis using linear programming (LP), which we explained in detail in Quantitative Module B.

**SOLUTION**

Using the data in Figure C.11 (p. 734), we write the objective function and constraints as follows:

Minimize total cost = $14X_{Atl,LA} + $11X_{Atl,NY} + $9X_{Tul,LA} + $12X_{Tul,NY} + $9X_{NO,LA} + $10X_{NO,NY}

Subject to:

\[ X_{Atl,LA} + X_{Atl,NY} \leq 600 \] (production capacity at Atlanta)

\[ X_{Tul,LA} + X_{Tul,NY} \leq 900 \] (production capacity at Tulsa)

\[ X_{NO,LA} + X_{NO,NY} \leq 500 \] (production capacity at New Orleans)

\[ X_{Atl,LA} + X_{Tul,LA} + X_{NO,LA} \geq 800 \] (Los Angeles demand constraint)

\[ X_{Atl,NY} + X_{Tul,NY} + X_{NO,NY} \geq 1200 \] (New York demand constraint)
DISCUSSION QUESTIONS

1. What are the three information needs of the transportation model?
2. What are the steps in the intuitive lowest-cost method?
3. Identify the three “steps” in the northwest-corner rule.
4. How do you know when an optimal solution has been reached?
5. Which starting technique generally gives a better initial solution, and why?
6. The more sources and destinations there are for a transportation problem, the smaller the percentage of all cells that will be used in the optimal solution. Explain.
7. All of the transportation examples appear to apply to long distances. Is it possible for the transportation model to apply on a much smaller scale, for example, within the departments of a store or the offices of a building? Discuss; create an example or prove the application impossible.

PROBLEMS*

1. Find an initial solution to the following transportation problem.
   a) Use the northwest-corner method.
   b) Use the intuitive lowest-cost approach.
   c) What is the total cost of each method?

<table>
<thead>
<tr>
<th>FROM</th>
<th>LOS ANGELES</th>
<th>CALGARY</th>
<th>PANAMA CITY</th>
<th>SUPPLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEXICO CITY</td>
<td>$ 6</td>
<td>$18</td>
<td>$ 8</td>
<td>100</td>
</tr>
<tr>
<td>DETROIT</td>
<td>$17</td>
<td>$13</td>
<td>$19</td>
<td>60</td>
</tr>
<tr>
<td>OTTAWA</td>
<td>$20</td>
<td>$10</td>
<td>$24</td>
<td>40</td>
</tr>
<tr>
<td>DEMAND</td>
<td>50</td>
<td>80</td>
<td>70</td>
<td></td>
</tr>
</tbody>
</table>

2. Using the stepping-stone method, find the optimal solution to Problem C.1. Compute the total cost.

3. a) Use the northwest-corner method to find an initial feasible solution to the following problem. What must you do before beginning the solution steps?
   b) Use the intuitive lowest-cost approach to find an initial feasible solution. Which is better?

<table>
<thead>
<tr>
<th>FROM</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>SUPPLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>$10</td>
<td>$18</td>
<td>$12</td>
<td>100</td>
</tr>
<tr>
<td>Y</td>
<td>$17</td>
<td>$13</td>
<td>$ 9</td>
<td>50</td>
</tr>
<tr>
<td>Z</td>
<td>$20</td>
<td>$18</td>
<td>$14</td>
<td>75</td>
</tr>
</tbody>
</table>

4. Find the optimal solution to Problem C.3 using the stepping-stone method.

5. Tharp Air Conditioning manufactures room air conditioners at plants in Houston, Phoenix, and Memphis. These are sent to regional distributors in Dallas, Atlanta, and Denver. The shipping costs vary, and the company would like to find the least-cost way to meet the demands at each of the distribution centers. Dallas

*Note: $P$ means the problem may be solved with POM for Windows; $\approx$ means the problem may be solved with Excel OM; and $\dagger$ means the problem may be solved with POM for Windows and/or Excel OM.
needs to receive 800 air conditioners per month, Atlanta needs 600, and Denver needs 200. Houston has 850 air conditioners available each month, Phoenix has 650, and Memphis has 300. The shipping cost per unit from Houston to Dallas is $8, to Atlanta $12, and to Denver $10. The cost per unit from Phoenix to Dallas is $10, to Atlanta $14, and to Denver $9. The cost per unit from Memphis to Dallas is $11, to Atlanta $8, and to Denver $12. How many units should owner Devorah Tharp ship from each plant to each regional distribution center? What is the total transportation cost? (Note that a “dummy” destination is needed to balance the problem.)

a) Complete the next iteration using the stepping-stone method.

b) Calculate the “total cost” incurred if your results were to be accepted as the final solution.

