1. Use the multifactor evaluation process in making decisions that involve a number of factors, where importance weights can be assigned.

2. Understand the use of the analytic hierarchy process in decision making.

3. Contrast multifactor evaluation with the analytic hierarchy process.

After completing this module, students will be able to:

**MODULE OUTLINE**

- **M1.1** Introduction
- **M1.2** Multifactor Evaluation Process
- **M1.3** Analytic Hierarchy Process
- **M1.4** Comparison of Multifactor Evaluation and Analytic Hierarchy Processes

**Appendix M1.1:** Using Excel for the Analytic Hierarchy Process
Many decision-making problems involve a number of factors. For example, if you are considering a new job, factors might include starting salary, career advancement opportunities, work location, the people you would be working with on the job, the type of work you would be doing, and assorted fringe benefits. If you are considering the purchase of a personal computer, there are a number of important factors to consider as well: price, memory, compatibility with other computers, flexibility, brand name, software availability, the existence of any user groups, and the support of the computer manufacturer and the local computer store. In buying a new or used car, such factors as color, style, make and model, year, number of miles (if it’s a used car), price, dealership or person you are purchasing the car from, warranties, and cost of insurance may be important factors to consider.

In *multifactor decision making*, individuals subjectively and intuitively consider the various factors in making their selection. For difficult decisions, a quantitative approach is recommended. All of the important factors can then be given appropriate weights and each alternative, such as a car, a computer, or a new job prospect, can be evaluated in terms of these factors. This approach is called the *multifactor evaluation process* (MFEP).

In other cases we may not be able to quantify our preferences for various factors and alternatives. We then use the *analytic hierarchy process* (AHP). This process uses pairwise comparisons and then computes the weighting factors and evaluations for us. We begin with a discussion of the MFEP.

**M1.2 Multifactor Evaluation Process**

With the MFEP, we start by listing the factors and their relative importance on a scale from 0 to 1.

Let’s consider an example. Steve Markel, an undergraduate business major, is looking at several job opportunities. After discussing the employment situation with his academic advisor and the director of the placement center, Steve has determined that the only three factors really important to him are salary, career advancement opportunities, and location of the new job. Furthermore, Steve has decided that career advancement opportunities are the most important to him. He has given this a weight of 0.6. Steve has placed salary next, with a weight of 0.3. Finally, Steve has given location an importance weight of 0.1. As with any MFEP problem, the importance weights for factors must sum to 1 (see Table M1.1).

At this time, Steve feels confident that he will get offers from AA Company, EDS, Ltd., and PW, Inc. For each of these jobs, Steve evaluated, or rated, the various factors on a 0 to 1 scale. For AA Company, Steve gave salary an evaluation of 0.7, career advancement an evaluation of 0.9, and location an evaluation of 0.6. For EDS, Steve evaluated salary as 0.8, career advancement as 0.7, and location as 0.8. For PW, Inc., Steve gave salary an evaluation of 0.9, career advancement an evaluation of 0.6, and location an evaluation of 0.9. The results are shown in Table M1.2.

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>IMPORTANCE (WEIGHT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary</td>
<td>0.3</td>
</tr>
<tr>
<td>Career advancement</td>
<td>0.6</td>
</tr>
<tr>
<td>Location</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Given this information, Steve can determine a total weighted evaluation for each of the alternatives or job possibilities. Each company is given a factor evaluation for the three factors, and then the factor weights are multiplied by the factor evaluation and summed to get a total weighted evaluation for each company. As you can see in Table M1.3, AA Company has received a total weighted evaluation of 0.81. The same type of analysis is done for EDS, Ltd., and PW, Inc., in Tables M1.4 and M1.5. As you can see from the analysis, AA Company received the highest total weighted evaluation; EDS, Ltd., was next, with a total weighted evaluation of 0.74. Using the multifactor evaluation process, Steve’s decision was to go with AA Company because it had the highest total weighted evaluation.

**TABLE M1.2**
Factor Evaluations

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>AA CO.</th>
<th>EDS, LTD.</th>
<th>PW, INC.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>Career advancement</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Location</td>
<td>0.6</td>
<td>0.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The company with the highest total weighted evaluation is selected.

**TABLE M1.3**
Evaluation of AA Company

<table>
<thead>
<tr>
<th>FACTOR NAME</th>
<th>FACTOR WEIGHT</th>
<th>FACTOR EVALUATION</th>
<th>WEIGHTED EVALUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary</td>
<td>0.3</td>
<td>0.7</td>
<td>0.21</td>
</tr>
<tr>
<td>Career</td>
<td>0.6</td>
<td>0.9</td>
<td>0.54</td>
</tr>
<tr>
<td>Location</td>
<td>0.1</td>
<td>0.6</td>
<td>0.06</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td></td>
<td>0.81</td>
</tr>
</tbody>
</table>

