Objective:

To learn how to properly apply the profitability measures described in Chapter 4 to select the best alternative out of a set of mutually exclusive alternatives (MEA).

Mutually Exclusive - the selection of one alternative excludes the consideration of any other alternative. Example: suppose you are shopping for a used automobile. You consider several cars, but will only buy one from a mutually exclusive set of choices.
Section 5.2 Fundamental Purpose of Capital Investment:
To obtain at least the MARR for every dollar invested.

Basic Rule:
Spend the least amount of capital possible unless the extra capital can be justified by the extra savings or benefits.

In other words, any increment of capital spent (above the minimum) must be able to pay its own way.
Section 5.2 Two types of decisions we'll face

1. Investment Alternatives - each alternative has an initial investment producing positive cash flows resulting from increased revenues, reduced costs, or both.

"Do nothing" (DN) is usually an implicit investment alternative.

If $\Sigma$ positive cash flows $> \Sigma$ negative cash flows, then IRR$>0$.
If $EW(MARR) \geq 0$, investment is profitable, or if $EW(MARR) < 0$, do nothing (DN) is better, where $EW$ refers to an equivalent worth method (e.g. PW).
Section 5.2. Two types of decisions we'll face

2. Cost Alternatives - have all negative cash flows except for the salvage value (if applicable). These alternatives represent “must do” situations, and DN is not an option

IRR not defined for cost alternatives. Can you explain why?
Section 5.3. The study period must be appropriate for the decision being made.

**Study Period**: The time interval over which service is needed to fulfill a specified function.

**Useful Life**: The period over time during which an asset is kept in productive operation.

Case 1: Study period = Useful life

Case 2: Study period ≠ Useful life

**Fundamental Principle**: Compare MEAs over the same period of time.
Section 5.4.1 (Case 1) Equivalent Worth (EW) Methods: PW, AW, FW

Procedure for Selecting the Best MEA using the EW method:

1. Compute the equivalent worth of each alternative, using the MARR as the interest rate.

2. Investment Alternatives: Select the alternative having the greatest equivalent worth.

   Note: If all equivalent worths are < 0 for investment alternatives, then "do nothing" is the best alternative.

3. Cost Alternatives: Select the alternative having the smallest equivalent cost (the one that is least negative).

All three equivalent worth methods (PW, AW, FW) will identify the same "best" alternative.

Example – Problem 5-1a (p 232)
Investment Alternatives; Study Period = Useful Life

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment cost (I)</td>
<td>$100,000</td>
<td>$152,000</td>
<td>$184,000</td>
<td>$220,000</td>
</tr>
<tr>
<td>Net annual receipts (A)</td>
<td>15,200</td>
<td>31,900</td>
<td>35,900</td>
<td>41,500</td>
</tr>
<tr>
<td>Salvage value (SV)</td>
<td>10,000</td>
<td>0</td>
<td>15,000</td>
<td>20,000</td>
</tr>
<tr>
<td>Useful life</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

If the MARR is 12%, use the PW method to select the best alternative.

\[
\text{PW}(12\%) = -I + A(P|A, 12\%, 10) + SV(P|F, 12\%, 10)
\]

\[
5.6502 \\
0.3220
\]
Problem 5-1a Solution

\[ PW_{DN} (12\%) = 0 \]

\[ PW_I (12\%) = -10,897 \]

\[ PW_{II} (12\%) = +28.241 \]

\[ PW_{III} (12\%) = +23,672 \]

\[ PW_{IV} (12\%) = +20,923 \]

Select Alternative ___ to maximize PW.
### Example – Problem 5-12a (p. 234)

Cost Alternatives; Study Period = Useful Life

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost (I)</td>
<td>-$85,600</td>
<td>-$63,200</td>
<td>-$71,800</td>
</tr>
<tr>
<td>Annual expenses, years 1-7 (AC)</td>
<td>-7,400</td>
<td>-12,100</td>
<td>-10,050</td>
</tr>
</tbody>
</table>

**MARR = 12%**

Use the AW method to choose the best alternative.