The three blood banks in Franklin County are coordinated through a central office that facilitates blood delivery to four hospitals in the region. The cost to ship a standard container of blood from each bank to each hospital is shown in the table below. Also given are the biweekly number of containers available at each bank and the biweekly number of containers of blood needed at each hospital. How many shipments should be made biweekly from each blood bank to each hospital so that total shipment costs are minimized?

<table>
<thead>
<tr>
<th>TO</th>
<th>FROM</th>
<th>HOSP. 1</th>
<th>HOSP. 2</th>
<th>HOSP. 3</th>
<th>HOSP. 4</th>
<th>SUPPLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>BANK 1</td>
<td>$8</td>
<td>$9</td>
<td>$11</td>
<td>$16</td>
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<td></td>
</tr>
<tr>
<td>BANK 2</td>
<td>$12</td>
<td>$7</td>
<td>$5</td>
<td>$8</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>BANK 3</td>
<td>$14</td>
<td>$10</td>
<td>$6</td>
<td>$7</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>DEMAND</td>
<td>90</td>
<td>70</td>
<td>40</td>
<td>50</td>
<td>250</td>
<td></td>
</tr>
</tbody>
</table>

In Solved Problem C.1 (page 734), Williams Auto Top Carriers proposed opening a new plant in either New Orleans or Houston. Management found that the total system cost (of production plus distribution) would be $20,000 for the New Orleans site. What would be the total cost if Williams opened a plant in Houston? At which of the two proposed locations (New Orleans or Houston) should Williams open the new facility?

For the following Karen-Reifsteck Corp. data, find the starting solution and initial cost using the northwest-corner method. What must you do to balance this problem?

<table>
<thead>
<tr>
<th>FROM</th>
<th>W</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>SUPPLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$132</td>
<td>$116</td>
<td>$250</td>
<td>$110</td>
<td>220</td>
</tr>
<tr>
<td>B</td>
<td>$220</td>
<td>$230</td>
<td>$180</td>
<td>$178</td>
<td>300</td>
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<tr>
<td>C</td>
<td>$152</td>
<td>$173</td>
<td>$196</td>
<td>$164</td>
<td>435</td>
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<tr>
<td>DEMAND</td>
<td>160</td>
<td>120</td>
<td>200</td>
<td>230</td>
<td></td>
</tr>
</tbody>
</table>
The Tara Tripp Clothing Group owns factories in three towns (W, Y, and Z), which distribute to three Walsh retail dress shops in three other cities (A, B, and C). The following table summarizes factory availabilities, projected store demands, and unit shipping costs:

| Walsh Clothing Group | | | |
|----------------------|-----------------|-----------------|-----------------|------------------|
| From | To | Dress Shop A | Dress Shop B | Dress Shop C | Factory availability |
| Factory W |  | $4 | $3 | $3 | 35 |
| Factory Y |  | $6 | $7 | $6 | 50 |
| Factory Z |  | $8 | $2 | $5 | 50 |
| Store demand | | 30 | 65 | 40 | 135 |

a) Complete the analysis, determining the optimal solution for shipping at the Tripp Clothing Group.
b) How do you know if it is optimal or not?

Consider the following transportation problem at Frank Timoney Enterprises in Clifton Park, NY.

<table>
<thead>
<tr>
<th>To</th>
<th>DESTINATION</th>
<th>DESTINATION</th>
<th>DESTINATION</th>
<th>SUPPLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESTINATION</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>SOURCE 1</td>
<td>$8</td>
<td>$9</td>
<td>$4</td>
<td>72</td>
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<tr>
<td>SOURCE 2</td>
<td>$5</td>
<td>$6</td>
<td>$8</td>
<td>38</td>
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<tr>
<td>SOURCE 3</td>
<td>$7</td>
<td>$9</td>
<td>$6</td>
<td>46</td>
</tr>
<tr>
<td>SOURCE 4</td>
<td>$5</td>
<td>$3</td>
<td>$7</td>
<td>19</td>
</tr>
<tr>
<td>DEMAND</td>
<td>110</td>
<td>34</td>
<td>31</td>
<td>175</td>
</tr>
</tbody>
</table>

a) Find an initial solution using the northwest-corner rule. What special condition exists?
b) Explain how you will proceed to solve the problem.
c) What is the optimal solution?

