Dating Stonehenge

Approximately eight miles north of Salisbury, Wiltshire, England, stands a large circular stone monument surrounded by an earthwork. This prehistoric structure is known throughout the world as Stonehenge. Its name is derived from the old English word *hengen*, referring to something hung up. In the case of the monument, this name refers to the large horizontal lintel stones. The monument consists of an outer ring of Sarsen stones, surrounding two inner circles of bluestones. The first and third circles are adorned with the familiar stone lintels. The Sarsen stones show signs of having been carefully shaped, suggesting a Mycenaean Greece and Minoan Crete influence. The entire structure is surrounded by a ditch and bank. Just inside the bank are 56 pits, named the Aubrey Holes, after their discoverer. These holes appear to have been filled shortly after their excavation.

Stonehenge is a mysterious, magnificent monument, which popularly has been associated with the Druids. However, there is no direct evidence supporting this connection. Recently, it has been discovered that a number of the stone alignments are associated with important solar and lunar risings and settings, suggesting that the site served as some sort of massive astronomical calendar. If this conclusion is accurate, then it seems likely that the monument might have been used as a temple for sky worshipers, although the exact nature of their religion is unknown.

Corinn Dillion is interested in dating the construction of the structure. Excavations at the site uncovered a number of unshed antlers, antler tines, and animal bones. Carbon-14 dating methods were used to estimate the ages of the Stonehenge artifacts. Carbon-14 is one of three carbon isotopes found in the Earth’s atmosphere. Carbon-12 makes up 99% of all the carbon dioxide in the air. Virtually all of the remaining one percent is composed of Carbon-13. By far, the rarest form of carbon isotope found in the atmosphere is Carbon-14. The ratio of Carbon-14 to Carbon-12 remains constant in living organisms. However, once the organism dies, the amount of Carbon-14 in the remains of the organism begins to decline, because it is radioactive, with half-life 5730 years (the “Cambridge half-life”). So, the decay of Carbon-14 into ordinary nitrogen makes possible a reliable estimate about the time of death of the organism. The counted Carbon-14 decay events are known to be normally distributed.

Dillon’s team used two different Carbon-14 dating methods to arrive at age estimates for the numerous Stonehenge artifacts. The Liquid Scintillation Counting (LSC) method utilizes benzene, acetylene, ethanol, methanol, or a similar chemical. Unlike the LSC method, the Accelerator Mass Spectrometry (AMS) technique offers direct Carbon-14 isotope counting. The AMS method’s greatest advantage is that it requires only milligram-sized samples for testing. The AMS method was used only on recovered artifacts that were of extremely small size.

Stonehenge’s main ditch was dug in a series of segments. Excavations at the base of the ditch uncovered a number of antlers, which bore signs of heavy use. These antlers could have been used by the builders as picks or rakes. The fact that no primary silt was discovered beneath the antlers suggests that they were buried in the ditch shortly after its completion. Another researcher, Phillip Corbin, using an archaeological markings approach, had previously claimed that the mean date for the construction of the ditch was 2950 B.C. A sample of nine age estimates from unshed antlers excavated from the ditch...
produced a mean of 3033.1 B.C., with standard deviation 66.9 years. Assume that the ages are normally distributed with no obvious outliers. At an $\alpha = 0.05$ significance level, is there any reason to dispute Corbin’s claim?

Four animal bone samples were discovered in the ditch terminals. These bones bore signs of attempts at artificial preservation and might have been in use for a substantial period of time prior to their being placed at Stonehenge. When dated, these bones had mean age 3187.5 B.C. and standard deviation 67.4 years. Assume that the ages are normally distributed with no obvious outliers. Use an $\alpha = 0.05$ significance level to test the claim that the population mean age of the site is different from 2950 B.C.

In the center of the monument are two concentric circles of igneous rock pillars, called bluestones. The construction of these circles was never completed. These circles are known as the Bluestone Circle and the Blue- stone Horseshoe. The stones in these two formations were transported to the site from the Prescelly Mountains in Pembrokeshire, southwest Wales. Excavation at the center of the monument revealed an antler, an antler tine, and an animal bone. Each of these artifacts was submitted for dating. It was determined that this sample of three artifacts had mean age 2193.3 B.C., with a standard deviation of 104.1 years. Assume that the ages are normally distributed with no obvious outliers. Use an $\alpha = 0.05$ significance level to test the claim that the population mean age of the Bluestone formations is different from Corbin’s declared mean age of the ditch, that is, 2950 B.C.