**TABLE M1.4**
Evaluation of EDS, Ltd.

<table>
<thead>
<tr>
<th>FACTOR NAME</th>
<th>FACTOR WEIGHT</th>
<th>FACTOR EVALUATION</th>
<th>WEIGHTED EVALUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary</td>
<td>0.3</td>
<td>0.8</td>
<td>0.24</td>
</tr>
<tr>
<td>Career</td>
<td>0.6</td>
<td>0.7</td>
<td>0.42</td>
</tr>
<tr>
<td>Location</td>
<td>0.1</td>
<td>0.8</td>
<td>0.08</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td></td>
<td>0.74</td>
</tr>
</tbody>
</table>

**TABLE M1.5**
Evaluation of PW, Inc.

<table>
<thead>
<tr>
<th>FACTOR NAME</th>
<th>FACTOR WEIGHT</th>
<th>FACTOR EVALUATION</th>
<th>WEIGHTED EVALUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary</td>
<td>0.3</td>
<td>0.9</td>
<td>0.27</td>
</tr>
<tr>
<td>Career</td>
<td>0.6</td>
<td>0.6</td>
<td>0.36</td>
</tr>
<tr>
<td>Location</td>
<td>0.1</td>
<td>0.9</td>
<td>0.09</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td></td>
<td>0.72</td>
</tr>
</tbody>
</table>
In situations in which we can assign evaluations and weights to the various decision factors, the MFEP described previously works fine. In other cases, decision makers may have difficulties in accurately determining the various factor weights and evaluations. In such cases, the analytic hierarchy process (AHP) can be used. This process was developed by Thomas L. Saaty and published in his 1980 book *The Analytic Hierarchy Process*.

The AHP process involves pairwise comparisons. The decision maker starts by laying out the overall hierarchy of the decision. This hierarchy reveals the factors to be considered as well as the various alternatives in the decision. Then, a number of pairwise comparisons are done, which result in the determination of factor weights and factor evaluations. They are the same types of weights and evaluations discussed in the preceding section and shown in Tables M1.1 through M1.5. As before, the alternative with the highest total weighted score is selected as the best alternative.

**Judy Grim’s Computer Decision**

To illustrate an example of this process, we take the case of Judy Grim, who is looking for a new computer system for her small business. She has determined that the most important overall factors are hardware, software, and vendor support. Furthermore, Judy has narrowed down her alternatives to three possible computer systems. She has labeled these SYSTEM-1, SYSTEM-2, and SYSTEM-3. To begin, Judy has placed these factors and alternatives into a decision hierarchy (see Figure M1.1).

The decision hierarchy for the computer selection has three different levels. The top level describes the overall decision. As you can see in Figure M1.1, this overall decision is to select the best computer system. The middle level in the hierarchy describes the factors that are to be considered: hardware, software, and vendor support. Judy could decide to use a number of additional factors, but for this example, we keep our factors to only three to show you the types of calculations that are to be performed using AHP. The lower level of the decision hierarchy reveals the alternatives. (Alternatives have also been called items or systems). As you can see, the alternatives include the three different computer systems.

The key to using AHP is pairwise comparisons. The decision maker, Judy Grim, needs to compare two different alternatives using a scale that ranges from equally preferred to extremely preferred.

**IN ACTION**

**R&D at Air Products and Chemicals, Inc.**

Without new ideas and products, a company can lose its competitive edge and profitability. A lack of research and development (R&D) can mean a loss of business and even bankruptcy for some organizations. Yet, spending money on R&D does not guarantee success. How scarce resources are allocated among diverse R&D projects can help a company develop leading products and sustain high profitability for years.

Realizing its importance, Air Products and Chemicals, Inc., identifies key issues for successful R&D investments. These issues are then communicated to those involved in R&D to improve project proposals and help increase the likelihood of successful R&D results. To determine the best R&D projects for funding, Air Products selects and weights criteria in a structured framework, using the analytic hierarchy process (AHP) discussed in this module. The AHP is used to determine the real strengths and weaknesses of proposed R&D projects. In addition, the AHP allows decision makers to determine a project ranking for each project. With AHP, Air Products fully funds the strong projects, denies funding to weak projects, and funds intermediate projects to some extent to resolve and overcome any weaknesses.

FIGURE M1.1
Decision Hierarchy for Computer System Selection

We use the following for pairwise comparison:
1—Equally preferred
2—Equally to moderately preferred
3—Moderately preferred
4—Moderately to strongly preferred
5—Strongly preferred
6—Strongly to very strongly preferred
7—Very strongly preferred
8—Very to extremely strongly preferred
9—Extremely preferred

Using Pairwise Comparisons
Judy begins by looking at the hardware factor and by comparing computer SYSTEM-1 with computer SYSTEM-2. Using the preceding scale, Judy determines that the hardware for computer SYSTEM-1 is moderately preferred to computer SYSTEM-2. Thus, Judy uses the number 3, representing moderately preferred. Next, Judy compares the hardware for SYSTEM-1 with SYSTEM-3. She believes that the hardware for computer SYSTEM-1 is extremely preferred to computer SYSTEM-3. This is a numerical score of 9. Finally, Judy considers the only other pairwise comparison, which is the hardware for computer SYSTEM-2 compared with the hardware for computer SYSTEM-3. She believes that the hardware for computer SYSTEM-2 is strongly to very strongly preferred to the hardware for computer SYSTEM-3, a score of 6. With these
pairwise comparisons, Judy constructs a pairwise comparison matrix for hardware. This is shown in the following table:

<table>
<thead>
<tr>
<th>HARDWARE</th>
<th>SYSTEM-1</th>
<th>SYSTEM-2</th>
<th>SYSTEM-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYSTEM-1</td>
<td>3</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>SYSTEM-2</td>
<td></td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>SYSTEM-3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This pairwise comparison matrix reveals Judy's preferences for hardware concerning the three computer systems. From this information, using AHP, we can determine the evaluation factors for hardware for the three computer systems.

Look at the upper-left corner of the pairwise comparison matrix. This upper-left corner compares computer SYSTEM-1 with itself for hardware. When comparing anything to itself, the evaluation scale must be 1, representing equally preferred. Thus, we can place the number 1 in the upper-left corner (see the next table) to compare SYSTEM-1 with itself. The same can be said for comparing SYSTEM-2 with itself and comparing SYSTEM-3 with itself. Each of these must also get a score of 1, which represents equally preferred.

In general, for any pairwise comparison matrix, we will place 1s down the diagonal from the upper-left corner to the lower-right corner. To finish such a table, we make the observation that if alternative A is twice as preferred as alternative B, we can conclude that alternative B is preferred only one-half as much as alternative A. Thus, if alternative A receives a score of 2 relative to alternative B, then alternative B should receive a score of $\frac{1}{2}$ when compared with alternative A. We can use this same logic to complete the lower-left side of the matrix of pairwise comparisons:

<table>
<thead>
<tr>
<th>HARDWARE</th>
<th>SYSTEM-1</th>
<th>SYSTEM-2</th>
<th>SYSTEM-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYSTEM-1</td>
<td>1</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>SYSTEM-2</td>
<td>$\frac{1}{3}$</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>SYSTEM-3</td>
<td>$\frac{1}{9}$</td>
<td>$\frac{1}{6}$</td>
<td>1</td>
</tr>
</tbody>
</table>

Look at this newest matrix of pairwise comparisons. You will see that there are 1s down the diagonal from the upper-left corner to the lower-right corner. Then, look at the lower left part of the table. In the second row and first column of this table, you can see that SYSTEM-2 received a score of $\frac{1}{3}$ compared with SYSTEM-1. This is because SYSTEM-1 received a score of 3 over SYSTEM-2 from the original assessment. Now look at the third row. The same has been done. SYSTEM-3 compared with SYSTEM-1, in row 3 column 1 of the table, received a score of $\frac{1}{9}$. This is because SYSTEM-1 compared with SYSTEM-3 received a score of 9 in the original pairwise comparison. In a similar fashion, SYSTEM-3 compared with SYSTEM-2 received a score of $\frac{1}{6}$ in the third row and second column of the table. This is because when comparing SYSTEM-2 with SYSTEM-3 in the original pairwise comparison, the score of 6 was given.
Evaluations for Hardware

Now that we have completed the matrix of pairwise comparisons, we can start to compute the evaluations for hardware. We start by converting the numbers in the matrix of pairwise comparisons to decimals to make them easier to work with. We then get column totals:

Once the column totals have been determined, the numbers in the matrix are divided by their respective column totals to produce the normalized matrix as follows:

To determine the priorities for hardware for the three computer systems, we simply find the average of the various rows from the matrix of numbers as follows:

Once the matrix is normalized, the numbers in each column will sum to one.

The priorities for each system are obtained by averaging the values in each row of the normalized matrix.

The results are displayed in Table M1.6. As you can see, the factor evaluation for SYSTEM-1 is 0.6583. For SYSTEM-2 and SYSTEM-3, the factor evaluations are 0.2819 and 0.0598. The same procedure is used to get the factor evaluations for all other factors, which are software and vendor support in this case. But before we do this, we need to determine whether our responses are consistent by determining a consistency ratio.

Determining the Consistency Ratio

To arrive at the consistency ratio, we begin by determining the weighted sum vector. This is done by multiplying the factor evaluation number for the first system by the first column of the original pairwise comparison matrix. We multiply the second factor evaluation by the second column, and so on. The results are displayed in Table M1.6.
Computing the weighted sum vector and the consistency vector.