Bell Mill Works (BMW) ships French doors to three building-supply houses from mills in Mountpelier, Nixon, and Oak Ridge. Determine the best shipment schedule for BMW from the data provided by Kelly Bell, the traffic manager at BMW. Use the northwest-corner starting procedure and the stepping-stone method. Refer to the following table. (Note: You may face a degenerate solution in one of your iterations.)

| Bell Mill Works | | | |
|-----------------|-----------------|-----------------|-----------------|------------------|
| From | To | Supply House 1 | Supply House 2 | Supply House 3 | Mill capacity (in tons) |
| Mountpelier |  | $3 | $3 | $2 | 25 |
| Nixon |  | $4 | $2 | $3 | 40 |
| Oak Ridge |  | $3 | $2 | $3 | 30 |
| Supply house demand (in tons) | 30 | 30 | 35 | 95 |

Captain Cabell Corp. manufacturers fishing equipment. Currently, the company has a plant in Los Angeles and a plant in New Orleans. David Cabell, the firm’s owner, is deciding where to build a new plant—Philadelphia or Seattle. Use the following table to find the total shipping costs for each potential site. Which should Cabell select?
Susan Helms Manufacturing Co. has hired you to evaluate its shipping costs. The following table shows present demand, capacity, and freight costs between each factory and each warehouse. Find the shipping pattern with the lowest cost.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Warehouse 1</th>
<th>Warehouse 2</th>
<th>Warehouse 3</th>
<th>Warehouse 4</th>
<th>Plant capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS ANGELES</td>
<td>$100</td>
<td>$75</td>
<td>$50</td>
<td>$150</td>
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<tr>
<td>NEW ORLEANS</td>
<td>$80</td>
<td>$60</td>
<td>$90</td>
<td>$225</td>
<td></td>
</tr>
<tr>
<td>PHILADELPHIA</td>
<td>$40</td>
<td>$50</td>
<td>$90</td>
<td>$350</td>
<td></td>
</tr>
<tr>
<td>SEATTLE</td>
<td>$110</td>
<td>$70</td>
<td>$30</td>
<td>$350</td>
<td></td>
</tr>
<tr>
<td>DEMAND</td>
<td>200</td>
<td>100</td>
<td>400</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Drew Rosen Corp. is considering adding a fourth plant to its three existing facilities in Decatur, Minneapolis, and Carbondale. Both St. Louis and East St. Louis are being considered. Evaluating only the transportation costs per unit as shown in the table, decide which site is best.

Using the data from Problem C.15 and the unit production costs in the following table, show which locations yield the lowest cost.

Duffy Pharmaceuticals enjoys a dominant position in the southeast U.S. with over 800 discount retail outlets. These stores are served by twice-weekly deliveries from Duffy’s 16 warehouses, which are in turn supplied daily by 7 factories that manufacture about 70% of all of the chain’s products.
It is clear to Tiffany Wendt, VP operations, that an additional warehouse is desperately needed to handle growth and backlogs. Three cities, Mobile, Tampa, and Huntsville, are under final consideration. The following table illustrates the current and proposed factory/warehouse capacities/demands and shipping costs per average box of supplies.

a) Based on shipping costs only, which city should be selected for the new warehouse?

b) One study shows that Ocala’s capacity can increase to 500 boxes per day. Would this affect your decision in part (a)?

c) Because of a new intrastate shipping agreement, rates for shipping from each factory in Florida to each warehouse in Florida drop by $1 per carton. How does this factor affect your answer to parts (a) and (b)?