\[
AW = I(A|P, 12\%, 7) + AC
\]

\[
0.2191
\]

\[
AW = I(A|P, 12\%, 7) + AC
\]
Example – Problem 5-12a Continued

\[ A W_A = -85,600(A|P, 12\%, 7) - 7,400 = -26,155 \]

\[ A W_B = \quad = -25,947 \]

\[ A W_C = \quad = -25,781 \]

Assuming one must be chosen (i.e., DN is not an option), select alternative C to minimize annual equiv. costs.
Section 5.4.2 Comparing MEAs - using the IRR method

Why not select the investment opportunity that maximizes IRR? Consider 2 alternatives.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>B-A(Δ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>-$100</td>
<td>-$10,000</td>
<td>-$9,900</td>
</tr>
<tr>
<td>Lump-Sum Receipt</td>
<td>$1,000</td>
<td>$15,000</td>
<td>$14,000</td>
</tr>
<tr>
<td>Next Year</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IRR</td>
<td>900%</td>
<td>50%</td>
<td>41.4%</td>
</tr>
</tbody>
</table>

If MARR = 20%, would you rather have A or B if comparable risk is involved?
Comparing MEAs - using the IRR method cont'd.

If MARR = 20%, PW_A = $733 and PW_B = $2,500.

* Never simply maximize the IRR.
* Never compare the IRR to anything except the MARR.

\[
\text{IRR}_{A\rightarrow B}: \text{PW}_{A\rightarrow B} = 0 = -9,900 + 14,000(P|F, i\%, 1)
\]

\[
9,900/14,000 = (P|F, i\%, 1)
\]

\[
i' = 41.4\% > \text{MARR}
\]
Section 5.4.2.1 Ranking Inconsistency with the IRR Method (Refer to page 200)

IRR$_A$: $0 = -$60,000 + $22,000(P/A, i_A \text{'}, 4)i_A \text{'} = 17.3\%$

IRR$_B$: $0 = -$73,000 + $26,225(P/A, i_B \text{'}, 4)i_B \text{'} = 16.3\%$

NEVER simply select the MEA that MAXIMIZES the IRR. We don't maximize rate of return. Look at the increment.

IRR$_{A \rightarrow B} : 0 = -$13,000 + $4,225(P/A, i_{A \rightarrow B}', 4)i_{A \rightarrow B}' = 11.4\%$

The rate of return on the increment, 11.4\%, >MARR. It is worth the additional investment to select Alternative B.
Section 5.4.2 Example – Problem 5-2 on page 232

Given three MEAs and MARR = 15% per year

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment (FC)</td>
<td>-28,000</td>
<td>-16,000</td>
<td>-23,500</td>
</tr>
<tr>
<td>Net Cash Flow/year</td>
<td>5,500</td>
<td>3,300</td>
<td>4,800</td>
</tr>
<tr>
<td>Salvage Value</td>
<td>1,500</td>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td>Useful Life</td>
<td>10 yrs</td>
<td>10 yrs</td>
<td>10 yrs</td>
</tr>
<tr>
<td>Study Period</td>
<td>10 yrs</td>
<td>10 yrs</td>
<td>10 yrs</td>
</tr>
</tbody>
</table>

Use the Incremental IRR procedure to choose the best alternative.
Example 5-2 Continued

Step 1. DN→2→3→1  (rank order alternatives from low capital investment to high capital investment)

Step 2. Compare DN→2

<table>
<thead>
<tr>
<th>Δ cash flows</th>
<th>(2 - DN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ Investment</td>
<td>-16,000 - 0 = -$16,000</td>
</tr>
<tr>
<td>Δ Annual Receipts</td>
<td>3,300 - 0 = 3,300</td>
</tr>
<tr>
<td>Δ Salvage Value</td>
<td>0 - 0 = 0</td>
</tr>
</tbody>
</table>