The next step is to determine the consistency vector. This is done by dividing the weighted sum vector by the factor evaluation values determined previously:

Consistency vector = \[
\begin{bmatrix}
(0.6583/0.6583) & (0.2819/0.2819) & (0.0598/0.0598)
\end{bmatrix}
\begin{bmatrix}
2.0423 \\
0.8602 \\
0.1799
\end{bmatrix} / \begin{bmatrix}
0.6583 \\
0.2819 \\
0.0598
\end{bmatrix} = \begin{bmatrix}
3.1025 \\
3.0512 \\
3.0086
\end{bmatrix}
\]

COMPUTING LAMBDA AND THE CONSISTENCY INDEX Now that we have found the consistency vector, we need to compute values for two more terms, lambda (\(\lambda\)) and the consistency index (CI), before the final consistency ratio can be computed. The value for lambda is simply the average value of the consistency vector. The formula for CI is

\[
CI = \frac{\lambda - n}{n - 1}
\]

where \(n\) is the number of items or systems being compared. In this case, \(n = 3\), for three different computer systems being compared. The results of the calculations are as follows:

\[
\lambda = \frac{3.1025 + 3.0512 + 3.0086}{3} = 3.0541
\]

\[
CI = \frac{\lambda - n}{n - 1} = \frac{3.0541 - 3}{3 - 1} = 0.0270
\]

COMPUTING THE CONSISTENCY RATIO Finally, we are now in a position to compute the consistency ratio. The consistency ratio (CR) is equal to the consistency index divided by the random index (RI), which is determined from a table. The random index is a direct function of the number of alternatives or systems being considered. This table is next followed by the final calculation of the consistency ratio:

<table>
<thead>
<tr>
<th>(n)</th>
<th>(RI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.58</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
</tr>
<tr>
<td>5</td>
<td>1.12</td>
</tr>
<tr>
<td>6</td>
<td>1.24</td>
</tr>
<tr>
<td>7</td>
<td>1.32</td>
</tr>
<tr>
<td>8</td>
<td>1.41</td>
</tr>
</tbody>
</table>
The consistency ratio tells us how consistent we are.

In general, the consistency ratio is given by:

\[ CR = \frac{CI}{RI} \]  

(M1-2)

In this case,

\[ CR = \frac{CI}{RI} = \frac{0.0270}{0.58} = 0.0466 \]

The consistency ratio tells us how consistent we are with our answers. A higher number means we are less consistent, whereas a lower number means that we are more consistent. In general, if the consistency ratio is 0.10 or less, the decision maker’s answers are relatively consistent. For a consistency ratio that is greater than 0.10, the decision maker should seriously consider reevaluating his or her responses during the pairwise comparisons that were used to obtain the original matrix of pairwise comparisons.

As you can see from the analysis, we are relatively consistent with our responses, so there is no need to reevaluate the pairwise comparison responses. If you look at the original pairwise comparison matrix, this makes sense. The hardware for SYSTEM-1 was moderately preferred to the hardware for SYSTEM-2. The hardware for SYSTEM-1 was extremely preferred to the hardware for SYSTEM-3. This implies that the hardware for SYSTEM-2 should be preferred over the hardware for SYSTEM-3. From our responses, the hardware for SYSTEM-2 was strongly to very strongly preferred over the hardware for SYSTEM-3, as indicated by the number 6. Thus, our original assessments of the pairwise comparison matrix seem to be consistent, and the consistency ratio that we computed supports our observations.

Although the calculations to compute the consistency ratio are fairly involved, they are an important step in using the AHP.

Evaluations for the Other Factors

So far, we have determined the factor evaluations for hardware for the three different computer systems along with a consistency ratio for these evaluations. Now, we can make the same calculations for other factors, namely software and vendor support. As before, we start with the matrix of pairwise comparisons. We perform the same calculations and end up with the various factor evaluations for both software and vendor support. We begin by presenting the matrix of pairwise comparisons for both software and vendor support:

<table>
<thead>
<tr>
<th>SOFTWARE</th>
<th>SYSTEM-1</th>
<th>SYSTEM-2</th>
<th>SYSTEM-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYSTEM-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SYSTEM-2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SYSTEM-3</td>
<td>8</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VENDOR SUPPORT</th>
<th>SYSTEM-1</th>
<th>SYSTEM-2</th>
<th>SYSTEM-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYSTEM-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SYSTEM-2</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SYSTEM-3</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
With the matrices shown, we can perform the same types of calculations to determine the factor evaluations for both software and vendor support for the three computer systems. The data for the three different systems are summarized in Table M1.7. We also need to determine the consistency ratios for both software and support. As it turns out, both consistency ratios are under 0.10, meaning that the responses to the pairwise comparison are acceptably consistent.

Note that the factor evaluations for the three factors and three different computer systems shown in Table M1.7 are similar to the factor evaluations in Table M1.2 for the job selection problem. The major difference is that we had to use the AHP to determine these factor evaluations using pairwise comparisons, because we were not comfortable in our abilities to assess these factors subjectively without some assistance.

### Determining Factor Weights

Next, we need to determine the factor weights. When we used the MFEP, it was assumed that we could simply determine these values subjectively. Another approach is to use the AHP and pairwise comparisons to determine the factor weights for hardware, software, and vendor support.

In comparing the three factors, we determine that software is the most important. Software is very to extremely strongly preferred over hardware (number 8). Software is moderately preferred over vendor support (number 3). In comparing vendor support to hardware, we decide that vendor support is more important. Vendor support is moderately preferred to hardware (number 3). With these values, we can construct the pairwise comparison matrix and then compute the weights for hardware, software, and support. We also need to compute a consistency ratio to make sure that our responses are consistent. As with software and vendor support, the actual calculations for determining the factor weights are left for you to make on your own. After making the appropriate calculations, the factor weights for hardware, software, and vendor support are shown in Table M1.8.