Table for Problem C.17

<table>
<thead>
<tr>
<th>FACTORY</th>
<th>ATLANTA, GA</th>
<th>NEW ORLEANS, LA</th>
<th>JACKSON, MS</th>
<th>BIRMINGHAM, AL</th>
<th>MONTGOMERY, AL</th>
<th>RALEIGH, NC</th>
<th>ASHEVILLE, NC</th>
<th>COLUMBIA, SC</th>
<th>CAPACITY (CARTONS PER DAY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valdosta, GA</td>
<td>$3</td>
<td>$5</td>
<td>$4</td>
<td>$3</td>
<td>$4</td>
<td>$6</td>
<td>$8</td>
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<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>200</td>
</tr>
<tr>
<td>Stuart, FL</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>200</td>
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<tr>
<td>Biloxi, MS</td>
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<td>1</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>600</td>
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<tr>
<td>Starkville, MS</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
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<td>6</td>
<td>5</td>
<td>6</td>
<td>400</td>
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<tr>
<td>Durham, NC</td>
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<td>8</td>
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<tr>
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<td>50</td>
<td>150</td>
<td>100</td>
<td>200</td>
<td>150</td>
<td>300</td>
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</tr>
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<table>
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<th>WILMINGTON, AL</th>
<th>CHARLOTTE, NC</th>
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<td>6</td>
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<td>2</td>
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<td>300</td>
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<td>Augusta, GA</td>
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<td>6</td>
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<td>9</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>400</td>
</tr>
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<td>3</td>
<td>5</td>
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<td>3</td>
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<td>9</td>
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<td>8</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>600</td>
</tr>
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<td>3</td>
<td>6</td>
<td>2</td>
<td>400</td>
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<td>5</td>
<td>1</td>
<td>2</td>
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<td>7</td>
<td>8</td>
<td>500</td>
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<tr>
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<td>300</td>
<td>100</td>
<td>150</td>
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</tbody>
</table>

INTERNET HOMEWORK PROBLEMS

See our Companion Web site at www.prenhall.com/heizer for these additional homework problems: C.18 through C.22.

CASE STUDY

Custom Vans, Inc.

Custom Vans, Inc., specializes in converting standard vans into campers. Depending on the amount of work and customizing to be done, the customizing can cost less than $1,000 to more than $5,000. In less than 4 years, Tony Rizzo was able to expand his small operation in Gary, Indiana, to other major outlets in Chicago, Milwaukee, Minneapolis, and Detroit.

Innovation was the major factor in Tony’s success in converting a small van shop into one of the largest and most profitable custom van operations in the Midwest. Tony seemed to have a special ability to design and develop unique features and devices that were always in high demand by van owners. An example was Shower-Rific, which was developed by Tony only 6 months after Custom Vans, Inc., was started. These small showers were completely self-contained, and they could be placed in almost any type of van and in a number of different locations within a van. Shower-Rific was made of fiberglass, and contained towel racks, built-in soap and shampoo holders, and a unique plastic door. Each Shower-Rific took 2 gallons of fiberglass and 3 hours of labor to manufacture.
Most of the Shower-Rifics were manufactured in Gary in the same warehouse where Custom Vans, Inc., was founded. The manufacturing plant in Gary could produce 300 Shower-Rifics in a month, but this capacity never seemed to be enough. Custom Van shops in all locations were complaining about not getting enough Shower-Rifics, and because Minneapolis was farther away from Gary than the other locations, Tony was always inclined to ship Shower-Rifics to the other locations before Minneapolis. This infuriated the manager of Custom Vans at Minneapolis, and after many heated discussions, Tony decided to start another manufacturing plant for Shower-Rifics at Fort Wayne, Indiana. The manufacturing plant at Fort Wayne could produce 150 Shower-Rifics per month.

The manufacturing plant at Fort Wayne was still not able to meet current demand for Shower-Rifics, and Tony knew that the demand for his unique camper shower would grow rapidly in the next year. After consulting with his lawyer and banker, Tony concluded that he should open two new manufacturing plants as soon as possible. Each plant would have the same capacity as the Fort Wayne manufacturing plant. An initial investigation into possible manufacturing locations was made, and Tony decided that the two new plants should be located in Detroit, Michigan; Rockford, Illinois; or Madison, Wisconsin. Tony knew that selecting the best location for the two new manufacturing plants would be difficult. Transportation costs and demands for the various locations would be important considerations.

The Chicago shop was managed by Bill Burch. This shop was one of the first established by Tony, and it continued to outperform the other locations. The manufacturing plant at Gary was supplying 200 Shower-Rifics each month, although Bill knew that the demand for the showers in Chicago was 300 units. The transportation cost per unit from Gary was $10, and although the transportation cost from Fort Wayne was double that amount, Bill was always pleading with Tony to get an additional 50 units from the Fort Wayne manufacturer. The two additional manufacturing plants would certainly be able to supply Bill with the additional 100 showers he needed. The transportation costs would, of course, vary, depending on which two locations Tony picked. The transportation cost per shower would be $30 from Detroit, $5 from Rockford, and $10 from Madison.