Compute $\Delta IRR_{DN→2}$

$PW(\Delta i') = 0 = -$16,000 + $3,300(P|A, \Delta i'\%, 10)$

$i'_{DN→2} \approx 15.9\%$

Step 3. Since $Di' > MARR$, keep alt. 2 (higher FC) as current best alternative. Drop DN from further consideration.
Example 5-2 Continued

Step 4. Compare 2→3

\[
\begin{align*}
\text{Δ cash flows} & \quad (3 \rightarrow 2) \\
\text{Δ Investment} & \quad -23,500 - (-16,000) = -7,500 \\
\text{Δ Annual Receipts} & \quad 4,800 - 3,300 = 1,500 \\
\text{Δ Salvage Value} & \quad 500 - 0 = 500 \\
\end{align*}
\]

Compute $\Delta IRR_{2\rightarrow3}$

\[
PW(\Delta i') = 0 = -$7,500 + 1,500(P|A, \Delta i'\%, 10) + 500(P|F, \Delta i'\%, 10)
\]

\[
i'_{2\rightarrow3} \approx 15.5\%
\]

Since $\Delta i' > MARR$, keep alt. 3 (higher FC) as current best alternative. Drop alt. 2 from further consideration.
**Example 5-2 Continued**

Next comparison: 3→1

\[
(1 - 3)
\]

\[\Delta \text{ cash flows}\]

<table>
<thead>
<tr>
<th>(\Delta)</th>
<th>(\text{Investment})</th>
<th>(\text{Annual Receipts})</th>
<th>(\text{Salvage Value})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta)</td>
<td>-28,000 – (-23,500) =</td>
<td>5,500 – 4,800 = 700</td>
<td>1,500 – 500 = 1,000</td>
</tr>
</tbody>
</table>

Compute \(\Delta IRR_{3→1}\)

\[
\text{PW}(\Delta i') = 0
\]

\[
= -4,500 + 700(P|A, \Delta i'\%, 10) + 1,000(P|F, \Delta i'\%, 10)
\]

\[
\Delta i'_{3→1} \approx 10.9\%
\]

Since \(\Delta i' < \text{MARR}\), keep alt. 3 (lower FC) as current best alternative. Drop alt. 1 from further consideration.

Step 5. All alternatives have been considered.

**Recommend alternative 3 for investment.**
Problem 5-12 Revisited with the IRR (incremental) method

Cost Alternatives; Study Period = Useful Life = 7 years, MARR = 12%

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment cost</td>
<td>-$85,600</td>
<td>-$63,200</td>
<td>-$71,800</td>
</tr>
<tr>
<td>Annual costs (years 1-7)</td>
<td>-7,400</td>
<td>-12,100</td>
<td>-10,050</td>
</tr>
<tr>
<td>Salvage Value</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Cost Only Procedure
Step 1: B → C → A
Step 2: Compare B → C

\[(C - B) \Delta \text{cash flows} = (C - B)\]

\[\Delta \text{Investment} = -71,800 - (-63,200) = -8,600\]
\[\Delta \text{Annual Costs} = -10,050 - (-12,100) = 2,050\]

Compute \(\Delta \text{IRR}_{\text{B→C}}\)

\[\text{AW}(\Delta i') = 0 = -8,600(A|P, \Delta i', 7) + 2,050\]
Problem 5-12, Comparing B→C Continued

\[ AW(12\%) = +166 \gt 0; \text{ therefore, } \Delta i' \text{ B→C} \gt \text{ MARR} \]

\[ (\Delta i' \text{ B→C} \approx 14.7\%) \]

Step 3: Keep C, Reject B

Step 4: Next comparison: C → A

\[
\begin{align*}
\begin{array}{lcc}
\text{(A - C)} & \Delta \text{cash flows} \\
\Delta \text{Investment} & -85,600 -(-71,800) = & -13,800 \\
\Delta \text{Annual Costs} & -7,400 -(-10,050) = & 2,650 \\
\end{array}
\end{align*}
\]

Compute \( \Delta \text{IRR}_{C\rightarrow A} \):

\[ AW(\Delta i') = 0 = -13,800(A|P, \Delta i', 7) + 2650 \]

\[ AW(12\%) = -374 \lt 0; \text{ therefore, } \Delta i'_{C\rightarrow A} \lt \text{ MARR} \]

\[ (\Delta i'_{C\rightarrow A} \approx 8\%) \]

Reject A, Keep C.