### Overall Ranking

After the factor weights have been determined, we can multiply the factor evaluations in Table M1.7 by the factor weights in Table M1.8. This is the same procedure that we used for the job selection decision in Section M1.2. It will give us the overall ranking for the three computer systems, which is shown in Table M1.9. As you can see, SYSTEM-3 received the highest final ranking and is selected as the best computer system.

### Using the Computer to Solve Analytic Hierarchy Process Problems

As you can see from the previous pages, solving AHP problems can involve a large number of calculations. Fortunately, computer programs are available to make AHP easier. Appendix M1.1

---

**TABLE M1.7**

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>SYSTEM-1</th>
<th>SYSTEM-2</th>
<th>SYSTEM-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardware</td>
<td>0.6583</td>
<td>0.2819</td>
<td>0.0598</td>
</tr>
<tr>
<td>Software</td>
<td>0.0874</td>
<td>0.1622</td>
<td>0.7504</td>
</tr>
<tr>
<td>Vendor</td>
<td>0.4967</td>
<td>0.3967</td>
<td>0.1066</td>
</tr>
</tbody>
</table>

**TABLE M1.8**

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>FACTOR WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardware</td>
<td>0.0820</td>
</tr>
<tr>
<td>Software</td>
<td>0.6816</td>
</tr>
<tr>
<td>Vendor</td>
<td>0.2364</td>
</tr>
</tbody>
</table>
Multifactor decision making has a number of useful and important applications. If you know or can determine with confidence and accuracy the factor weights and factor evaluations, MFEP is preferred. If not, you should use AHP. As it turns out, AHP also gives the factor weights and factor evaluations from which the final selection can be made. The only difference is that with the AHP, we compute the factor weights and factor evaluations from a number of pairwise comparison matrices. We also compute a consistency ratio to make sure that our responses to the original pairwise comparison matrix are consistent and acceptable. If they are not, we should go back and perform the pairwise comparison again. Although AHP involves a larger number of calculations, it is preferred to MFEP in cases in which you do not feel confident or comfortable in determining factor weights or factor evaluations without making pairwise comparisons.

**TABLE M1.9**

<table>
<thead>
<tr>
<th>SYSTEM OR ALTERNATIVE</th>
<th>TOTAL WEIGHTED EVALUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYSTEM-1</td>
<td>0.2310</td>
</tr>
<tr>
<td>SYSTEM-2</td>
<td>0.2275</td>
</tr>
<tr>
<td>SYSTEM-3*</td>
<td>0.5416</td>
</tr>
</tbody>
</table>

* SYSTEM-3 is selected.

demonstrates how Excel can be used for the AHP calculations shown in this module. A commercial package called Expert Choice for Windows can also be used to solve the types of AHP problems discussed in this module. It is also possible to use AHP with group decision making. Team Expert Choice helps groups brainstorm ideas, structure their decisions, and evaluate alternatives.

**M1.4 Comparison of Multifactor Evaluation and Analytic Hierarchy Processes**

Multifactor decision making has a number of useful and important applications. If you know or can determine with confidence and accuracy the factor weights and factor evaluations, MFEP is preferred. If not, you should use AHP. As it turns out, AHP also gives the factor weights and factor evaluations from which the final selection can be made. The only difference is that with the AHP, we compute the factor weights and factor evaluations from a number of pairwise comparison matrices. We also compute a consistency ratio to make sure that our responses to the original pairwise comparison matrix are consistent and acceptable. If they are not, we should go back and perform the pairwise comparison again. Although AHP involves a larger number of calculations, it is preferred to MFEP in cases in which you do not feel confident or comfortable in determining factor weights or factor evaluations without making pairwise comparisons.

**IN ACTION Using AHP in a Pilot Study of Organ Transplantation**

Getting organs for transplantation and deciding who gets scarce organs has been a topic of discussion for decades. There have been cases of celebrities, retired athletes, and movie stars receiving organs ahead of perhaps more deserving people. Some believe the decision about who gets organs can be political instead of medical. There have also been charges that some countries harvest organs from living healthy prisoners.

At the Hospital for Sick Children in Toronto, AHP has been used in a pilot study to help determine who should get organs. The goal of the pilot study was to develop a set of consistent and broadly acceptable criteria. Developing good criteria was difficult. For example, what priority should a child with Down syndrome be given to receive an organ transplant? Using pairwise comparisons, a set of criteria for children receiving organ transplants was evaluated using the AHP framework. The criteria included intelligence, survival expectations, physical dependence on others, the need for long-term financial support, the need for long-term health support, parent activities required, the ability of the child to return to a full schedule of school activities, and other similar factors.