Finally, three additional antler samples were uncovered at the Y and Z holes. These holes are part of a formation of concentric circles 11 meters and 3.7 meters, respectively, outside of the Sarsen Circle. The sample mean age of these antlers is 1671.7 B.C. with a standard deviation of 99.7 years. Assume that the ages are normally distributed with no obvious outliers. Use an $\alpha = 0.05$ significance level to test the claim that the population mean age of the Y and Z holes is different from Corbin’s stated mean age of the ditch—that is, 2950 B.C.

From your analysis, does it appear that the mean ages of the artifacts for the ditch, the ditch terminals, the Bluestones, and the Y and Z holes dated by Dillion are consistent with Corbin’s claimed mean age of 2950 B.C. for the construction of the ditch? Can you use the results from your hypothesis tests to infer the likely construction order of the various Stonehenge structures? Explain.

Using Dillion’s data, construct a 95% confidence interval for the population mean ages of the various sites. Do these confidence intervals support Corbin’s claim? Can you use these confidence intervals to infer the likely construction order of the various Stonehenge structures? Explain.

Which statistical technique, hypothesis testing or confidence intervals, is more useful in assessing the age and likely construction order of the Stonehenge structures? Explain.

Discuss the limitations and assumptions of your analysis. Is there any additional information that you would like to have before publishing your findings? Would another statistical procedure be more useful in analyzing these data? If so, which one? Explain. Write a report to Corinn Dillion detailing your analysis.

Source: This fictional account is based upon information obtained from Archaeometry and Stonehenge (http://www.eng.h.gov.uk/stoneh). The means and standard deviations used throughout this case study were constructed by calculating the statistics from the midpoint of the calibrated date range supplied for each artifact.
What Does It Really Weigh?

Many of the consumer products that we purchase have labels that describe the net weight of the contents. For example, the net weight of a candy bar might be listed as 4 ounces. Choose any consumer product that reports the net weight of the contents on the packaging.

(a) Obtain a random sample of size 8 or more of the consumer product. We will treat the random purchases as a simple random sample. Weigh the contents.

(b) If your sample size is less than 30, verify that the population from which the sampling was drawn is normal and that the sample does not contain any outliers.

(c) As the consumer, you are concerned only with situations in which you are getting “ripped off.” Determine the null and alternative hypotheses from the point of view of the consumer.

(d) Test the claim that the consumer is getting “ripped off” at the $\alpha = 0.05$ level of significance. Are you getting “ripped off”? What makes you say so?

(e) Suppose you are the quality-control manager. How would you structure the alternative hypothesis? Test this claim at the $\alpha = 0.05$ level of significance. Is there anything wrong with the manufacturing process? What makes you say so?
Eyeglasses are part medical device and part fashion statement, a marriage that has always made them a tough buy. Aside from the thousands of different frames the consumer has to choose from, there are various lens materials and coatings that can add to the durability, and the cost, of a pair of eyeglasses. One manufacturer even goes so far as to claim that its lenses are “the most scratch-resistant plastic lenses ever made.” With a claim like that, we had to test the lenses (June 2001).

One test involved tumbling the lenses in a drum containing scrub pads of grit of varying size and hardness. Afterward, readings of the lenses’ haze were taken on a spectrometer to determine how scratched they had become. To evaluate their scratch resistance, we measured the difference between the haze reading before and after tumbling.

The graphic illustrates the difference between an uncoated lens (on the left) and the manufacturer’s “scratch-resistant” lens (on the right).

The following table contains the haze measurements both before and after the scratch resistance test for this manufacturer. Haze difference is measured by subtracting the before score from the after score; in other words haze difference is computed as “After”−“Before”.

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
<th>Difference</th>
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</thead>
<tbody>
<tr>
<td>0.18</td>
<td>0.72</td>
<td>0.54</td>
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<tr>
<td>0.16</td>
<td>0.85</td>
<td>0.69</td>
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<tr>
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<td>0.55</td>
</tr>
<tr>
<td>0.21</td>
<td>0.51</td>
<td>0.30</td>
</tr>
</tbody>
</table>

(a) Suppose it is known that the closest competitor to the manufacturer’s lens has a mean haze difference of 1.0. Do the data support the manufacturer’s scratch resistance claim?

(b) Write the null and alternative hypotheses, letting $\mu_{\text{diff}}$ represent the mean haze difference for the manufacturer’s lens.

(c) We used Minitab (release 13.1) to perform a one-sample t-test. The results are shown below.

Using the Minitab output, answer the following questions:
1. What is the value of the test statistic?
2. What is the $P$-value of the test?
3. What is the conclusion of this test? Write a paragraph for the readers of Consumer Reports magazine that explains your findings.

Note to Readers: In many cases, our test protocol and analytical methods are more complicated than described in these examples. The data and discussions have been modified to make the material more appropriate for the audience.