Wilma Jackson, manager of the Custom Van shop in Milwaukee, was the most upset about not getting an adequate supply of showers. She had a demand for 100 units, and at the present time, she was only getting half of this demand from the Fort Wayne manufacturing plant. She could not understand why Tony didn’t ship her all 100 units from Gary. The transportation cost per unit from Gary was only $20, while the transportation cost from Fort Wayne was $30. Wilma was hoping that Tony would select Madison for one of the manufacturing locations. She would be able to get all the showers needed, and the transportation cost per unit would only be $5. If not in Madison, a new plant in Rockford would be able to supply her total needs, but the transportation cost per unit would be twice as much as it would be from Madison. Because the transportation cost per unit from Detroit would be $40, Wilma speculated that even if Detroit became one of the new plants, she would not be getting any units from Detroit.

Custom Vans, Inc., of Minneapolis was managed by Tom Poanski. He was getting 100 showers from the Gary plant. Demand was 150 units. Tom faced the highest transportation costs of all locations. The transportation cost from Gary was $40 per unit. It would cost $10 more if showers were sent from the Fort Wayne location. Tom was hoping that Detroit would not be one of the new plants, as the transportation cost would be $60 per unit. Rockford and Madison would have a cost of $30 and $25, respectively, to ship one shower to Minneapolis.

The Detroit shop’s position was similar to Milwaukee’s—only getting half of the demand each month. The 100 units that Detroit did receive came directly from the Fort Wayne plant. The transportation cost was only $15 per unit from Fort Wayne, while it was $25 from Gary. Dick Lopez, manager of Custom Vans, Inc., of Detroit, placed the probability of having one of the new plants in Detroit fairly high. The factory would be located across town, and the transportation cost would be only $5 per unit. He could get 150 showers from the new plant in Detroit and the other 50 showers from Fort Wayne. Even if Detroit was not selected, the other two locations were not intolerable. Rockford had a transportation cost per unit of $35, and Madison had a transportation cost of $40.

Tony pondered the dilemma of locating the two new plants for several weeks before deciding to call a meeting of all the managers of the van shops. The decision was complicated, but the objective was clear—to minimize total costs. The meeting was held in Gary, and everyone was present except Wilma.

Tony: Thank you for coming. As you know, I have decided to open two new plants at Rockford, Madison, or Detroit. The two locations, of course, will change our shipping practices, and I sincerely hope that they will supply you with the Shower-Rifics that you have been wanting. I know you could have sold more units, and I want you to know that I am sorry for this situation.

Dick: Tony, I have given this situation a lot of consideration, and I feel strongly that at least one of the new plants should be located in Detroit. As you know, I am now only getting half of the showers that I need. My brother, Leon, is very interested in running the plant, and I know he would do a good job.

Tom: Dick, I am sure that Leon could do a good job, and I know how difficult it has been since the recent layoffs by the auto industry. Nevertheless, we should be considering total costs and not personalities. I believe that the new plants should be located in Madison and Rockford. I am farther away from the other plants than any other shop, and these locations would significantly reduce transportation costs.

Dick: That may be true, but there are other factors. Detroit has one of the largest suppliers of fiberglass, and I have checked prices. A new plant in Detroit would be able to purchase fiberglass for $2 per gallon less than any of the other existing or proposed plants.

Tom: At Madison, we have an excellent labor force. This is due primarily to the large number of students attending the University of Wisconsin. These students are hard workers, and they will work for $1 less per hour than the other locations that we are considering.

Bill: Calm down, you two. It is obvious that we will not be able to satisfy everyone in locating the new plants. Therefore, I would like to suggest that we vote on the two best locations.

Tony: I don’t think that voting would be a good idea. Wilma was not able to attend, and we should be looking at all of these factors together in some type of logical fashion.

Discussion Question
Where would you locate the two new plants? Why?

### ADDITIONAL CASE STUDIES

Internet Case Studies: Visit our Companion Web site at [www.prenhall.com/heizer](http://www.prenhall.com/heizer) for these free case studies:

- **Consolidated Bottling (B)**: This case involves determining where to add bottling capacity.
- **Northwest General Hospital**: This case involves minimizing the time to distribute hot food in a hospital.

### BIBLIOGRAPHY