The results of the AHP study differed from standard surveys conducted in Canada and the United States. The AHP study, for example, determined that such factors as the ability to pay, the presence of medical insurance, or a patient’s financial or economic status should not be considered in making a transplant decision. This may have been a result of Canada’s national health care system, which assures health care for all Canadian citizens. It was also determined that physical limitations, such as being disabled, should not be a determining factor for an organ transplant. A low intelligence, such as an IQ of 70 or lower, was also not as important in the AHP study as it had been in earlier surveys. The AHP study determined that the most important criteria was the organ transplant patient’s ability to survive the difficult transplant process, accept the difficult transition process following organ transplant, and lead a relatively normal life after the organ transplant. Overall, the study was able to take into account ethical, qualitative, and quantitative factors to determine who should receive organ transplants.

Summary

Multifactor decision making is appropriate when an individual, a group, or an organization faces a number of factors in a decision-making situation. With the MFEP, a decision maker assigns an importance weight to each factor. The weights can, for example, range from 0 to 1. Then, for each alternative, all factors are evaluated. The factor weights are multiplied by each factor evaluation for a given alternative and summed. The alternative with the highest overall score is selected.

With the AHP, the decision maker performs a number of pairwise comparisons between each pair of alternatives for each factor to determine the factor evaluations. A pairwise comparison is also performed between each pair of factors to determine the factor weights. This information is used to determine a total weighted evaluation for each alternative. The alternative with the highest total weighted evaluation is selected. The AHP approach also allows for the computation of a consistency ratio to help decision makers determine if their pairwise comparisons are consistent.

Glossary

Analytic Hierarchy Process (AHP) A process that uses pairwise comparisons to determine factor evaluations and factor weights in a multifactor decision-making environment.

Factor Evaluations Evaluations that indicate one’s preference for a particular factor for a particular alternative or item.

Factor Weights Weights that give the relative importance of one factor to another.

Multifactor Decision Making A decision-making environment in which multiple factors are to be considered in making the final selection.

Multifactor Evaluation Process (MFEP) A multifactor decision-making approach in which the factor weights and factor evaluations can be accurately determined and used in the decision-making process.

Key Equations

\[(M1-1) \quad CI = \frac{\lambda - n}{n - 1} \]
Consistency index.

\[(M1-2) \quad CR = \frac{CI}{RI} \]
Consistency ratio.

Solved Problems

Solved Problem M1-1

Tom Schmid is thinking about buying a Nordic ski machine. The three factors important to him are price, ease of use, and the ability to store the exercise equipment in a closet when he is done using it. Given the following data, help Tom determine the best machine for him:

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>IMPORTANCE WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>0.9</td>
</tr>
<tr>
<td>Ease of use</td>
<td>0.75</td>
</tr>
<tr>
<td>Storage</td>
<td>0.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>PROFESSIONAL NORDIC SKIER</th>
<th>ECONO NORDIC SKIER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Ease of use</td>
<td>0.95</td>
<td>0.6</td>
</tr>
<tr>
<td>Storage</td>
<td>0.9</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Solution

Given these data, we can multiply the weights by the evaluations for each skier and then sum the results. The results are shown in the following table:

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>PROFESSIONAL NORDIC SKIER</th>
<th>ECONO NORDIC SKIER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>(0.5)(0.9) = 0.45</td>
<td>(0.8)(0.9) = 0.72</td>
</tr>
<tr>
<td>Ease of use</td>
<td>(0.95)(0.75) = 0.7125</td>
<td>(0.6)(0.75) = 0.45</td>
</tr>
<tr>
<td>Storage</td>
<td>(0.9)(0.6) = 0.54</td>
<td>(0.7)(0.6) = 0.42</td>
</tr>
<tr>
<td>Total</td>
<td>1.70</td>
<td>1.59</td>
</tr>
</tbody>
</table>

Given this analysis, Tom should select the Professional Nordic Skier.

Solved Problem M1-2

Gretchen Little has used AHP to determine factor evaluations. The consistency index for her problem is 0.0988. The number of factors in her problem is four. Can you draw any conclusions from these data?

Solution

Using a value of 4 for \( n \), we look in the table in this module to get the random index (RI). From the table with a value of 4 for \( n \), we see that RI is 0.90. From this information we can compute the consistency ratio as follows:

\[
CR = \frac{CI}{RI} = \frac{0.0988}{0.9} = 0.10978
\]

Because CR is close to but greater than 0.10, her pairwise comparisons may not have been consistent. It is recommended that she re-solve the problem carefully and recompute the consistency ratio.
Self-Test

1. In the MFEP
   a. the factor weights must sum to 1.
   b. the factor evaluations must sum to 1.
   c. the factor weights must sum to 10.
   d. the factor evaluations must sum to 10.

2. In the AHP, the pairwise comparisons use a scale that ranges from equally preferred to extremely preferred. The numerical range used for these comparisons are
   a. 0 to 10.
   b. 1 to 10.
   c. 0 to 9.
   d. 1 to 9.