Step 5. All alternatives have been considered. Recommend alternative C to minimize total equivalent cost to the consumer.
Example – Problem 5-1 Revisited with the IRR method

Investment Alternatives, MARR = 12%, Study Period = Useful Life = 10 years

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
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</thead>
<tbody>
<tr>
<td>Investment cost (I)</td>
<td>-$100,000</td>
<td>-$152,000</td>
<td>-$184,000</td>
<td>-$220,000</td>
</tr>
<tr>
<td>Net annual receipts (A)</td>
<td>15,200</td>
<td>31,900</td>
<td>35,900</td>
<td>41,500</td>
</tr>
<tr>
<td>Salvage value (SV)</td>
<td>10,000</td>
<td>0</td>
<td>15,000</td>
<td>20,000</td>
</tr>
</tbody>
</table>

1. DN → I → II → III → IV

2. Compare DN → I

(I - DN)  
Δ cash flows

Δ Investment →  
-100,000 - 0 = -100,000

Δ Annual Receipts →  
15,200 - 0 = 15,200

Δ Salvage Value →  
10,000 - 0 = 10,000

Compute ΔIRR_{DN→I}

PW(Δi') = 0

= -100,000 + 15,200(P|A, Δi'%, 10) + 10,000(P|F, Δi'%, 10)

Δi'_{DN→I} ≈ 9.4% < MARR
Problem 5-1 Revisited Continued

Step 3. Since $\Delta i' < \text{MARR}$, keep DN (lower FC) as current best alternative. Drop I from further consideration.

Step 4. Next comparison: DN $\rightarrow$ II

\[
\begin{array}{ccc}
\text{Category} & \Delta \text{Cash Flows} \\
\text{II} - \text{DN} & \Delta I & -152,000 - 0 = -152,000 \\
& \Delta AR & 31,900 - 0 = 31,900 \\
& \Delta SV & 0 - 0 = 0 \\
\end{array}
\]

Compute $\Delta IRR_{\text{DN} \rightarrow \text{II}}$

\[
\text{PW}(\Delta i') = 0 = -152,000 + 31,900(\text{P|A, } \Delta i'\%, 10)
\]

$\Delta i'_{\text{DN} \rightarrow \text{II}} \approx 16.4\% > \text{MARR}$

Keep Alternative II, Drop DN
Problem 5-1 Revisited Continued

Step 4. Next comparison: II → III

\[
\frac{(III - II)}{\Delta \text{ cash flows}}
\]

\[\begin{align*}
\Delta I & \quad -184,000 - (-152,000) = -32,000 \\
\Delta \text{ AR} & \quad 35,900 - 31,900 = 4,000 \\
\Delta \text{ SV} & \quad 15,000 - 0 = 15,000
\end{align*}\]

\[\text{PW}(\Delta i') = 0\]

\[= -$32,000 + $4,000(P|A, \Delta i', 10) + $15,000(P|F, \Delta i', 10)\]

\[\Delta i'_{II \rightarrow III} \approx 9\% < \text{MARR}\]

Keep Alternative II, Drop III from further consideration
Problem 5-1 Revisited Continued

Step 4. Next comparison: II → IV

\[(IV - II)\]
\[\Delta \text{ cash flows}\]

\(\Delta I\)
-220,000 - (-152,000) = -$68,000

\(\Delta AR\)
41,500 - 31,900 = 9,600

\(\Delta SV\)
20,000 - 0 = 20,000

\[\text{PW}(\Delta i') = 0\]

\[= -68,000 + 9,600(P|A, \Delta i', 10) + 20,000(P|F, \Delta i', 10)\]

\[\Delta i'_{II \rightarrow IV} \approx 9.6\% < \text{MARR}\]

Keep Alternative II, Drop IV from further consideration

5. All have been considered. Select II as preferred alternative.