3. In the pairwise comparison matrix used in the AHP, each position on the diagonal from the upper left to the lower right of the matrix will be assigned a value of
   a. 0.
   b. 1.
   c. \( \frac{1}{2} \).
   d. 9.

4. In a normalized matrix, the sum of the numbers
   a. in each row must be 1.
   b. in each column must be 1.
   c. in each column must equal the consistency index.
   d. in each row must equal the consistency index.

5. The priorities in the AHP are found by averaging the numbers in
   a. the rows in the pairwise comparison matrix.
   b. the columns in the pairwise comparison matrix.
   c. the rows of the normalized matrix.
   d. the columns of the normalized matrix.

6. A decision maker’s decisions are relatively consistent if the consistency ratio is
   a. greater than or equal to 0.1.
   b. less than or equal to 0.1.
   c. greater than or equal to 1.0.
   d. less than or equal to 1.0.

7. If the consistency ratio indicates that a set of decisions is inconsistent, the decision maker should
   a. normalize the pairwise comparison matrix again.
   b. develop a new pairwise comparison matrix.
   c. eliminate some of the alternatives.
   d. eliminate some of the factors.

8. Pairwise comparisons are made with
   a. the MFEP.
   b. the AHP.
   c. both the MFEP and the AHP.

Discussion Questions and Problems

Discussion Questions

M1-1 Describe decision situations in which multifactor decision making is appropriate. What decision-making situations do you face that could benefit from the multifactor decision-making approach?

M1-2 Briefly describe the MFEP.

M1-3 When should the AHP be used compared with the MFEP?

Problems

M1-4 George Lyon is about to buy a compact stereo cassette player. He is currently considering three brands—Sun, Hitek, and Surgo. The important factors to George are the price, color, warranty, size of the unit, and brand name. George has determined factor weights of 0.4, 0.1, 0.1, 0.1, and 0.3, respectively. Furthermore, George has determined factor evaluations for all of the factors for the three different manufacturers of the unit he is considering. The Sun unit has factor evaluations of 0.7, 0.9, 0.8, 0.8, and 0.9 for the price, color, warranty, size, and brand-name factors. The Hitek unit has factor evaluations of 0.6, 0.9, 0.9, 0.8, and 0.9 for these factors. Finally, Surgo has factor evaluations of 0.4, 0.4, 0.2, and 0.6 for the same factors of price, color, warranty, size, and brand name. Determine the total weighted evaluation for the three manufacturers. Which one should George select?
Linda Frieden (Problem M1-5) is also concerned about the warranty for the three cars she is considering. The second car is moderately preferred to car 1 in terms of warranty, but car 1 is moderately preferred to car 3 in terms of warranty. Furthermore, car 2 is very to extremely strongly preferred over car 3. Determine the factor evaluations for style concerning the three cars and compute the necessary ratio.

Linda Frieden (Problems M1-5 to M1-7) now must determine the relative weights for the three factors of price, warranty, and style. She believes that the price is equally to moderately preferred over warranty and that price is extremely preferred to style. She also believes that the car warranty is strongly to very strongly preferred over the style. Using this information, determine the weights for these three factors. Also determine the consistency ratio to make sure that the values are consistent enough to use in the analysis. In Problems M1-5 to M1-7, Linda has determined factor evaluations for price, warranty, and style for the three cars. Using the information you determined in this problem along with the solutions to the three preceding problems, determine the final rankings for each car. Which car should be selected?

Gina Fox is a student who will be graduating soon, and she is planning to attend graduate school to work toward an MBA. Gina has been accepted into the graduate programs at three universities. Now she must decide which one to attend. Gina has rated each one on the cost, reputation of the program, and quality of life at the university. These ratings are summarized as follows (1 is a poor rating and 10 is perfect):

<table>
<thead>
<tr>
<th>UNIVERSITY</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>4</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Reputation</td>
<td>9</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Quality of life</td>
<td>7</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

Linda Frieden (Problems M1-5 and M1-6) would like to consider style as an important factor in making a decision to purchase a new car. Car 2 is moderately preferred to car 1 in terms of style, but car 1 is moderately preferred to car 3 in terms of style. Furthermore, car 2 is very to extremely strongly preferred over car 3. Determine the factor evaluations or priorities for the three cars for car warranty. Compute the consistency ratio.

Linda Frieden (Problems M1-5 to M1-7) now must determine the relative weights for the three factors of price, warranty, and style. She believes that the price is equally to moderately preferred over warranty and that price is extremely preferred to style. She also believes that the car warranty is strongly to very strongly preferred over the style. Using this information, determine the weights for these three factors. Also determine the consistency ratio to make sure that the values are consistent enough to use in the analysis. In Problems M1-5 to M1-7, Linda has determined factor evaluations for price, warranty, and style for the three cars. Using the information you determined in this problem along with the solutions to the three preceding problems, determine the final rankings for each car. Which car should be selected?

Upon reevaluating the situation, Gina Fox (see Problem M1-9) is not comfortable with her ratings. Therefore, she has decided to compare the universities two at a time. On cost, B is strongly preferred to A; B is moderately preferred to C, and C is moderately preferred to A. On reputation, A is very strongly preferred to B; C is moderately preferred to B; and A is strongly preferred to C. On quality of life, A and B are equally preferred; A is strongly preferred to C; and B is very strongly preferred to C. On the three factors, cost is very strongly preferred to quality of life; cost is moderately preferred to reputation; and reputation is equally to moderately preferred to quality of life.

Gina has decided that cost is the overriding factor. She has given cost a weight of 0.6, reputation a weight of 0.2, and quality of life a weight of 0.2. Which university should Gina select?

Gina select?

Jim Locke, an undergraduate student in the ESU College of Business, is trying to decide which microcomputer to purchase with the money his parents gave him for Christmas. He has reduced the number of computers he has been considering to three, call them system 1 (S1), system 2 (S2), and system 3 (S3). For each computer, he would like to consider the price, the brand name, the memory capacity, speed, flexibility, and compatibility with IBM PCs.

To make the correct decision, he has decided to make pairwise comparisons for all the factors. For price, the first computer system is equally to moderately preferred over the second computer system and very to extremely strongly preferred over the third computer system. The second computer system is strongly preferred over the third computer system. The second computer system is strongly preferred over the third computer system. For brand name, the first computer system is equally preferred to the second computer system, and the first computer system is strongly to very strongly preferred over the third computer system. The second computer system is moderately to strongly preferred over the third computer system.

When it comes to memory, the second computer system is equally to moderately preferred over the first computer system, and the third computer system is very strongly preferred over the first computer system. Furthermore, the third computer system is strongly to very strongly preferred over the second computer system. For speed, the second computer system is moderately preferred to the first computer system, but the first computer system is equally to moderately preferred over the third computer system. Furthermore, the second computer system is strongly preferred over the third computer system.

For the flexibility factor, the third computer system is very to extremely strongly preferred over
the first computer system, and the second computer system is equally to moderately preferred over the first computer system. The third computer system is also moderately to strongly preferred over the second computer system.

Finally, Jim has used pairwise comparisons to look at how compatible each computer system is with the IBM PC. Using this analysis, he has determined that the first computer system is very to extremely strongly preferred over the second computer system when it comes to compatibility. The first computer system is moderately to strongly preferred over the third computer system, and the third computer system is moderately preferred over the second computer system.

When it comes to comparing the factors, Jim has used pairwise comparisons to look at price, brand name, memory, speed, flexibility, and compatibility. Here are the results of the analysis. Price is extremely preferred to brand name, moderately to strongly preferred to memory, strongly preferred to speed, moderately preferred to flexibility, and equally to moderately preferred to PC compatibility. In other words, price is a very important factor. The computer’s memory is equally to moderately preferred to brand name, speed is equally preferred to brand name, flexibility is moderately to strongly preferred to brand name, and PC compatibility is strongly preferred to brand name. In looking at memory, Jim has determined that memory is equally to moderately preferred to speed. PC compatibility, however, is strongly to very strongly preferred to memory, and overall flexibility is equally to moderately preferred to the computer’s memory. PC compatibility is strongly to very strongly preferred to speed, and flexibility is moderately preferred to speed. Finally, Jim has determined that PC compatibility is equally to moderately preferred to flexibility.

Using all of these preferences for pairwise comparisons, determine the priorities or factor evaluations, along with the appropriate consistency ratios for price, brand name, memory, speed, flexibility, and PC compatibility for the three different computer systems. In addition, determine the overall weights for each of the factors. Which computer system should be selected?

Bibliography


Appendix M1.1: Using Excel for the Analytic Hierarchy Process

Excel can be used to perform the calculations in the AHP. Program M1.1A (columns A–I) and Program M1.1B (columns J–P) give the formulas that are used to develop the normalized matrices, the weighted sum vectors, the consistency indices, and the consistency ratios for the example in Section M1.3. The only inputs required in this example are the number of alternatives (cell D1), the upper right halves of the pairwise comparison matrices, and the table for the random index (Cells A26 to B33). When using the MMULT command (cells J27–J29 and four times in column L), highlight the cells and type this function. Then press the CTRL-SHIFT-ENTER keys simultaneously to enter the function into all of these cells. Program M1.2 gives the output for this example.
APPENDIX M1.1: USING EXCEL FOR THE ANALYTIC HIERARCHY PROCESS

PROGRAM M1.1A  Partial Excel Spreadsheet Formulas for AHP

Enter the upper right-hand half of each comparison matrix.

The normalized matrices are in columns G–I.

This matrix gives the priorities for each factor. These were computed in column J.

PROGRAM M1.1B  Additional Excel Spreadsheet Formulas for AHP

The keys Ctrl, Shift, and Enter are simultaneously pressed to enter the MMULT function.

The factor weights (J21–J23) are multiplied by the individual factor evaluations to give the overall evaluations.

The LOOKUP command is used to find the random index (RI).
There are three factors and three systems in this example.

The normalized matrices are used to obtain the priorities for each individual factor.

System 3 has the highest overall rating.